With Friends Like These, Who Needs Enemies?

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Abstract

Why are political leaders often attacked by their ideological allies? The paper addresses this puzzle by presenting a model in which the conflict between the incumbent and his allies is ideological, dissent is electorally costly, and voters are learning about their own policy preferences over time. Here, by dissenting against the incumbent (and thereby harming the party in the upcoming election), the allies can change his incentives to choose more or less extreme policies, which affects the amount of voter learning. This induces a trade-off between winning the current election and inducing the party leadership to pursue the allies' all-things-considered more-preferred policy. Optimally balancing this trade-off sometimes involves active dissent that damages the party in the short-run. In equilibrium dissent arises precisely because it is electorally costly.

Introduction

'Renzi is not apt for his role, he does not have the stature of a leader' (Cuperlo 2016). 'He says a lot of things, that do not always coincide with the truth' (Bersani 2015). These are public statements made by prominent Italian politicians about Matteo Renzi, former prime minister and leader of the Democratic Party (PD), the biggest Left-wing party in the country. And these are not isolated examples: Renzi was often publicly accused of being a liar, incompetent and even 'worse than the devil' (D'Alema 2016). Quite surprisingly, the authors of these public attacks were not members of the opposition. Renzi's worst critics were in fact all members of his own party: the leaders of the so called "Minoranza Dem", the extreme minority faction within the PD.

Similar phenomena have emerged in other European countries as well as in the US. In the UK, the Labour Party is currently undergoing a 'civil war' (Jones 2016). The members of the party's minority often openly denigrate the leader Jeremy Corbyn whom, they argue, 'literally has no idea (...) how to conduct himself as a leader' (Mandelson 2017). In France, a group of rebel Socialist MPs (the Frondeurs) regularly manifested their dissent against President and party leader Francois Holland. Similarly, divisions within the US Republican party are apparent. Prominent Republican lawmakers have publicly attacked President Trump, arguing that 'he shows a growing inability, and even unwillingness, to separate truth from lies' (McCain 2017).

Interestingly, this phenomenon is not solely an intra-party issue. Media outlets often denigrate political leaders with whom they are ideologically aligned. The right-leaning Evening Standard has openly attacked UK Conservative prime minister Theresa May, depicting her cabinet as 'stale' and 'enfeebled' (Urwin 2017). Similarly The Guardian, historically left-leaning, has described Labour leader Corbyn as 'dismal, lifeless, spineless' (Toynbee 2016).

These examples show that political leaders are often publicly attacked and denigrated by their own ideological allies. Yet, evidence indicates that this form of dissent typically damages a party's electoral chances, since voters dislike parties that appear divided (Greene and Haber 2016; YouGov 2016; Kam 2009; Groeling 2010). As such, public dissent hurts both the leader and the dissenters themselves. This raises the question: why would a leader's ideological allies choose to publicly attack him despite this being electorally harmful?

In this paper, I argue that this form of public dissent emerges precisely because it is electorally costly, in order to induce a policy response. Dissent hurts the leader's chances of winning re-election. This, in turn, changes his incentives to take policy gambles. As such, when gambles take the form of more extreme policies (as they will do *endogenously* in my framework), the allies face a potential trade-off between maximizing the probability that the leader wins the upcoming elections and inducing him to adopt a policy more in line with their own ideological preferences. If the gain from changing today's equilibrium policy is sufficiently large, public dissent emerges in equilibrium.

Focusing on dissent against an incumbent, I micro-found this argument by presenting a model with four key ingredients. First, the incumbent and his allies come from the same side of the political spectrum, but do not have exactly the same policy preferences. The allies can represent a minority faction within the party, a media outlet, an external donor or even a special interest group: any actor whose policy preferences are closer to the incumbent's than the challenger's. Second, dissent is electorally costly: it generates a negative valence shock that potentially damages the party's electoral prospect. Dissent can entail publicly criticizing the party leader, dispraising his policy choices, revealing a scandal or even 'mechanically' reducing his electoral chances (for example, a donor may choose to reduce its electoral contributions). Third, the model assumes that voters face uncertainty about their ideal policy. For example, voters may not know which policy is most likely to produce their desired outcome. Finally, a crucial feature of the model is that voters can reduce their uncertainty by learning through experience. In particular, I propose a new framework to think about policy experimentation. Voters learn about the optimal decision for the future by observing how much they like the outcome of today's policy. The presence of a random shock complicates their inference problem. Within this framework, I show, the amount of voter learning depends on the location of the implemented policy along the left-right spectrum. The more extreme the policy is, the more the voters learn about their ideal platform. Suppose that an extreme policy is implemented. If a voter obtains a high (low) payoff from the resulting outcome, the policy is likely (unlikely) to be in line with her true preferences. Conversely, because of the presence of the random shock, the outcome of a moderate policy is much less informative.

In this setting, the incumbent has incentives to engage in information control. His equilibrium policy choice maximizes the trade-off between implementing his bliss point today and generating the optimal amount of information in order to be re-elected tomorrow. This, I show, is a function of the incumbent's ex-ante electoral strength. A leading incumbent, who is going to be re-elected even if the voters receive no new information, has incentives to implement moderate platforms that prevent information generation. In contrast, a trailing one will want to engage in extreme policies that increase the amount of voter learning, in hopes of improving his electoral prospects. Finally, an incumbent who can never be re-elected (irrespective of what the voters learn) is indifferent with respect to the amount of information that is generated, and will simply follow his ideological preferences.

Within this framework, dissent may allow the allies to solve the ideological conflict with the incumbent. By dissenting, the allies generate a negative valence shock against the incumbent, thereby reducing his ex-ante electoral strength. This, in turn, creates incentives to implement more or less informative (i.e. extreme) policies. As such, dissent changes the incumbent's equilibrium policy choice, while also harming the party electorally. This generates a potential trade-off for the incumbent's allies, between ensuring that their preferred party wins the upcoming election and inducing the incumbent to implement a policy more in line with their own ideological preferences. Optimally balancing this trade-off sometimes involves active dissent that damages the party in the short run. Thus, dissent emerges precisely because it is electorally costly, and it produces unity of interests between the incumbent and his allies even if no player actually cares about unity per se. Surprisingly, the analysis reveals that improving the incumbent's electoral prospects or reducing his ideological conflict with the allies may make dissent more likely to emerge.

Further, the results highlight that the presence of an extreme ally to the incumbent party may be welfare improving for the voters. In the model, voters benefit from informative policies being implemented as this increases the probability of making the correct electoral decision in the future. However, under some conditions, electoral accountability has the perverse consequence of inducing lower levels of policy experimentation relative to both the incumbent's ideological preferences and the voter's optimum. The incumbent's extreme ally may mitigate such inefficiency. By dissenting,

the ally can create incentives for the incumbent to implement extreme policies that allow the voters to learn. If the value of acquiring new information is sufficiently large, this strictly increases the voters' welfare.

The results of the model also have an important implication for empirical research on the topic. Existing estimates of the electoral rewards of party unity, that are obtained by comparing treated and control units (i.e. parties that do and do not experience dissent), are inevitably biased. In addition, it is hard to know ex-ante what the direction of the bias will be. However, this does not imply that the model is not falsifiable. Indeed, the theory suggests where else to look in order to empirically investigate the electoral consequences of dissent. The model generates testable comparative statics regarding parties' electoral performance conditional on experiencing dissent. Focusing on this restricted sample, and thereby avoiding the problem of selection bias described above, researchers can empirically investigate the conditions under which dissent is expected to hurt parties the most. The theory predicts that parties' performance conditional on dissent should be positively correlated with variables such as the level of education, news media consumption and political engagement in the electorate. Finally, I discuss how the model's comparative static predictions may allow us to distinguish it from other possible explanations for the emergence of dissent.

Related Literature

This paper relates first and foremost to the literature on intra-party politics. In the formal literature, the interaction between different factions is typically analysed as a bargaining game. Mutlu-Eren (2015) considers how the threat of a split influences the party's behaviour in the legislature. Similarly, Hortala-Vallve and Mueller (2015) consider a model in which the threat of defection by the minority can induce the party leadership to democratize the candidates' selection process. In these papers, the threat is credible when the faction is sufficiently likely to win the upcoming election if running alone after a split. Turning to the empirical literature, we find similar references to the competing factions bargaining over a prize. In Parties and Party Systems, Sartori (1976) describes factions as blackmailing the leadership, and seeking side payments. Belloni (1976)

and Boucek (2009) express similar ideas. More recently Budge et al. (2010) explain parties' policy shifts away from the center as a result of the minority faction vetoing a moderation.

Yet this approach has some issues when we consider dissent as *public attacks* against the leader, rather than dissent as formal defection. In a bargaining game dissent would be used as a threat, to be executed *after* the incumbent has made his policy choice. However, at this point dissent has no effect but to reduce the probability that the party wins the upcoming election. This strictly decreases both the incumbent's and the ally's expected payoff. As such, the threat can never be credible and we should never observe dissent in equilibrium. Further, even beyond the issue of credibility, in a bargaining game the materialization of the threat typically lies off the equilibrium path. Hence, this is arguably not an appropriate framework to understand why political parties so often experience open dissent.

A second strand of literature considers politicians' incentives to pander to their own individual constituencies (e.g., Carey and Shugart 1995 and Buisseret and Prato 2018). Politicians may face a trade-off between the national party's electoral fortunes and their own success. In particular, this trade-off may emerge if a politician's local constituency is opposed to the national party line. Within this framework, dissent may serve the purpose of signaling the politician's misalignment with the leadership (and thus alignment with the constituency's preferences).

This argument is certainly intuitive (and potentially applies to formal defections as well as public attacks against the leadership). However, both anecdotal and systematic evidence seem to suggest that it does not fully capture the rationale behind this phenomenon. In particular, according to this framework we should expect the individual dissenters to be in a relatively weak electoral position (i.e., not to come from safe constituencies). However, Proksch and Slapin (2015) analyze data from the UK and Germany and show that, if anything, the opposite holds. Public attacks against the leadership are (weakly) more likely to come from members of parliament elected with a larger margin. Further, the authors exploit the features of the German mixed-member proportional electoral system, and show that members elected with a party-list vote are as likely to dissent as those elected with a constituency vote. Similarly hard to reconcile with the 'pandering' argument is the emergence of dissent under a closed-list system such as the Italian one,

where the party leader controls the list composition and as such the dissenter's electoral fate. This calls for a theory that allows us to make sense of this phenomenon even when the party members' individual electoral motives do not provide incentives (or worse, provide disincentives) to attack the leadership.

Thus, this paper presents a substantially different type of model, in which dissent precedes rather than following the party leader's strategic choice (in contrast with a bargaining set-up), and emerges in order to induce a policy response (in contrast with the pandering set-up).

The core ingredient of the model is the voters' uncertainty over their optimal choice. Given the symmetric lack of information, such uncertainty may only be resolved via experience. This connects the paper with the research on learning and experimentation. The key intuition therein is that, when deciding which policy to implement today, politicians consider how the outcome will influence the voters' future beliefs. Most extant works assume that the voters must learn about the incumbent's type, i.e. his ability or competence. The incumbent chooses between a safe and a risky policy, with a success on the latter being conditional on the politician being a 'good type' (see for example Dewan and Hortala-Vallve 2016; Majumdar and Mukand 2004). Under the assumption of symmetric uncertainty, a risky policy is always a gamble. This paper differentiates itself from the extant literature by considering a setting in which the incumbent's incentives to gamble arise endogenously from his allies' strategic behaviour. Further, the voters must learn about their own policy preferences (i.e. the state of the world), and not about the office holder's competence.

In this perspective, the paper is closely related to recent work by Callander (2011). The author considers a world in which players face uncertainty about how policies map into outcomes: they know the slope of the mapping function (representing the state of the world), but try to fine-tune their predictions by learning about the exact realization of the variance. The nature of the uncertainty is reversed in this paper: the voters must learn the fundamental underlying state of the world. This generates the result that extreme policies, rather than small incremental changes as in Callander, produce more information. As such, this paper provides a new framework to think about policy experimentation. Additionally, Callander focuses on the statically optimal choice for

a decision maker. He thus chooses to abstract from dynamic considerations, by assuming either myopic players (Callander 2011) or exogenous retention probabilities (Callander and Hummel 2014). In contrast, the focus of this paper is precisely on the incumbent's dynamic incentives to control information, and on how these impact his policy choices and the conflict with his ideological allies.

Finally, the paper relates to the literature on Bayesian Persuasion, originated from the work of Austen-Smith (1998) and Kamenica and Gentzkow (2011). In my model, as in the Bayesian Persuasion framework, the incumbent can engage in information control by manipulating the receiver's posterior distribution. In the Bayesian Persuasion framework the mechanism through which this occurs is somewhat black-boxed. The assumption is that the persuader can choose any desired signal (i.e., realization space and conditional probability distributions) by designing a 'test' or a 'policy experiment'. The results of the test do not directly influence the players' payoffs, therefore the persuader must be able to credibly commit to truthfully revealing them to the public. The key innovation of my paper is therefore to explicitly model *how* the incumbent can engage in information control, by looking at the impact that the *implemented policy* has on voter learning.

The Model Set-Up

Dissent is analysed within the framework of a principal-agent model, under the assumption that the voters face uncertainty over their ideal policy (the state of the world) and learn by experience. I focus on dissent within the incumbent party. The players are therefore the incumbent (I), his ideological ally (A), a challenger (C), and a representative voter (V). The incumbent's ally can represent a minority faction within the party, media outlets ideological close to the incumbent, donors or even interest groups: any actor whose ideological preferences are closer to the incumbent's than the challenger's.

At the beginning of the game, the incumbent's ally chooses whether to dissent against him. The choice is binary: $D \in \{0, 1\}$. Dissent may entail publicly criticizing the incumbent's personality, or manifesting a disagreement with the party line. After observing his ally's choice, the incumbent implements a policy x_1 along the real line. The voter chooses whether to retain the incumbent

or replace him with his challenger. The second-period office-holder implements a new policy x_2 (under the assumption of no credible commitment).

The voter faces uncertainty over the exact value of her ideal policy x^v.[1] One way to interpret this assumption is that the voter does not know which policy is most likely to produce her preferred outcome. Thus, her uncertainty refers to the slope of the function mapping policies into outcomes. An alternative interpretation would consider a voter that knows how different policies map into outcomes, but can not perfectly anticipate how these outcomes will impact her payoffs. The voter's ideal policy can take one of two values: $x^v \in \{\underline{\alpha}, \bar{\alpha}\}$. For simplicity (but without loss of generality) I assume $\underline{\alpha} = -\bar{\alpha} < 0$. The qualitative results survive if $\underline{\alpha}$ and $\bar{\alpha}$ have the same sign, that is if the voter knows whether her ideal policy is a left-wing one or a right-wing one, but faces uncertainty over its exact location.

The model features no asymmetry of information: no player knows the true value of x^v, and all players assign the same prior probability γ to the voter's ideal policy being a right-wing one ($\gamma = \text{prob}(x^v = \bar{\alpha})$). Given this symmetric uncertainty, learning only happens via experience. The voter observes how much she liked (or disliked) the first period policy, and updates on the true value of x^v by using Bayes rule.[2] Formally, the voter's payoff realization is a noisy signal of the state of the world:

$$U_t^v = -(x^v - x_t)^2 + \epsilon_t - \mathbb{I}\delta \tag{1}$$

$$\epsilon_t \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$$

As I will discuss in more details below, the assumption that the random shock ϵ is uniformly distributed is not necessary for the results. The parameter δ captures the observation that,

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[1]While the model only considers a representative voter, the results do not require all voters to face such uncertainty. Indeed, some voters may be ideological and have well-defined policy preferences. The results presented below go through as long as the 'uncertain' voters are pivotal in determining the electoral outcome.

[2]Whether the incumbent, his allies, and the challenger also observe the voter's payoff realization is inconsequential for the equilibrium results.

everything else being equal, voters dislike parties that appear divided: if the incumbent experiences dissent in the first period, the voter's expected utility from re-electing him is reduced by δ ($\mathbb{I} = 1$ if $D = 1$ and the incumbent is re-elected and $\mathbb{I} = 0$ otherwise). In other words, I assume that dissent generates an endogenous valence shock against the party. In order to simplify the analysis and presentation of the results, I leave the cost of dissent black-boxed. I will discuss possible micro-foundations of this assumption in a separate section.

Finally, I A and C are policy motivated, and their bliss points are common knowledge:[3]

$$U_t^i = -(x^i - x_t)^2 \quad \forall i \in \{I, A, C\} \tag{2}$$

Without loss of generality, I will consider a right-wing incumbent and a left-wing challenger: $x^C \leq 0 \leq x^I$. For simplicity, I also assume that the candidates' bliss points are symmetric around 0: $x^I = -x^C \geq 0$. The incumbent and his ally come from the same side of the ideological spectrum (i.e. are both right-wing), but do not have exactly the same bliss point. However, the ally's preferences are always closer to the incumbent's than to the challenger's:

$$|x^A - x^I| < |x^A - x^C| \tag{3}$$

In the main body of the paper I will focus on the case of an extreme ally ($x^A > x^I$).
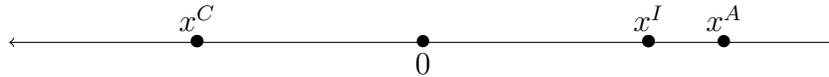
In the Appendix, I show that within this framework dissent can emerge even when the ally is more moderate than the incumbent, and identify the conditions under which this occurs in equilibrium.

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[3]For purposes of presentation (and to focus on learning via experience) the model assumes away any asymmetry of information. However, the key insights of the paper would (under some conditions) survive if we allow the incumbent to have private information about his own preferences or the voter's bliss point.

Timing

1. Nature determines the value of $x^v \in \{\underline{\alpha}, \bar{\alpha}\}$

2. The Incumbent's Ally chooses whether to dissent against him: $D \in \{0, 1\}$

3. The Incumbent implements a policy $x_1 \in \mathbb{R}$

4. The Voter's first-period payoffs realize

5. The Voter chooses whether to re-elect the Incumbent or replace him with the Challenger

6. The second-period office holder implements policy $x_2 \in \mathbb{R}$ (no credible commitment)

7. Second-period payoffs realize and game ends

The equilibrium concept is Perfect Bayesian Equilibrium. In order to avoid trivial results, I assume that when indifferent the incumbent's ally chooses not to dissent. This is formally equivalent to assuming an infinitely small material cost of dissenting.

In order to isolate the impact of ideological disagreements, I do not include office rents in the players' utility function. Whenever the incumbent and his ally do not attach the same value to winning office per se, office rents would in fact represent a second source of conflict. Suppose for example that the ally represents a minority faction within the party. Should the party win the upcoming election, the incumbent (i.e. the leader of the majority faction) would arguably grab a larger share of the office rents relative to his ally. This potentially translates into different risk appetite in policy making, thereby increasing the conflict of preferences between the incumbent and his allies. Hence, as long as the value of office is not too large, including office payoffs would make dissent even easier to sustain in equilibrium.

Equilibrium Analysis

As usual, we proceed by backwards induction, starting from the second period's office holder's choice. Politicians have no credible commitment ability. As such, given the absence of re-elections incentives, the second period office-holder will always implement his preferred platform. The voter

therefore faces a selection problem. Her electoral choice will then be determined by the (posterior) beliefs that her own ideal policy is aligned with the incumbent's preferred platform, as well as by the presence or absence of dissent within the incumbent party. Specifically, in any PBE of the game, the voter re-elects the right wing incumbent if and only if the posterior probability of being ideologically aligned with him ($\mu = prob(x^v = \bar{\alpha})$) is sufficiently high:

$$\mu > \frac{\mathbb{I}\delta + 4\bar{\alpha}x^I}{8\bar{\alpha}x^I} \tag{4}$$

The indifference breaking assumption is without loss of generality. Notice that, absent dissent, the incumbent is always re-elected as long as $\mu > \frac{1}{2}$. When the incumbent experiences dissent, the higher the cost δ, the higher the voter's posterior needs to be to guarantee re-election

Learning and Experimentation

Moving one step backwards, consider the voter's inference problem. The voter observes how much she liked or disliked the first period policy, and updates her beliefs on her ideal policy by using Bayes' rule. The analysis reveals a crucial feature of the learning process: the amount of information obtained by the voter depends on the location of the policy implemented in the first period. Specifically, the voter learns more from more extreme policies. As the implemented policy becomes more extreme, the distance in the expected outcomes as a function of the true state increases. As a consequence, each signal is more informative. In more substantive terms, if the voter likes (dislikes) the outcome of an extreme policy, such policy is likely (unlikely) to be in line with her true ideology. However, given the presence of the random shock, the outcome of a moderate policy is much less informative. This feature emerges in a very stark form in a world in which the shock is drawn from a uniform distribution.

Lemma 1: *The voter learning satisfies the following properties:*

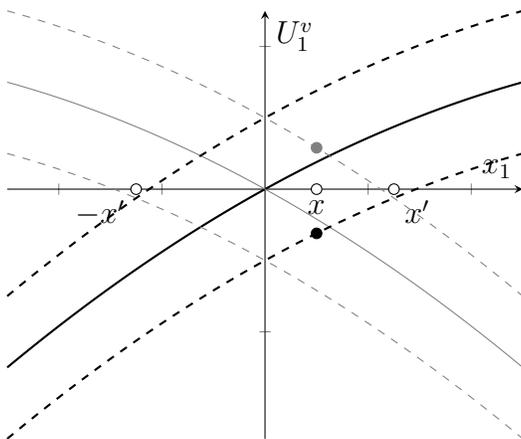*(i) Her posterior μ takes one of three values: $\mu \in \{0, \gamma, 1\}$;*

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Figure 1: Voter's payoff realization as a function of first-period policy. The thick increasing (thin decreasing) curves represent the case in which $x_V = \bar{\alpha}$ ($x_V = \underline{\alpha}$). The solid curves represent the voter's expected payoff $E[U_1^v]$, while the dashed ones represent $E[U_1^v] - \frac{1}{2\psi}$ and $E[U_1^v] + \frac{1}{2\psi}$

(ii) The more extreme the policy implemented in the first period x_1, the higher the probability that $\mu \neq \gamma$;

(iii) There exists a policy x' such that if $|x_1| \geq |x'|$, then $\mu \neq \gamma$ with probability 1.

Lemma 1 tells us that the voter either learns everything or nothing. Further, the probability that the voter discovers her true preferences increases as the implemented policy becomes more extreme. While a formal proof of this Lemma is presented in the Appendix A, the underlying reasoning is easy to illustrate graphically. In Figure 1, the solid lines represent the voter's expected period 1 payoff as a function of the implemented policy, for the two possible values of x^v. Thus, the thick increasing solid curve is $-(x_1 - \bar{\alpha})^2$ and the thin decreasing solid curve is $-(x_1 - \underline{\alpha})^2$. The dashed curves instead represent the maximum and minimum possible values of the payoff realization when we take the random shock into account. Thus, the thick increasing dashed curves (representing the state of the world in which $x^v = \bar{\alpha}$) are, respectively, $-(x_1 - \bar{\alpha})^2 + \frac{1}{2\psi}$ and $-(x_1 - \bar{\alpha})^2 - \frac{1}{2\psi}$.

The presence of the shock creates a partial overlap in the support of the payoff realization for a positive and negative state of the world: for any given policy $x_1 \in (-x', x')$, there exist values of the voter's payoff that may be observed whatever her true bliss point. Consider, for example, policy x as represented in the graph. Any payoff realization falling between the gray and black

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bullets may be observed with positive probability under both states of the world. Clearly, if the payoff realization falls outside this range of overlap, it constitutes a fully informative signal. There is only one state of the world that could have generated that specific realization: the voter simply likes the policy too much, or too little, for this to be justified as a consequence of the shock. Thus, upon observing her payoff, the voter learns the true state (i.e. discovers the true value of x_V). Conversely, any payoff realization that falls inside the range of overlap is completely uninformative. Due to the assumption that the shock is uniformly distributed, any such realization has exactly the same probability of being observed under the two states of the world. Thus, the voter learns nothing and must go back to her prior beliefs. As the implemented policy becomes more extreme, the gray and black bullets get closer and closer to each other. The range of overlap becomes smaller, and the voter is more likely to learn the true value of her ideal policy.

Let me emphasize that the results presented below are robust to alternative assumptions about the distribution of the shock, as long as extreme policies are more informative than moderate ones. Consider for example a world in which the shock is normally distributed with full support. The learning process would be much smoother: any outcome realization would be somewhat informative, but never fully so. However, it would still be the case that extreme policies generate more information. As the implemented policy becomes more extreme, the distance in the expected outcomes as a function of the state increases. This in turn increases each signal's informativeness. Generally speaking, the mechanism that I uncover in this paper relies on the fact that the policy choice influences the amount of information the voter receives. This is what allows a dissenting ally to influence the equilibrium policy. As such, the main insights of the paper would survive in a world in which more moderate (rather than more extreme) policies are more informative.

The Incumbent

The voter's posterior beliefs determine her electoral decision, as shown in Lemma 1. Since the amount of information the voter observes is a function of the implemented policy, the incumbent has an incentive to engage in information control. The incumbent cannot control exactly which signal the voter will observe, but he can determine the expected probability of such a signal being

informative. In other words, he cannot influence the voter's expected posterior (which is indeed always equal to the prior), but can influence its 'decomposition'. Hence, the first period equilibrium policy maximises the incumbent's trade-off between implementing his bliss point today and generating the optimal amount of information in order to get re-elected tomorrow. The way that this trade-off is optimised depends on the incumbent's ex-ante electoral strength. Define a *leading* incumbent as one who is guaranteed re-election if the voter receives no new information (condition (4) is satisfied at $\mu = \gamma$), and a *trailing* incumbent as one who will only be re-elected if the voter updates in his favour (condition (4) fails at $\mu = \gamma$ but is satisfied at $\mu = 1$). A *certain loser* is an incumbent who is replaced even if the voter updates in his favour (condition (4) fails to be satisfied at $\mu = 1$). The following Lemma holds:

Lemma 2: *In any PBE of the game*

- *A **certain loser** implements his bliss point*
  $(x_1^* = x^I)$

- *A **leading incumbent** implements a policy weakly more moderate than his bliss point*
  $(x_1^* \leq x^I)$

- *A **trailing incumbent** implements a policy weakly more extreme than his bliss point*
  $(x_1^* \geq x^I)$

For the incumbent, information revelation is risky. Even if $\gamma > \frac{1}{2}$, i.e. information is more likely to help him than hurt him, there is still a chance that the voter will instead learn that he own ideal policy is aligned with the challenger's (i.e. that $x^v = \underline{\alpha}$). A leading incumbent has no reason to accept the risk since he is guaranteed re-election when the voter does not update. Thus, he has incentives to prevent the voter from learning, and will always implement a policy that is (weakly) more moderate than his bliss point. Following Dewan and Hortala-Vallve (2017), I say that a leading incumbent experiences *fear of failure*. On the contrary, a trailing incumbent needs the voter to update (in his favour) in order to be re-elected. No matter how small the probability of success, a trailing incumbent always wants to engage in policy experimentation, so as to generate

as much information as possible and improve its electoral prospects. Borrowing terminology from the IR literature (Downs and Rocke 1994), I say that this incumbent has incentives to *gamble for resurrection*, and always implements a policy (weakly) more extreme than his bliss point.[4] A certain loser trivially has no reason to engage in information control, since he cannot change the electoral outcome. Hence, he will always implement exactly his bliss point.[5] The exact policies adopted by a leading and a trailing incumbent are calculated in the Appendix. Such policies are a function of the bliss point x^I, the prior γ, and the probability of learning for any given policy $(4\alpha\psi)$. The following Lemma defines the relationship between the equilibrium policy and the relevant parameters.

In Lemma 3 and the remainder of the paper I will be assuming that $x^I < x'$, where x' is the smallest (positive) policy that produces an informative signal with probability 1. The assumption is without loss of generality, and imposed in order to reduce the number of cases under consideration.

Lemma 3:

- *A trailing incumbent's equilibrium policy*

  1. *becomes (weakly) more extreme as his disadvantage decreases (γ increases)*

  2. *becomes (weakly) more extreme as his bliss point increases*

- *A leading incumbent's equilibrium policy*

  1. *becomes more extreme as his lead (γ) increases*

  2. *is always increasing in his bliss point when he enjoys a large lead ($\gamma > \frac{3}{4}$). When his lead is small ($\gamma < \frac{3}{4}$), the policy is non monotonic and concave in the bliss point*

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[4]Notice that the incumbent's behaviour is reminiscent of the results in Groseclose (2001), despite the two models considering very different settings. In both papers a leading incumbent moderates in order to maximise his electoral advantage, while a trailing one moves to the extreme in order to exploit the variance in the distribution - of expected outcomes in this paper, of voters' bliss points in Groseclose.

[5]The same would apply to an incumbent is always re-elected, for all values of μ and $\mathbb{I}\delta$. However, given the symmetry assumption, such a case never occurs.

The lower γ, the lower the probability that information will be in the incumbent's favor. As such, a leading incumbent's incentives to prevent information generation are stronger when γ is small, and a trailing incumbent's willingness to gamble is stronger when γ is large. Consider now the incumbent's bliss point. A trailing incumbent's policy choice is always increasing in his bliss point: as the incumbent becomes more extreme gambling becomes less costly and more valuable (since losing is more costly). Instead, a leading incumbent faces a trade off. As his bliss point becomes more extreme, preventing information generation becomes more costly today, but also more valuable for the future (as the challenger is further away and the payoff from winning increases). When the incumbent is too moderate, incentives to prevent information generation are weak since the gain from winning the next election is small. The direct effect dominates, and the equilibrium policy increases in the bliss point. Conversely, when the incumbent is too extreme, and the prior γ is sufficiently low, the electoral impact of the policy choice becomes dominant. As the incumbent's bliss point increases, winning the next election is more valuable, and the equilibrium policy becomes more moderate.

Dissent by an Extreme Ally

Moving one step back, we can now focus on the ally's decision whether to dissent against the incumbent. First of all, I establish that in equilibrium dissent is always harmful for the party's expected electoral performance, even if the incumbent best responds by modifying his policy choice precisely with the aim of minimizing this effect (as discussed in Lemma 3).

Lemma 4: *In equilibrium dissent always reduces the probability that the incumbent will be re-elected.*

Thus, by dissenting the ally reduces both his own and the incumbent's expected second period payoff. Nonetheless, dissent is sometimes observed in equilibrium. I show that, under some conditions, the ally faces a trade-off between maximizing the incumbent's electoral chances and inducing him to implement a policy more in line with his (i.e the ally's) own preferences.

The incumbent's ideological preferences are always more moderate than his ally's. However, by dissenting (and thereby harming the party in the upcoming election), the ally can induce him to implement a more extreme policy. Consider a leading incumbent. Absent dissent, he would always implement a policy (weakly) more moderate than his bliss point, in order to reduce the probability that the voter updates her beliefs about her true preferences. Suppose now that the incumbent's ally chooses to dissent against him. If the electoral cost is sufficiently large, this turns the leading incumbent into a trailing one. As Lemma 2 indicates, this creates incentives for the incumbent to gamble on resurrection: engage in extreme policies that increase the amount of voter learning. Thus, electorally costly dissent would move the incumbent's equilibrium policy choice to the extreme, closer to the ally's own preferences. When the gain is sufficiently large relative to the cost of losing the upcoming election, the ally chooses to dissent in equilibrium. Proposition 1 identifies necessary and sufficient conditions for this to occur (the proofs can be found in Appendix A). [6]

Proposition 1: *There exist* $\underline{\gamma}$, $\overline{\gamma}$, $\underline{x^A}$ *and* $\underline{x^I}$ *such that the incumbent's extreme ally chooses to dissent if and only if:*

- *Absent dissent, the incumbent is leading, but his advantage is not too large*

  $\underline{\gamma} < \gamma < \overline{\gamma}$, *where* $\underline{\gamma} \geq \frac{1}{2}$

- *The electoral cost of dissent is sufficiently high that it turns the leading incumbent into a trailing one, but not so high that the incumbent loses for sure*

  $(2\gamma - 1)4\bar{\alpha}x^I \leq \delta < 4\bar{\alpha}x^I$

- *Both the incumbent and his ally are sufficiently extreme*

  $x^I > \underline{x^I}$ *and* $x^A > \underline{x^A} > x^I$

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[6]In this paper I focus on dissent against an incumbent. However, the model can also be applied to explain the emergence of dissent within challenger parties. The challenger's ideological allies may want to openly attack him, thereby damaging the party in the upcoming election, so as to alter the incumbent's incentives to engage in information control. Within this framework, the challenger's allies use dissent to modify the incumbent's strategic choice, rather than to solve an ideological conflict within their own party. As such, dissent can emerge even absent any ideological disagreement, i.e. if the challenger and his allies have perfectly aligned preferences.

The thresholds in Proposition 1 are a function of the other parameters in the model.

Intuition may suggest that dissent is more likely to materialize during periods of electoral crisis. The party is expected to perform poorly, and the ensuing internal turmoil degenerates into an open manifestation of conflict. The first result shows that, in the case of an extreme ally, the opposite is true. Suppose the incumbent is trailing even without experiencing dissent. Absent dissent, he will implement a policy that is weakly more extreme than his bliss point: he needs to generate information in order to be re-elected. Dissent either has no impact on his policy choice (if δ is so small that it does not affect the voter's electoral decision), or induces him to implement exactly his bliss point (if δ is sufficiently large to turn him into a sure loser). Hence, by dissenting the ally causes the incumbent to adopt a (weakly) more moderate policy, while also (weakly) reducing his own future expected payoff. Then, dissent is never observed in equilibrium. It is only when the incumbent is leading (i.e. $\gamma > \frac{1}{2}$) that the ally (potentially) gains from dissent by creating incentives to gamble for resurrection.

The second set of results refers to the electoral cost of dissent (δ). Quite intuitively, dissent never emerges in equilibrium when its electoral cost is so large that it makes the incumbent lose for sure. In this scenario the expected loss would be maximized, while the gain for the extreme ally would be minimized. Recall that an incumbent who is a sure loser has no reason to control information, and will always implement exactly his bliss point. Thus, while dissent would be somewhat effective in modifying the equilibrium policy, it could not induce the incumbent to move beyond his bliss point. The policy gain would be too small for the incumbent's ally to be willing to pay the cost of losing the upcoming election for sure. However, perhaps more surprisingly, the analysis also reveals that for dissent to be observed in equilibrium its electoral cost (δ) cannot be too small either. Recall that an incumbent is leading if the voter would choose to re-elect him upon receiving no new information. If δ is too small (relative to the prior γ), then the incumbent is still leading even after experiencing dissent. In this case, dissent has no effect on the voter's electoral choice and therefore no impact on the equilibrium policy. Trivially, the incumbent's ally has no reason to dissent in the first place. Thus, the electoral cost of dissent (δ) must be sufficiently large so as to turn a leading incumbent into a trailing one.
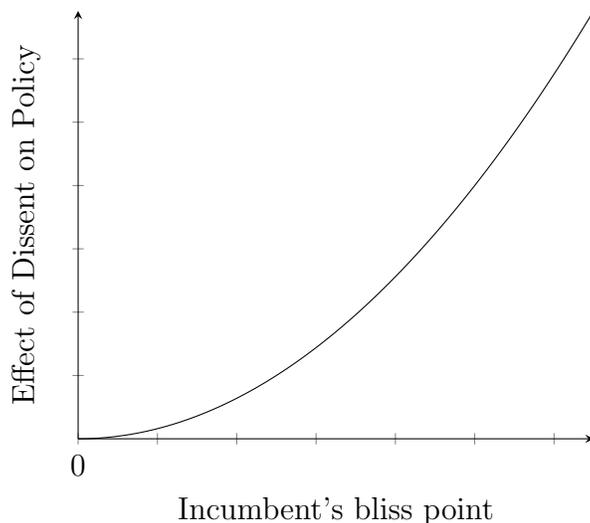
Figure 2: Effect of dissent on the equilibrium policy $((x_1^*|D=1) - (x_1^*|D=0))$

The second set of conditions on the prior γ ($\underline{\gamma} < \gamma < \overline{\gamma}$) ensures that the ally's gain from dissent outweighs the future expected cost. Recall that γ is the probability that the voter's true preferences are aligned with the incumbent's (i.e. $x^v = \overline{\alpha}$). As such, the higher γ, the lower a leading incumbent's incentives to prevent the voter from learning by implementing a moderate policy. As a consequence, the effect of dissent on the equilibrium policy is (weakly) decreasing in γ. When γ is too large (i.e. the incumbent enjoys a large lead) dissent therefore has a very small impact on the equilibrium policy, and the ally has no reason to pay the associated electoral cost. Conversely, if γ is too small the probability that the party would win the election after experiencing dissent is too low (recall that a trailing incumbent is re-elected only if the voter updates in his favor). Losing the upcoming election is very costly for the incumbent's extreme ally, therefore dissent is never observed in equilibrium.

Finally, let us now focus on the ideological misalignment between the incumbent and his allies. Such misalignment represents the only source of conflict in the model. Yet, Proposition 1 shows that increasing the ideological distance between the incumbent and his ally does not always make dissent more likely to emerge. Indeed, while only a sufficiently extreme ally may be willing to dissent, for this to occur in equilibrium the incumbent himself must be sufficiently extreme. Dissent cannot force the incumbent to implement any specific policy. The incumbent's ally can only

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influence his equilibrium choice by creating incentives to gamble on resurrection by engaging in policy experimentation. However, if the incumbent is too moderate, such incentives are too weak: gambling is too costly, and not very valuable. It is costly as it entails implementing extreme policies, potentially very far from the incumbent's bliss point. It is not very valuable since for a moderate incumbent the gain from winning the upcoming election is small (the distance from the opposition is small). Thus, as Figure 2 shows, the impact of dissent on the incumbent's choice is increasing in his bliss point (the vertical axis represents the difference between the equilibrium policy with and without dissent). If the incumbent is too moderate dissent will have a very small effect on the equilibrium policy. This reduces the ally's gain, and hence incentives to dissent in the first place. This result highlights the peculiar nature of dissent. in this model, which brings about unity even if no player actually cares about unity per se. Dissent serves the purpose of realigning the interests of the incumbent and his ally, thereby recomposing the existing ideological conflict. However, for dissent to be effective, such conflict cannot be too deep.

Comparative Statics: the Ambiguous Impact of the Ideological Conflict between the Incumbent and His Allies

Proposition 1 indicates that a necessary condition for dissent to occur in equilibrium is that γ falls within a certain range. The larger this range, the 'more likely' it is that dissent will be observed in equilibrium (in the sense of set inclusion). Proposition 2 describes how the size of this range (and therefore the likelihood of observing dissent) varies with the incumbent's and his ally's bliss points.

Proposition 2:

- *The likelihood of observing dissent (weakly) increases as the ally becomes more extreme*

- *There exists a unique $\widehat{x^I}(x^A) > \underline{x}^I$ such that if $x^I < \widehat{x^I}(x^A)$, then the likelihood of observing dissent increases as the incumbent becomes more extreme*

In line with the above discussion, Proposition 2 further highlights that the ideological conflict between the incumbent and his ally (i.e the distance between their bliss points) has an ambiguous

effect on the probability of observing dissent. Increasing the ideological conflict either increases or decreases the likelihood of dissent, depending on whether the incumbent becomes more moderate or his ally more extreme. When the ideological misalignment increases due to the incumbent's ally becoming more extreme, dissent always becomes more likely. The more extreme the ally is, the more he gains by moving the equilibrium policy closer to his bliss point. However, the same is not necessarily true when the ideological conflict deepens due to the incumbent becoming more moderate. The intuition is exactly the same as discussed in relation to Proposition 1. As the incumbent becomes more extreme both a direct and indirect effects emerge. The direct effect is straightforward: the distance in the policy preferences of the incumbent and his ally decreases. This reduces the ally's incentives to dissent. The indirect effect goes in the opposite direction. As the incumbent becomes more extreme, dissent has a larger impact on his equilibrium policy choice. This in turn increases the ally's gain from dissent. If the incumbent's bliss point is sufficiently close to zero, this indirect effect dominates, and dissent is more likely to emerge as the ideological conflict decreases.

Welfare Analysis

Can the presence of the incumbent's extreme ally be welfare improving for the voter? In the model, the voter values policy experimentation as it increases the probability that she will make the correct electoral decision. As such, her first-period preferred platform maximizes the trade-off between her ex-ante ideological preferences (as dictated by her prior beliefs) and the need to learn about her ideal policy. However, the results presented above indicate that, under some conditions, electoral accountability may have the perverse consequence of inducing a lower lever of experimentation than what is optimal for the voter. The incumbent's extreme ally may mitigate such inefficiency, by inducing the incumbent to engage in extreme policies that increase the amount of voter learning. If the value of acquiring new information is sufficiently large, this strictly increases the voter's expected utility in the whole game.

Proposition 3 identifies sufficient conditions for this to be true.

Proposition 3: *In equilibrium the voter benefits from the presence of an extreme ally to the incumbent party if:*

- *The cost of dissent δ is sufficiently large that it turns the leading incumbent into a trailing one, but not so large that it always hurts the voter ex ante ($\underline{\delta} < \delta < \overline{\delta_w}$)*

- *The value of information is sufficiently high*

  - *The prior (γ) is sufficiently close to $\frac{1}{2}$ ($\frac{1}{2} < \gamma < \overline{\gamma_w}$)*

  - *Incumbent and challenger are moderately polarized ($\underline{x_w^I} < x^I < \overline{x_w^I}$)*

  - *Learning the true state has a sufficiently large impact on the voter's preferences ($\bar{\alpha} > \bar{\alpha}_w$)*

- *The incumbent's ally is sufficiently extreme ($x^A > \underline{x_w^A}$)*

The first two conditions are intuitive: the voter must not dislike dissent too much, and obtaining new information must be sufficiently valuable. For this to be true, the voter's prior must be sufficiently uninformative (i.e. close to $\frac{1}{2}$), and the value of making the correct electoral decision must to be large enough. The third condition seems more puzzling: as the ally becomes more extreme the ideological misalignment with the voter increases. However, recall that the ally's bliss point has no direct effect on the equilibrium policy choice, thus on the voter's welfare. The effect is only an indirect one, through the ally's willingness to dissent. Since the first conditions impose further restrictions on the parameters, for the incumbent's ally to be willing to dissent when such conditions are satisfied (and therefore dissent is beneficial for the voter) he must be sufficiently extreme.

The normative implications of the results presented above are reminiscent of the 'case for responsible parties', presented by Bernhardt, Duggan and Squintani (2009). The authors find that, in a world in which the exact location of voters' preferences is unknown, all voters ex-ante prefer some degree of platform divergence between competing parties. However, electoral incentives may induce an excessive convergence in parties' platforms, thus ultimately hurting the

voters. Therefore, as in this paper, a positive role of ideological extremism in the political elite emerges. In Bernhardt, Duggan and Squintani, all voters benefit from a moderate degree of parties' ideological extremism (polarization), that can guarantee an optimal level of platform divergence. In this paper, the incumbent's extreme ally can mitigate the perverse consequences created by electoral incentives, inducing an optimal level of policy experimentation.[7]

These results also speak to the debate on the normative evaluation of party factions. The debate dates back to the 19th century. As noted by Boucek (2009), negative perceptions of factionalism originated with Hume (1877) and are still predominant. The main argument within this tradition is that factions 'exacerbate non-cooperative behaviour and so are antithetical to achievement of common goals' (Dewan and Squintani 2015, 861). A 'defence of factions' comes from the claim that organized and ideologically cohesive subgroups within political parties facilitate deliberation and pooling of valuable information, and therefore enhance the quality of the party's policy proposals. The argument is advanced initially by Bouceck herself (2009), investigated empirically by McAllister (1991), and proven formally by Dewan and Squintani (2015). The more or less implicit assumption is that factions engage in accommodative rather than disruptive activities (McAllister 1991). This paper moves one step further, showing that factionalism may have a positive value even when factions engage in 'disruptive' activities.

Micro Founding the Electoral Cost of Dissent

A key assumption of the paper is that dissent is electorally costly: everything else being equal, dissent reduces the probability that the incumbent is re-elected. In the model this cost is black-boxed, as this substantially simplifies both the analysis and interpretation of the results. However, it is worth discussing about potential ways to micro-found this assumption. Why do voters dislike parties that experience dissent?

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[7]It is important to highlight that other mechanisms through which politicians' ideological polarization may prove welfare improving have also been identified. Van Weelden (2015) for example shows that polarization in the candidates' preferences decreases rent-seeking in equilibrium, unambiguously increasing voters' welfare for appropriate parameters.

One possibility is that dissent 'mechanically' reduces voters' appreciation of political parties. In this sense, the parameter δ would represent a behavioral 'bias' in the voters' preferences. It is well recognized that voters tend to like charismatic leaders (Groseclose 2001). Perhaps when the incumbent is publicly criticized by his own allies, or ridiculed by the media, this negatively affects voters' perception of the party.

However, dissent may be electorally costly even if voters are fully rational. Dissent may harm the party electorally because it conveys negative information to the voters. Voters do not dislike divided parties per se, rather the observation of dissent causes them to negatively update their beliefs over the incumbent's honesty, competence, etc. Dissent may convey such information in two different ways.

First, if the incumbent's allies have access to *verifiable* information, by dissenting they can expose him as a liar, corrupt or incompetent. The specification and results of this model would be exactly as presented above, with δ representing the electoral value of competence (net of the probability that the challenger is a good type). Under the conditions identified in Proposition 1, the allies choose to dissent whenever they can reveal evidence that the incumbent is a bad type. If the conditions are not met, the allies always keep quiet. The only difference with the model presented here is that, because verifiable information cannot be fabricated, dissent can never emerge in equilibrium if the incumbent is a good type.

Alternatively, we may assume that the incumbent's allies do not have access to such verifiable evidence. Nonetheless, they may have an informational advantage with respect to the voters. For example, the allies may scrutinize the incumbent's previous actions and performance, thereby obtaining additional information about his true competence (see Caillaud and Tirole 1999, Fox and Van Weelden 2010). As such, the allies can engage in a signaling game with the electorate. Dissent is electorally costly when, *in equilibrium*, it constitutes a negative signal of the incumbent's type. However, for dissent to emerge when it is electorally costly (i.e. under separation or semi-separation), the gain from changing the incumbent's policy choice must be sufficiently large. The qualitative results would then be as in the reduced-form model presented above.[8]

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[8]Some additional conditions are however required to sustain separation. The ally must care sufficiently about

In concluding this section let me emphasize that, while the assumption of electorally costly dissent is motivated by both empirical evidence and the theoretical reasoning presented above, the mechanism identified in this paper relies only on the voters not being indifferent to dissent (i.e. $\delta \neq 0$). Indeed, it would survive in a world in which dissent produces a positive valence shock (i.e. $\delta < 0$), thus improving rather than damaging a party's electoral prospects. Clearly, under such an assumption the puzzle would be reversed: if dissent is beneficial to a party, how do we explain cases in which we do not observe dissent? The mechanism identified in this paper would provide a potential answer. An extreme ally may choose not to improve its party's electoral prospects, in order to preserve the incumbent's incentives to gamble for resurrection.

Extension: What if the Ally Has Bargaining Power?

So far I have assumed that the incumbent is essentially a policy dictator. His allies have no formal bargaining power, and dissent is the only tool to influence the equilibrium policy. This assumption is plausible if we consider dissent by ideological allies external to the party (such as ideologically aligned media), but perhaps less so if we focus on factional dissent. To be sure, institutional features such as the vote of confidence procedure (in parliamentary systems) may grant large discretion to the party leader. However, it is also possible that the minority faction will have some bargaining power (perhaps due to the credible threat of a formal defection/party split) and therefore influence over policy.

Are the results of this paper robust to assuming that the incumbent's allies have bargaining power over the policy making process? To address this question, I analyse the model under the assumption that, in the first period, the incumbent maximises a weighted average of his own and his ally's utility:

$$U_1^W = (1 - \beta)U^I + \beta U^A \tag{5}$$

quality, and he cannot be too extreme, as otherwise he would have incentives to dissent even when the incumbent is a good type.

This is equivalent to considering (in a reduced form) a game in which *after* the incumbent's ally chooses whether to dissent against him, the two engage in in a bargaining game over the policy choice. β thus represents the ally's influence over policy making in the first period. There are two reasons to consider a setting in which the ally has bargaining power in the first period only. First, it is plausible to argue that faction's bargaining power comes from the threat of a formal defection. Such a threat is credible when the ally (i.e. the dissenting faction) has a sufficiently high chance of winning the upcoming election if running alone after a party split (as for example in Mutlu-Eren 2015; Hortala-Vallve and Mueller 2015). Since there is no election after the second period, the ally has no way to make a credible threat. Additionally, assuming that the ally has bargaining power only over the first period policy is a way to obtain a meaningful comparison with the baseline model. Suppose that the second period policy is also determined via a bargaining process. Recall that dissent occurs in equilibrium only if the incumbent is leading. In the baseline model this requires $\gamma > \frac{1}{2}$ (since incumbent and challenger are assumed to be symmetric). If the extreme ally has formal bargaining power over the second period policy, the condition becomes $\gamma > \frac{(\beta x^A + (1-\beta)x^I + \alpha)^2 - (x^I - \alpha)^2}{4\alpha(x^I + \beta x^A + (1-\beta)x^I)} > \frac{1}{2}$. Therefore, when comparing the bargaining extension to the baseline model, I would not only be altering the β parameter, but also imposing further conditions on γ, which would make the comparison less meaningful.

It is straightforward to see why bargaining power and dissent are, to a certain extent, substitutes. Dissent is a tool to influence the incumbent's equilibrium policy choice. When the ally has some formal control over policy making ($\beta > 0$), the incentives to pay the electoral cost of dissent are weaker. Indeed, in the limiting case in which the ally is given full authority over policy ($\beta = 1$), he will never choose to dissent against himself.

However, the results uncover a second and more subtle effect. Bargaining power and dissent will sometimes complement rather than substitute each other, so that dissent is more likely to be observed compared to the case in which the incumbent is a policy dictator. Recall that, in the no-bargaining baseline, dissent emerges in equilibrium only if the incumbent is sufficiently extreme; if the incumbent is too moderate, the incentives to gamble are too weak and dissent has too little an effect on the equilibrium policy. However, if the ally is given formal authority over policy

making, it can effectively 'compensate' for an excessively moderate incumbent, so that dissent can emerge in equilibrium for every (positive) value of the incumbent's bliss point. The following holds:

Proposition 4: *For all $x^I \geq 0$, there exist non-measure zero sets $\Gamma(x^I)$ and $B(x^I)$ such that if $\gamma \in \Gamma(x^I)$ and $\beta \in B(x^I)$ then dissent by an extreme ally occurs in equilibrium*

The sets $B(x^I)$ and $\Gamma(x^I)$ also depend on the other parameters. Proposition 4 shows that the mechanism uncovered in the paper is robust to assuming that the ally has formal bargaining power, arising for example from a credible threat of defection or party split. Additionally – as discussed above – when the incumbent is sufficiently moderate (x^I is sufficiently close to zero) $\beta > 0$ is a necessary condition to observe dissent in equilibrium. The following corollary also holds:

Corollary 1: *Suppose that $\frac{1}{8\overline{\alpha}\psi} < x^I$ and $\frac{1}{4\overline{\alpha}\psi} < x^A < \frac{1}{4\overline{\alpha}\psi(1-2\overline{\alpha}\psi x^I)}$. Then, for all $\beta \in [0,1)$, there exists a non-measure zero set $\Gamma(\beta)$ such that if $\gamma \in \Gamma(\beta)$ dissent by an extreme ally occurs in equilibrium*

The corollary shows that even if the ally is granted almost full discretion over the first period policy (β approaches 1), he will still choose to dissent under some conditions.

Empirical Implications and Falsifiability

Before concluding, it is important to explore the theory's implications for empirical research, and to discuss how the model's predictions may allow us to adjudicate between competing explanations for the emergence of dissent.

Several scholars have recognized that, as highlighted in this paper, party unity may have a crucial impact on electoral outcomes. Trying to quantify the electoral cost of dissent is therefore an important empirical exercise, useful to complete our understanding of electoral competition and accountability. The strategy employed in the extant literature is to regress the probability of winning (or other measures of electoral success) at time t on a binary variable indicating whether
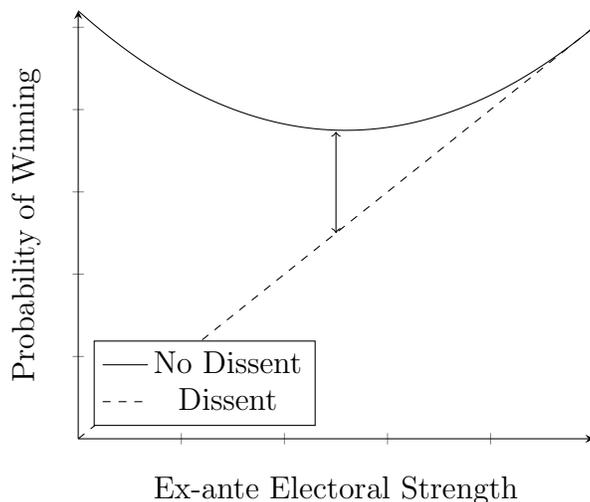
Figure 3: Probability of Winning as Function of Ex-ante Strength (γ).
The blue line represents the probability of winning when the incumbent experiences no dissent.
The red line represents the probability of winning conditional on experiencing dissent.

the party experienced dissent at time $t-1$ (e.g. Clark 2009, Kam 2009, Groeling 2010):

$$prob(W_i = 1) = \alpha + \beta_1 X_i + \beta_2 D_i + \epsilon_i \tag{6}$$

Where X_i is a vector of covariates, and β_2 is the coefficient of interest. Graphically, the quantity of interest is the average distance between the two curves in Figure 3, representing the probability of winning as a function of the party's ex-ante electoral strength (γ), with and without dissent.

The results of the model have two key implications. First of all, they show that it is impossible to isolate the direct effect, i.e. voter's dislike of parties that experience dissent. The incumbent modifies his policy choice precisely with the aim of mitigating the electoral cost of dissent. Thus, any estimate would at best reflect the equilibrium effect of dissent on electoral success: the cost mediated through the incumbent's best response. Additionally, the model shows that any such estimate would inevitably suffer from selection bias. Proposition 1 shows that whether parties experience dissent depends precisely on their ex-ante electoral strength (γ needs to be moderately high). Thus, it is impossible to observe both treated and control units for the same level of ex-ante electoral strength. Figure 4 represents what the researcher can actually observe: treated units, i.e. parties that experience dissent, at moderately high levels of electoral strength and untreated ones
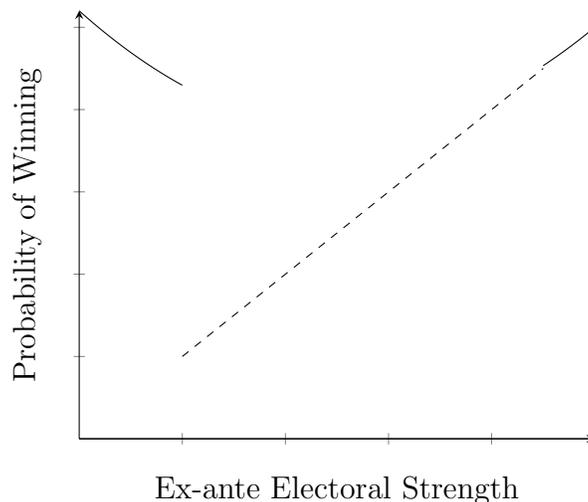
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Figure 4: Probability of Winning as Function of Ex-ante Strength (γ) - Observable

at γ close to $\frac{1}{2}$ and 1. Comparing parties that experience dissent with their untreated counterparts means comparing parties with different levels of ex-ante electoral strength. Thus, it is impossible to recover an unbiased estimate of the (equilibrium) effect of dissent on parties' electoral performance.

Further, it is hard to know ex-ante what the direction of the bias in the results will be. In the example of Figure 4, the direction of the bias would be upward: the estimated electoral cost of dissent would be higher than the true one. However, under different parameter values, the dissenting region shifts. Consider for example Figure 5, obtained by increasing the ally's bliss point: dissent emerges only at higher values of γ (compared to the case illustrated in Figure 4). In this case the direction of the bias is no longer clear. Indeed, the estimate may even have the wrong sign. Due to the selection bias, parties that experience dissent may perform better than 'control' units. Thus, even if we are aware of the existence of the bias, it is hard to interpret the results of this type of analysis.

However, this does not imply that the theory is not falsifiable. Indeed, the model generates predictions regarding the electoral performance of parties that do experience dissent. If we focus on this restricted sample, thereby avoiding the problem of selection bias described above, we can still say something on when dissent is expected to harm parties the most.

When a party experiences dissent, winning the upcoming election requires the voter to discover her true preferences (and update in favour of the party). The probability of the voter learning
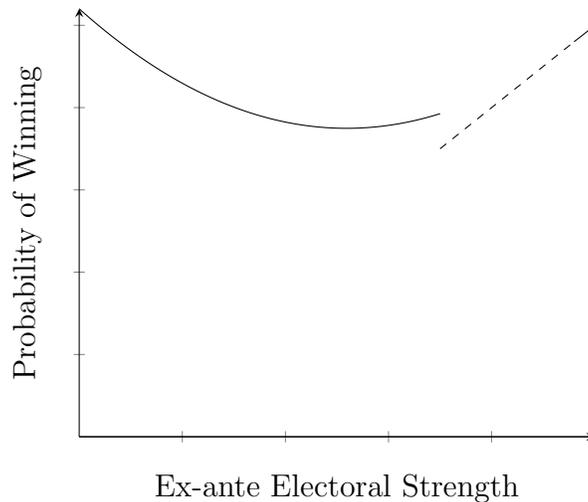
Figure 5: Probability of Winning as Function of Ex-ante Strength, $x^A > \underline{x^A}$

is obviously higher in high information environments. The larger the amount of information received by the voters, and their ability to interpret such information, the higher the probability of winning conditional on experiencing dissent. Thus, when considering a regression with the probability of winning conditional on dissent as dependent variable, we should expect to see a positive and statistically significant coefficient for variables such as news media consumption, education or political engagement in the population. Additionally, irrespective of the noisiness of the information environment, the incumbent must be willing to gamble and engage in policy experimentation. Thus, conditional on experiencing dissent, we should expect the party's electoral success to be increasing in the leader's ideological extremism. The more extreme the party leader is, the more he will be willing to gamble (gambling is both less costly and more valuable), the more likely it is that the policy outcome will be informative.

Finally, it is important to discuss whether the theory's empirical implications may allow us to distinguish it from alternative explanations for the emergence of electorally costly dissent.One possibility is that dissent emerges when politicians face a trade-off between the national party's electoral fortune and their individual success. This theory, and the differences with the argument presented here, have already been discussed in the literature review. Here, it is important to add that the mechanism identified in this paper applies not only to intra-party dissent but also to cases in which the party leader is attacked by ideological allies external to the party (such as media

outlets, as discusses in the introduction). The same does not seem to hold for the explanation relying on the trade off between individual and collective reputation.

Another possibility is that the dissenters are trying to damage the leader so as to make it easier to depose him. Within this framework, dissent should emerge when the leader is expected to perform poorly in the upcoming elections. Yet, this is not always the case. In the Italian example, dissent exploded against a leader who was expected to bring the party to electoral success. This is in line with the predictions of Proposition 1, according to which dissent by an extreme ally emerges when the incumbent is leading and expected to win with a sufficiently high probability. Finally, it is important to stress that if the dissenters' goal is to replace the dominant faction and take over the party (rather than simply depose the incumbent leader), then this argument complements the one proposed in this paper. For an extreme faction to take over, it has to believe that it has a chance of winning the election. When the electorate is too moderate, this requires changing voters' policy preferences. That is, the faction has incentives to force the incumbent leader to experiment, just like in the present model. As such, the framework presented here could explain dissent for pure policy-motivated reasons (as in the current paper) or for both policy *and* instrumental reasons (taking over). In this perspective allowing for replacement does not alter (and if anything strengthens) the qualitative insights presented here.

Conclusion

Political leaders often experience dissent by their own ideological allies despite this being electorally harmful. In order to address this puzzle, I have presented a political agency model in which voters are learning about their own policy preferences over time. The first contribution of the paper is to provide a new framework to study policy experimentation. Within this framework, the amount of voter learning depends on the location of the implemented policy, with extreme policies generating more information. As a consequence, leading incumbents have incentives to implement moderate platforms, while trailing ones want to engage in extreme policies that generate more information.

Within this setting, dissent emerges precisely because it is electorally costly. By dissenting (and thereby harming the party in the upcoming election) the incumbent's ally can change his

incentives to choose more or less extreme policies, which affects the amount of voter learning. This creates a trade-off between winning the upcoming election and inducing the incumbent to pursue the ally's all-things-considered more-preferred policy. Optimally balancing this trade-off sometimes involves active dissent that damages the party in the short-run. We observe dissent in equilibrium when the ensuing electoral cost is sufficiently high, the incumbent's bliss point is sufficiently biased in the direction of his ally, and the voter's prior that her preferences are in line with the incumbent's is moderately high. The results are robust to assuming that the allies have formal bargaining power over the first period policy. Further, the results indicate that the presence of extremists within the incumbent party may be welfare improving for the voter, inducing an optimal amount of policy experimentation by the office holder. The theory also has relevant implications for empirical research, showing that existing estimates of the electoral rewards of party unity obtained by comparing treated and control units are inevitably biased. However, the model generates testable predictions regarding parties' electoral performance conditional on dissent.

In this paper I have assumed that an ideological conflict underlies the emergence of this form of dissent. However, within the same framework dissent can arise even absent any ideological disagreement. For example, the incumbent and his ally may have the same bliss point but different access to (or evaluation of) office rents. This induces different risk appetite in policy making, thereby potentially generating a conflict in preferences. Similarly, the two actors may disagree about their beliefs over the voters' ideal policy. Suppose, for example, that the ally assigns a higher probability to the voter's true preferences being aligned with the party's. Then, the ally would always prefer a (weakly) more extreme policy relative to the incumbent. As in the model presented here, electorally costly dissent would therefore serve the purpose of incentivizing the incumbent to engage in policy experimentation, (re)creating unity of interests with his ideological allies.

Finally, while this paper has focused on intra-party conflict, the mechanism it uncovers applies more generally. Indeed, it can capture the dynamics of the interaction between political actors in any strategic situation that can be described as a principal agent model with two key features. First, there is some (common) uncertainty on what is the principal's optimal retention decision,

and the amount of information that is generated is a function of the agent's action. The principal's uncertainty can refer to her ideal policy, as in the model presented here, or to the agent's type, e.g. his ability or competence. Second, the agent's ally (i.e. an actor whose payoff is higher when the agent is retained than when he is replaced) can take an action that, everything else constant, changes the probability that the agent is retained. In this setting, the ally can choose to manipulate the probability that the agent is retained in order to alter his incentives to generate more or less information. This creates a trade off between ensuring that the agent is retained, and inducing him to take an action closer to the ally's own preferences. The agent's action may refer to a level of effort, a point in the policy space, the degree of reform, or even the amount of rent extraction. As such, this framework can be applied to several different settings, encompassing both developed and developing democracies, as well as authoritarian countries.

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Appendix A: Proofs

Lemma 1: *The voter learning satisfies the following properties:*

(i) Her posterior μ takes one of three values: $\mu \in \{0, \gamma, 1\}$;

(ii) The more extreme the policy implemented in the first period x_1, the higher the probability that $\mu \neq \gamma$;

(iii) There exists a policy x' such that if $|x_1| \geq |x'|$, then $\mu \neq \gamma$ with probability 1.

Proof. The proof of Claims 1 and 2 below is necessary and sufficient to prove Lemma 1.

Claim 1: *Let $x_t \geq 0$.*

(i) A payoff realization $U_t^v \notin [-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$ is fully informative. Upon observing $U_t^v > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}$, the players form posterior beliefs that $x^v = \bar{\alpha}$ with probability 1. Similarly, upon observing $U_t^v < -(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}$ the players form beliefs that $x^v = \underline{\alpha}$ with probability 1.

(ii) A payoff realization $U_t^v \in [-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$, is uninformative. Upon observing U_t^v, players confirm their prior belief that $x^v = \bar{\alpha}$ with probability γ.

Symmetric results apply when $x_t < 0$.

Proof. The proof of part (i) is trivial given the boundedness of the distribution of ϵ, and is therefore omitted. Part (ii) follows straightforwardly from applying Bayes rule. Recall that the voter's payoff realization U_t^v is a function of the implemented policy (x_t) the voter's true bliss point (x^v) and the noise term (ϵ): $U_t^v = -(x^v - x_t)^2 + \epsilon$. Denote as $f(\cdot)$ the PDF of ϵ. Then,

$$prob(x^v = \bar{\alpha}|U_t^v) = \frac{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma}{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma + f(U_t^v + (x_t - \underline{\alpha})^2)(1 - \gamma)} \tag{7}$$

Given the assumption that ϵ is uniformly distributed

$$f(U_t^v + (x_t - \bar{\alpha})^2) = f(U_t^v + (x_t - \underline{\alpha})^2) \tag{8}$$

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Therefore the above simplifies to

$$prob(x^v = \bar{\alpha}|U_t^v) = \gamma \tag{9}$$

This concludes the proof of Claim 1. $\square$

Claim 1 proves that players either observe an uninformative or a fully informative signal. Claim 2 shows that the policy choice determines the expected probability that the signal will be informative. The more extreme the implemented policy, the higher such probability.

Claim 2: *Let L be a binary indicator, taking value 1 if the players learn the true value of x^v at the end of period 1, and 0 otherwise. There exists $x' = \frac{1}{4\bar{\alpha}\psi}$ such that*

- *For all $|x_1| \geq |x'|$*

$$Prob(L = 1|x_1) = 1 \tag{10}$$

- *For all $x_1 \in [0, x')$*

$$Prob(L = 1|x' \geq x_1 \geq 0) = 4\bar{\alpha}\psi x_1 \tag{11}$$

- *For all $x_1 \in (-x', 0]$*

$$Prob(L = 1| - x' \leq x_1 \leq 0) = -4\bar{\alpha}\psi x_1 \tag{12}$$

Proof. Let me first prove the existence of point x'. From Claim 1, x' is the point such that for any policy $|x| \geq |x'|$, the interval $[-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$ is empty. This requires

$$- (x_t - \underline{\alpha})^2 + \frac{1}{2\psi} + (x_t - \bar{\alpha})^2 + \frac{1}{2\psi} \leq 0 \tag{13}$$

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Recall that $\bar{\alpha} = -\underline{\alpha}$, thus the above reduces to

$$x \geq \frac{1}{4\bar{\alpha}\psi} = x' \tag{14}$$

To complete the proof, assume $x_1 \in [0, x']$. The expected probability of the realized outcome being informative is

$$Prob(L = 1|\gamma, 0 < x_1 < x') =$$

$$\gamma[Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \tfrac{1}{2\psi})] + (1 - \gamma)[Prob(-(x_t - \underline{\alpha})^2 + \epsilon_1 < -(x_t - \bar{\alpha})^2 - \tfrac{1}{2\psi})] \tag{15}$$

Given the symmetry

$$Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}) = Prob(-(x_t - \underline{\alpha})^2 + \epsilon_1 < -(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}) \tag{16}$$

(15) simplifies to

$$Prob(L = 1|x_1 > 0) = Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi})) = 4\bar{\alpha}\psi x_1 \tag{17}$$

Similar calculations produce the result for $x_1 \in (-x', 0]$.

This concludes the proof of Claim 2 □

This concludes the proof of Lemma 1 □

In what follows I will assume that $x^I < \frac{1}{4\bar{\alpha}\psi}$. This assumption is without loss of generality, and imposed in order to reduce the number of cases under consideration; results for the case in which $x^I > \frac{1}{4\bar{\alpha}\psi}$ are available upon request.

Lemma 2: *In any PBE of the game*

- A **certain loser** implements his bliss point

  $(x_1^* = x^I)$

- A **leading incumbent** implements a policy weakly more moderate than his bliss point

  $(x_1^* \leq x^I)$

- A **trailing incumbent** implements a policy weakly more extreme than his bliss point

  $(x_1^* \geq x^I)$

Proof. The proof of the first point is trivial: a certain loser is never re-elected, hence his policy choice does not influence his future payoff. He maximises his immediate utility by implementing his bliss point x^I. Leading and trailing incumbents will instead consider the expected informativeness of the policy, and how it influences their probability of re-election.

Consider first a trailing incumbent. Electoral concerns create incentives to implement an informative policy. The equilibrium policy solves the following maximisation problem:

$$
\begin{aligned}
\underset{x_1}{\text{maximise}} \quad & -(x_1 - x^I)^2 - (1 - 4\bar{\alpha}\psi x_1 \gamma)(x^I + x^I)^2 - 4\bar{\alpha}\psi x_1 \gamma (x^I - x^I)^2 \\
\text{subject to} \quad & x_1 \leq \frac{1}{4\bar{\alpha}\psi}
\end{aligned}
\tag{18}
$$

Hence:

$$
x_1^* = \min\{x^I + 8\bar{\alpha}\psi(x^I)^2\gamma, \frac{1}{4\bar{\alpha}\psi}\}
\tag{19}
$$

The condition that $x_1 \leq \frac{1}{4\alpha\psi}$ derives from the fact that any policy weakly more extreme than $x' = \frac{1}{4\alpha\psi}$ is fully informative, therefore the leading incumbent would have no reason to move beyond x' (recall that we assume $x^I < x'$).

Consider now a leading incumbent. The equilibrium policy will maximise the trade-off between implementing his true bliss point today, and generating as little information as possible, so as to increase the probability of being re-elected tomorrow. The equilibrium policy solves the following maximisation problem:

$$\underset{x_1}{\text{maximise}} \quad -(\mathrm{x}_1 - \mathrm{x^I})^2 - 4\bar{\alpha}\psi \mathrm{x}_1(1-\gamma)(\mathrm{x^I} + \mathrm{x^I})^2 - (1 - 4\bar{\alpha}\psi \mathrm{x}_1(1-\gamma))(\mathrm{x^I} - \mathrm{x^I})^2 \tag{20}$$

$$\text{subject to} \quad x_1 \geq 0$$

Hence:

$$x_1^* = \max\{x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma), 0\} \tag{21}$$

An incumbent may only be leading if $\gamma > \frac{1}{2}$. Additionally, $x^I < \frac{1}{4\bar{\alpha}\psi}$ by assumption. Thus, $x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) > 0$ and

$$x_1^* = x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) \tag{22}$$

$\square$

Lemma 3:

- *A trailing incumbent's equilibrium policy*

    1. *becomes (weakly) more extreme as his disadvantage decreases (γ increases)*

    2. *becomes(weakly) more extreme as his bliss point increases*

- *A leading incumbent's equilibrium policy*

    1. *becomes more extreme as his lead (γ) increases*

    2. *always becomes more extreme as his bliss point increases, when he enjoys a large lead ($\gamma > \frac{3}{4}$). When the lead is moderate ($\gamma < \frac{3}{4}$), the policy is non monotonic and concave in the bliss point*

The proof is omitted since it follows straightforwardly from Lemma 2.

Lemma 4: *In equilibrium dissent always (weakly) reduces the probability that the incumbent will be re-elected.*

Proof. Let x^d be the incumbent's policy choice after dissent, and x the policy that he would choose otherwise. Consider first of all a leading incumbent. We must distinguish between three cases: (i) $\delta < \underline{\delta}$ such that the incumbent's initial advantage is not outweighed (i.e. $\gamma > \frac{\delta + 4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi x^I}$). In this case dissent does not modify the incumbent's policy choice nor the voter's electoral decision. (ii) $\delta \geq \bar{\delta}$ such that the incumbent always loses after dissent (i.e $\delta \geq 4\bar{\alpha}x^I$). The claim follows straightforwardly. (iii) $\delta \in [\underline{\delta}; \bar{\delta})$, such that the incumbent wins if and only if the voter updates in his favour (i.e. dissent turns the leading incumbent into a trailing one). The following holds. Let $\pi(x_1)$ be the probability of the voter observing an informative signal at the end of period 1, as a function of the implemented policy. The probability of the incumbent being re-elected absent dissent is $1 - \pi(x)(1-\gamma)$. The probability of the incumbent being re-elected after dissent is instead $\pi(x^d)\gamma$. $1 - \pi(x)(1-\gamma) \geq \pi(x^d)\gamma$, since the LHS is at least $1 - (1-\gamma) = \gamma$ and the RHS is at most γ. Finally, consider a trailing incumbent. There are only two possibilities: (i) $\delta > \underline{\delta}$ such that after experiencing dissent the incumbent loses for sure. The claim follows trivially (ii) $\delta \leq \underline{\delta}$ such that the incumbent is still trailing even after experiencing dissent. Dissent has no impact on the policy choice, nor on the voter's electoral decision. $\qquad\square$

Proposition 1: *There exist $\underline{\gamma}$, $\bar{\gamma}$, $\underline{x^A}$ and $\underline{x^I}$ such that the incumbent's extreme ally chooses to dissent if and only if:*

- *Absent dissent, the incumbent is leading, but his advantage is not too large*

  $\underline{\gamma} < \gamma < \bar{\gamma}$, *where* $\underline{\gamma} \geq \frac{1}{2}$

- *The electoral cost of dissent is sufficiently high that it turns the leading incumbent into a trailing one, but not so high that the incumbent loses for sure*

  $(2\gamma - 1)4\bar{\alpha}x^I \leq \delta < 4\bar{\alpha}x^I$

- *Both the incumbent and his ally are sufficiently extreme*

  $x^I > \underline{x^I}$ *and* $x^A > \underline{x^A} > x^I$

Proof. Let me first prove that dissent is never observed in equilibrium if the incumbent is trailing.[9]

Absent dissent, a trailing incumbent implements policy $x_1^* = \min \in \{x^I + 8\bar{\alpha}\psi(x^I)^2\gamma, \frac{1}{4\bar{\alpha}\psi}\}$. Dissent has no impact on his policy choice (and therefore never emerges in equilibrium) if $\delta < 4\bar{\alpha}x^I$. Suppose instead that $\delta \geq 4\bar{\alpha}x^I$. Then, after experiencing dissent the incumbent is a sure loser: even if the voter learns that $x^v = \bar{\alpha}$, she will still choose to replace the incumbent with his challenger. As a consequence, in the first period the incumbent would always implement exactly his bliss point x^I upon experiencing dissent.

Therefore, there are two pairs of equilibrium policies that we must consider:

- $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = \frac{1}{4\bar{\alpha}\psi}$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$

- $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$

I will analyse each case separately, conjecturing the existence of an equilibrium in which the ally chooses to dissent.

Case 1: $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = \frac{1}{4\bar{\alpha}\psi}$

The equilibrium conditions for the incumbent are

$$\gamma \leq \frac{1}{2} \tag{23}$$

$$\gamma > \frac{1}{8\bar{\alpha}\psi x^I}\left(\frac{1}{4\bar{\alpha}\psi x^I} - 1\right) \tag{24}$$

Additionally, the equilibrium condition for the faction is

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > -\left(\frac{1}{4\bar{\alpha}\psi} - x^A\right)^2 - \gamma(x^I - x^A)^2 - (1 - \gamma)(x^I + x^A)^2 \tag{25}$$

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[9]The reason why we must consider this case is that, even if dissent would always induce a trailing incumbent to moderate his policy choice, it may be the case that the ally's bliss point lies between the platforms that the incumbent would implement with and without dissent. It is therefore possible that $[x^A - (x_1^*|D = 1, \gamma < \frac{1}{2})]^2 < [x^A - (x_1^*|D = 0, \gamma < \frac{1}{2})]^2$ even if $(x_1^*|D = 1, \gamma < \frac{1}{2}) < (x_1^*|D = 0, \gamma < \frac{1}{2})$. We must therefore exclude that the ally's gain from dissenting against a trailing incumbent is larger than the cost.

Which reduces to

$$\gamma < \frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} \tag{26}$$

Thus, we need to identify conditions under which

$$\frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} > \frac{1}{8\bar{\alpha}\psi x^I}\left(\frac{1}{4\bar{\alpha}\psi x^I} - 1\right) \tag{27}$$

Let $T = 4\bar{\alpha}\psi$. The above can be rearranged as

$$\frac{1 - 2x^A T(1 - x^I T) - (x^I T)^2}{2x^A} > \frac{1 - x^I T}{x^I} \tag{28}$$

Which reduces to

$$2x^A(1 - (x^I T)^2) < x^I(1 - (x^I T)^2) \tag{29}$$

Since $(x^I T)^2 = (4\bar{\alpha}\psi x^I)^2 < 1$, the above can never be satisfied when $x^A > x^I$.

Case 2: $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$

The equilibrium condition for the faction is

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > \tag{30}$$

$$-(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 - (1 - 4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(x^I + x^A)^2$$

$$-4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - x^A)^2$$

Denoting $\Delta = 8\bar{\alpha}\psi\gamma(x^I)^2$, the above can be rearranged as

$$0 > -\Delta^2 - 2\Delta(x^I - x^A) + 16\bar{\alpha}\psi\gamma(x^I + \Delta)x^I x^A \tag{31}$$

Substituting $\Delta = 8\bar{\alpha}\psi\gamma(x^I)^2$ and dividing by $16\bar{\alpha}\psi\gamma(x^I)^2$, the above reduces to

$$x^A < \frac{x^I}{2} \tag{32}$$

Given $x^A > x^I$, the condition can never be satisfied.

Thus, dissent never emerges in equilibrium if $x^A > x^I$ and $\gamma < \frac{1}{2}$.

Consider now the conditions on the cost of dissent δ. The proof of the first condition ($\delta > 4\bar{\alpha}x^I(2\gamma - 1)$) is presented in the main body of the paper. Here, I present a formal proof of the second condition. Let $\delta \geq 4\bar{\alpha}x^I$. After experiencing dissent, the incumbent would turn into a sure loser. Therefore:

$$(x_1^* | D = 1) = x^I \tag{33}$$

Conversely, (from Lemma 2) if the leading incumbent experiences no dissent:

$$(x_1^* | D = 0) = x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma) \tag{34}$$

Dissent strictly increases the ally's utility if and only if:

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > \tag{35}$$
$$-(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma) - x^A)^2 - (1 - 4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma)))(x^I - x^A)^2$$
$$-4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(x^I + x^A)^2$$

This reduces to

$$x^A[1 - 2(4\bar{\alpha}\psi x^I(1 - \gamma))(1 - 4\bar{\alpha}\psi x^I(1 - \gamma))] + 4\bar{\alpha}\psi(x^I)^2(1 - \gamma)(1 - 4\bar{\alpha}\psi x^I(1 - \gamma)) < 0 \tag{36}$$

The LHS is increasing in x^A and never satisfied at $x^A = 0$. Hence, dissent by an extremist ally emerges only if $\delta < 4\bar{\alpha}x^I$.

Finally, I must prove that there exist unique $\underline{\gamma}$, $\overline{\gamma}$, $\underline{x}^I$ and $\underline{x}^A$ such that dissent by an extremist ally emerges in equilibrium only if $\underline{\gamma} < \gamma < \overline{\gamma}$, $x^I > \underline{x}^I$ and $x^A > \underline{x}^A$.

Suppose that $4\bar{\alpha}x^I(2\gamma-1) < \delta < 4\bar{\alpha}x^I$ and $\gamma > \frac{1}{2}$, and conjecture the existence of an equilibrium in which the ally chooses to dissent. We must consider two cases:

1. $(x_1^*|D = 1) = \frac{1}{4\bar{\alpha}\psi}$ and $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(1-\gamma)(x^I)^2$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$

2. $(x_1^*|D = 1) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$ and $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(1-\gamma)(x^I)^2$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$

I will analyse each of the two cases separately.

Case 1: $(x_1^*|D = 1) = \frac{1}{4\bar{\alpha}\psi}$ **and** $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(1-\gamma)(x^I)^2$

From Lemma 2, the equilibrium conditions for the incumbent are

$$\gamma > \frac{1}{2} \tag{37}$$

$$\gamma > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \tag{38}$$

Additionally, the equilibrium condition for the faction is

$$-(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1 - \gamma)(x^I + x^A)^2 > \tag{39}$$
$$-(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma) - x^A)^2 - [1 - 4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))](x^I - x^A)^2$$
$$-4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(x^I + x^A)^2$$

Let $I = 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))$. We can rewrite the above condition as:

$$(1 - \gamma)(1 - I)((x^I - x^A)^2 - (x^I + x^A)^2) > (\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - (\frac{I}{4\bar{\alpha}\psi} - x^A)^2 \tag{40}$$

Which is equivalent to

$$(1 - \gamma)(1 - I)(-4x^A x^I) > \frac{-x^A}{2\bar{\alpha}\psi}(1 - I) + \frac{1}{(4\bar{\alpha}\psi)^2}(1 + I)(1 - I) \tag{41}$$

By substituting $I = 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))$ and solving for γ we get the following condition:

$$\gamma > \frac{1 + (2x^A - x^I)(8\bar{\alpha}\psi x^I - 1)(4\bar{\alpha}\psi)}{2x^I(4\bar{\alpha}\psi)^2(2x^A - x^I)} \tag{42}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\gamma > \underline{\gamma} = max \quad \{\frac{1}{2}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1), \frac{1 + (2x^A - x^I)(8\bar{\alpha}\psi x^I - 1)(4\bar{\alpha}\psi)}{2x^I(4\bar{\alpha}\psi)^2(2x^A - x^I)}\}$

2. $x^I > \frac{1}{8\bar{\alpha}\psi}$

3. $x^A > \frac{1}{8\bar{\alpha}\psi} + \frac{x^I}{2}$

Where the conditions on x^I and x^A ensure that the range $[\underline{\gamma}, 1]$ exists.

Case 2: $(x_1^*|D = 1) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$ **and** $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(1 - \gamma)(x^I)^2$

From Lemma 4, the equilibrium conditions for the incumbent are

$$\gamma > \frac{1}{2} \tag{43}$$

$$\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \tag{44}$$

Additionally, the the equilibrium condition for the faction is

$$-(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 \tag{45}$$

$$-(1 - 4\bar{\alpha}\psi(x^I + 8\alpha\psi(x^I)^2\gamma)\gamma)(x^I + x^A)^2 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)\gamma(x^I - x^A)^2 >$$

$$-(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma) - x^A)^2 - (1 - 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(1 - \gamma))(x^I - x^A)^2$$

$$-4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(1 - \gamma)(x^I + x^A)^2$$

Let $I = 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and $x^D = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$. We can rewrite the above condition as:

$$-(x^D - x^A)^2 - (1 - \gamma I)(x^I + x^A)^2 - \gamma I(x^I - x^A)^2 > \tag{46}$$

$$-(x^D - x^A - 8\bar{\alpha}\psi(x^I)^2)^2 - (1 - (1 - \gamma)(I - 4\bar{\alpha}\psi(8\bar{\alpha}\psi(x^I)^2)))(x^I - x^A)^2$$

$$-(1 - \gamma)(I - 4\bar{\alpha}\psi(8\bar{\alpha}\psi(x^I)^2))(x^I + x^A)^2$$

By expanding, letting $x^D = \frac{I}{4\bar{\alpha}\psi}$ and dividing both sides by $4x^I$ we get:

$$\gamma I x^A > x^A - x^A((1 - \gamma)(I - 2(4\bar{\alpha}\psi x^I)^2) + x^I(I - 4\bar{\alpha}\psi x^A - (4\bar{\alpha}\psi x^I)^2) \tag{47}$$

Which is equivalent to:

$$x^A + I(x^I - x^A) - 4\bar{\alpha}\psi x^I x^A + (4\bar{\alpha}\psi x^I)^2(2x^A(1 - \gamma) - x^I) < 0 \tag{48}$$

By substituting $I = 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and solving for γ we get the following condition:

$$\gamma > \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)} \tag{49}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)}\} < \gamma < min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\} = \bar{\gamma}$

2. $\frac{\sqrt{3}-1}{8\bar{\alpha}\psi} < x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$

3. $x^A > \frac{x^I(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1)}{2(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1) - 1}$

Where the conditions on x^I and x^A ensure that the range $[\underline{\gamma}, \bar{\gamma}]$ exists.

This concludes the proof of Proposition 1. $\qquad\square$

The following corollary also holds, with respect to Case 2:

Corollary 1A: $\underline{\gamma} = \frac{1}{2} \implies \overline{\gamma} < 1$

Proof. Corollary 1A tells us that it can never be the case that (i) $\underline{\gamma} = \frac{1}{2}$ and (ii) $\overline{\gamma} = 1$. For (i) to be true we need:

$$\frac{1}{2} > \frac{4\bar{\alpha}\psi x^I (2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2 (x^I)^2 (2x^A - x^I)} \tag{50}$$

Which reduces to:

$$x^A[1 - 8\bar{\alpha}\psi x^I] + 4\bar{\alpha}\psi(x^I)^2 < 0 \tag{51}$$

Which clearly requires:

$$x^I > \frac{1}{8\bar{\alpha}\psi} \tag{52}$$

For (ii) to be true we need:

$$1 < \frac{1}{8\bar{\alpha}\psi x^I}\left(\frac{1}{4\bar{\alpha}\psi x^I} - 1\right) \tag{53}$$

Which reduces to

$$x^I < \frac{1}{8\bar{\alpha}\psi} \tag{54}$$

Clearly the two conditions can never be simultaneously satisfied.

$\square$

Proposition 2:

- *The likelihood of observing dissent (weakly) increases as the ally becomes more extreme*

- *There exists a unique $\widehat{x^I}(x^A) > \underline{x}^I$ such that if $x^I < \widehat{x^I}(x^A)$, then the likelihood of observing dissent increases as the incumbent becomes more extreme*

Proof. Denote $\Gamma(x^I, x^A, \bar{\alpha}\psi)$ the set of values of γ such that dissent is an equilibrium strategy iff $\gamma \in \Gamma$. From the proof of Proposition 1 is easy to verify that Γ is always weakly increasing in x^A. Analysing Cases 1 and 2, Γ is (weakly) increasing in x^I if and only if either one of the following sets of conditions is satisfied:

1. Case 1: $\underline{\gamma} = 1$. This requires $\frac{\sqrt{5}-1}{8\bar{\alpha}\psi} < x^I < \frac{1}{4\bar{\alpha}\psi}$ and $x^A > \frac{1+4\bar{\alpha}\psi x^I (1-4\bar{\alpha}\psi x^I)}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)}$

2. Case 1: $\underline{\gamma} = \frac{1-4\bar{\alpha}\psi x^I}{2(4\bar{\alpha}\psi x^I)^2}$ which requires $x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$ and $x^A > \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2}$. It is easy to verify that when $\underline{\gamma} = \frac{1-4\bar{\alpha}\psi x^I}{2(4\bar{\alpha}\psi x^I)^2}$ in Case 1, irrespective of the bounds in Case 2 Γ will be weakly increasing in x^I

3. Case 2: $\underline{\gamma} = \frac{4\bar{\alpha}\psi x^I(2x^A-x^I)(4\bar{\alpha}\psi x^I-1)+x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A-x^I)}$ and $\overline{\gamma} = 1$, which requires $x^I < \frac{1}{8\bar{\alpha}\psi}$.

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Thus, necessary and sufficient condition for Γ to be increasing in x^I is that $x^I < \widehat{x^I}(x^A)$.
$x^A > \frac{1+4\bar{\alpha}\psi x^I(1-4\bar{\alpha}\psi x^I)}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)} \implies \widehat{x^I}(x^A) = \frac{1}{4\bar{\alpha}\psi}$, $\frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2} < x^A < \frac{1+4\bar{\alpha}\psi x^I(1-4\bar{\alpha}\psi x^I)}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)} \implies \widehat{x^I}(x^A) = \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$, and $x^A < \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2} \implies \widehat{x^I}(x^A) = \frac{1}{8\bar{\alpha}\psi}$ $\qquad\square$

Proposition 3: *In equilibrium the voter benefits from the presence of an extreme ally to the incumbent party if:*

- *The cost of dissent δ is sufficiently large that it turns the leading incumbent into a trailing one, but not so large that it always hurts the voter ex ante ($\underline{\delta} < \delta < \overline{\delta_w}$)*

- *The value of information is sufficiently high*

    - *The prior (γ) is sufficiently close to $\frac{1}{2}$ ($\frac{1}{2} < \gamma < \overline{\gamma_w}$)*

    - *Incumbent and challenger are moderately polarized ($\underline{x^I_w} < x^I < \overline{x^I_w}$)*

    - *Learning the true state has a sufficiently large impact on the voter's preferences ($\bar{\alpha} > \bar{\alpha}_w$)*

- *The incumbent's ally is sufficiently extreme ($x^A > \underline{x^A_w}$)*

Proof. In order to identify *sufficient* conditions for the voter to benefit from dissent, suppose that $\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$. Then, $(x_1^*|D=1) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$ and $(x_1^*|D=0) = x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma)$. Dissent increases the voter's welfare if and only if the following condition is satisfied:

$$-4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)\gamma\delta - \gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - \bar{\alpha})^2 \tag{55}$$

$$-(1-\gamma)(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma + \bar{\alpha})^2 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - \bar{\alpha})^2$$

$$-(1 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(\gamma(x^I + \bar{\alpha})^2 + (1-\gamma)(x^I - \bar{\alpha})^2) >$$

$$-\gamma(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) - \bar{\alpha})^2 - (1-\gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) + \bar{\alpha})^2$$

$$-4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma))(x^I - \bar{\alpha})^2$$

$$-(1 - 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma)))(\gamma(x^I - \bar{\alpha})^2 + (1-\gamma)(x^I + \bar{\alpha})^2)$$

Which reduces to:

$$\delta < \frac{(1-2\gamma)(1 - 8\bar{\alpha}\psi x^I + 2(4\bar{\alpha}\psi x^I)^2 + 16\bar{\alpha}\psi^2(x^I)^3) - 4\psi(x^I)^2 + 4(4\bar{\alpha}\psi x^I\gamma)^2}{\psi\gamma(1 + 8\bar{\alpha}\psi x^I\gamma)} = \overline{\delta_w} \tag{56}$$

If the above is satisfied, the voter benefits from dissent. However, we need to make sure that dissent would indeed emerge in equilibrium (given the incumbent's equilibrium policy choices with and without dissent). From the proof of Proposition 1 we know that this requires the following conditions:

1. $4\bar{\alpha}x^I(2\gamma - 1) \leq \delta < 4\bar{\alpha}x^I$

2. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)}\} < \gamma < min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\} = \overline{\gamma}$

3. $\frac{\sqrt{3}-1}{8\bar{\alpha}\psi} < x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$

4. $x^A > \frac{x^I(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1)}{2(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1) - 1}$

Thus, for the voter to benefit the presence of the incumbent's extreme ally, both (46) and

conditions 1 to 4 above must be satisfied. This requires $\overline{\delta_w} > 4\bar{\alpha}x^I(2\gamma - 1)$, which reduces to:

$$(1 - 2\gamma)(1 + 4\bar{\alpha}\psi x^I(8\bar{\alpha}\psi x^I - 2 + 4\psi(x^I)^2 + 8\bar{\alpha}\psi x^I\gamma^2 + \gamma)) - 4\psi(x^I)^2 + 4(4\bar{\alpha}\psi x^I\gamma)^2 > 0 \quad (57)$$

The LHS is decreasing in γ, therefore the above establishes an upper bound $\overline{\gamma_w}$. For the condition to be possible to satisfy in equilibrium we need $\overline{\gamma_w} > \underline{\gamma}$. From the proof of Case 2 we can verify that $\underline{\gamma} = \frac{1}{2}$ when $x^I > \frac{1}{8\bar{\alpha}\psi}$ and $x^A > \frac{4\bar{\alpha}\psi(x^I)^2}{8\bar{\alpha}\psi x^I - 1}$. Additionally, given Corollary 1A $\underline{\gamma} = \frac{1}{2} \implies \overline{\gamma} = \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$. Thus, the voter benefits from the presence of the incumbent's extreme ally if:

1. $4\bar{\alpha}x^I(2\gamma - 1) \leq \delta < \overline{\delta_w}$

2. $\gamma < \overline{\gamma_w}$

3. $\underline{\gamma} = \frac{1}{2} < \gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) = \overline{\gamma}$

4. $\frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) > \frac{1}{2}$

5. $\overline{\gamma_w} > \frac{1}{2}$

6. $x^I > \frac{1}{8\bar{\alpha}\psi} = \underline{x_w^I}$

7. $x^A > \frac{4\bar{\alpha}\psi(x^I)^2}{8\bar{\alpha}\psi x^I - 1} = \underline{x_w^A}$

$\overline{\gamma_w} > \frac{1}{2}$ if and only if the following is satisfied:

$$-4\psi(x^I)^2 + 4(2\bar{\alpha}\psi x^I)^2 > 0 \quad (58)$$

Which reduces to

$$\alpha > \frac{1}{2\sqrt{\psi}} = \bar{\alpha}_w \quad (59)$$

$\frac{1}{8\bar{\alpha}\psi x^I}\left(\frac{1}{4\bar{\alpha}\psi x^I} - 1\right) > \frac{1}{2}$ if and only if

$$x^I < \frac{\sqrt{5} - 1}{8\bar{\alpha}\psi} = \overline{x_w^I} \tag{60}$$

Thus we can rewrite the sufficient conditions for the voter to benefit from dissent in equilibrium as:

1. $\underline{\delta} < \delta < \overline{\delta_w}$

2. $\frac{1}{2} < \gamma < min\{\overline{\gamma}, \overline{\gamma_w}\}$

3. $\underline{x_w^I} < x^I < \overline{x_w^I}$

4. $x^A > \underline{x_w^A}$

5. $\bar{\alpha} > \bar{\alpha}_w$

$\square$

Extension: What if the Ally Has Bargaining Power?

In this section I will consider the case in which the incumbent's ally has bargaining power over the first period policy making. I will thus assume that in the first period the incumbent maximises a weighted average of his own and the ally's utility:

$$U_1^W = \beta[-(x_1 - x^A)^2 + U_2^A(x_1, x_2, x^A)] + (1 - \beta)[-(x_1 - x^I)^2 + U_2^I(x_1, x_2, x^I)] \tag{61}$$

This is equivalent to analysing a game in which, after the ally chooses wether to dissent, it engages in a bargaining stage with the incumbent to determine the policy to be implemented in the first period. Therefore, the parameter β represents, in this reduced form, the ally's bargaining power in the first period. As in the baseline model, I assume $x^C = -x^I \leq 0$ and $x^I < \frac{1}{4\alpha\psi}$. Additionally, I assume that in the second period the ally has no bargaining power. A discussion

of the necessity and significance of this assumption is in the main body of the paper.

We can determine the equilibrium policy choice of the incumbent, proceeding as in the proof of Lemma 3.

Consider first a trailing incumbent. The following holds:

- Let $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}a\psi}$, then $x_1^* = \beta x^A + (1-\beta)x^I$

- Let $\beta x^A + (1-\beta)x^I < \frac{1}{4\bar{\alpha}\psi}$, then $x_1^* = \quad \min \quad \{\frac{1}{4\bar{\alpha}\psi}; [\beta x^A + (1-\beta)x^I][1 + 8\bar{\alpha}\psi x^I\gamma]\}$

Consider now a leading incumbent:

- Let $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}\psi}$.
  Then $x_1^* = \beta x^A + (1-\beta)x^I$ if $\gamma > \frac{1+4\bar{\alpha}\psi[(\beta x^A+(1-\beta)x^I)(4\alpha\psi x^I-1)]}{(4\bar{\alpha}\psi)^2[x^I(\beta x^A+(1-\beta)x^I)]}$, and $x_1^* = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi x^I(1-\gamma)]$ otherwise[10]

- Let $\beta x^A + (1-\beta)x^I < \frac{1}{4\bar{\alpha}\psi}$, then $x_1^* = [\beta x^A + (1-\beta)x^I][1 - 8\alpha\psi x^I(1-\gamma)]$

Proposition 4: *For all $x^I \geq 0$, there exist non-measure zero sets $\Gamma(x^I)$ and $B(x^I)$ such that if $\gamma \in \Gamma(x^I)$ and $\beta \in B(x^I)$ then dissent by an extreme ally occurs in equilibrium*

Proof. I proceed as in the proof of Proposition 1. Suppose that $4\bar{\alpha}x^I(2\gamma - 1) < \delta < 4\bar{\alpha}x^I$ and $\gamma > \frac{1}{2}$, and conjecture the existence of an equilibrium in which the ally chooses to dissent. We must consider three cases:

1. $(x_1^*|D = 1) = \beta x^A + (1-\beta)x^I$ and $(x_1^*|D = 0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\bar{\alpha}}\psi(1-\gamma)x^I]$

2. $(x_1^*|D = 1) = \frac{1}{4\bar{\alpha}\psi}$ and $(x_1^*|D = 0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\bar{\alpha}}\psi(1-\gamma)x^I]$

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[10]When $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}\psi}$ the leading incumbent's overall utility as a function of the first period policy has two maxima: one at $\beta x^A + (1-\beta)x^I$ and a second at $[\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi x^I(1-\gamma)]$. The condition on γ identifies which one of the two is the global maximum.

3. $(x_1^*|D=1) = [\beta x^A + (1-\beta)x^I][1 + 8\bar{\alpha}\psi\gamma x^I]$ and $(x_1^*|D=0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi(1-\gamma)x^I]$

I will analyse each of the three cases separately.

Case 1: $(x_1^*|D=1) = \beta x^A + (1-\beta)x^I$, $(x_1^*|D=0) = (\beta x^A + (1-\beta)x^I)(1 - 8\alpha\psi x^I(1-\gamma))$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{62}$$

$$\beta \geq \frac{1 - 4\bar{\alpha}\psi x^I}{4\alpha\psi(x^A - x^I)} \tag{63}$$

$$\gamma < \frac{1 + 4\bar{\alpha}\psi((\beta x^A + (1-\beta)x^I)(4\bar{\alpha}\psi x^I - 1))}{(4\bar{\alpha}\psi)^2 x^I(\beta x^A + (1-\beta)x^I)} \tag{64}$$

Additionally, the equilibrium condition for the faction is

$$-(\beta x^A + (1-\beta)x^I - x^A)^2 - \gamma(x^I - x^A)^2 \tag{65}$$

$$-(1-\gamma)(x^I + x^A)^2 > -[(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma)) - x^A]^2$$

$$-[1 - 4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I - x^A)^2$$

$$-[4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I + x^A)^2$$

Let $x^D = \beta x^A + (1-\beta)x^I$ and $x^D - \Delta = (\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$ where $\Delta = (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)$. The above reduces to

$$-\Delta^2 + 2\Delta(x^D - x^A) + 4x^I x^A(1-\gamma) - 16\bar{\alpha}\psi(1-\gamma)x^I x^A(x^D - \Delta) < 0 \tag{66}$$

Substituting $\Delta = (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)$ and dividing for $4x^I(1-\gamma)$ gives

$$-x^I(4\bar{\alpha}\psi)^2(1-\gamma)(\beta x^A + (1-\beta)x^I)^2 + 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(x^D - x^A) \tag{67}$$

$$+x^A - 4\bar{\alpha}\psi x^A(x^D - (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)) < 0$$

Substituting $x^D = \beta x^A + (1-\beta)x^I$ and solving for γ gives us condition:

$$\gamma > 1 + \frac{x^A - 4\bar{\alpha}\psi[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}{(4\bar{\alpha}\psi)^2 x^I[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]} \tag{68}$$

Thus, the conjectured equilibrium exist if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, 1 + \frac{x^A - 4\bar{\alpha}\psi[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}{(4\bar{\alpha}\psi)^2 x^I[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}\} < \gamma < \frac{1 + 4\bar{\alpha}\psi((\beta x^A + (1-\beta)x^I)(4\bar{\alpha}\psi x^I - 1))}{(4\bar{\alpha}\psi)^2 x^I(\beta x^A + (1-\beta)x^I)} = \overline{\gamma}$

2. $\underline{\beta} = \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \leq \beta < min \quad \{1, \frac{1 + 4\bar{\alpha}\psi x^I(2\bar{\alpha}\psi x^I - 1)}{4\bar{\alpha}\psi(x^A - x^I)(1 - 2\bar{\alpha}\psi x^I)}\} = \overline{\beta}$

3. $x^A > \frac{1}{4\bar{\alpha}\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x^A ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 2: $(x_1^*|D = 1) = \frac{1}{4\bar{\alpha}\psi}$, $(x_1^*|D = 0) = (\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma)$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{69}$$

$$\beta < \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \tag{70}$$

$$\gamma > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)} - 1) \tag{71}$$

Additionally, the equilibrium condition for the faction is

$$-(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 > \tag{72}$$

$$-[(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma)) - x^A]^2$$

$$-[1 - 4\bar{\alpha}\psi(1 - \gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma))](x^I - x^A)^2$$

$$-[4\bar{\alpha}\psi(1 - \gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma))](x^I + x^A)^2$$

Let $I = 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$. The above can be rewritten as:

$$-(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 > \tag{73}$$

$$-(\frac{I}{4\bar{\alpha}\psi} - x^A)^2 - (1 - I(1-\gamma))(x^I - x^A)^2 - I(1-\gamma)(x^I + x^A)^2$$

Which reduces to

$$(1-I)(\frac{x^A}{2\bar{\alpha}\psi} - 4x^I x^A(1-\gamma) - \frac{1+I}{(4\bar{\alpha}\psi)^2}) > 0 \tag{74}$$

By substituting $I = 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$ and solving for γ we get condition:

$$1 + \frac{-1 + 4\bar{\alpha}\psi(2x^A - x^I - \beta(x^A - x^I))}{-2(4\bar{\alpha}\psi)^2 x^I(2x^A - x^I - \beta(x^A - x^I))} < \gamma < 1 \tag{75}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, 1 + \frac{-1 + 4\bar{\alpha}\psi(2x^A - x^I - \beta(x^A - x^I))}{-2(4\bar{\alpha}\psi)^2 x^I(2x^A - x^I - \beta(x^A - x^I))}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)} - 1)\} < \gamma < 1 = \overline{\gamma}$

2. $\underline{\beta} = max \quad \{0, \frac{1 - 4\bar{\alpha}\psi x^I - 2(4\bar{\alpha}\psi x^I)^2}{4\bar{\alpha}\psi(x^A - x^I)(8\bar{\alpha}\psi x^I + 1)}\} < \beta < \overline{\beta} = min \quad \{\frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)}, \frac{4\bar{\alpha}\psi(2x^A - x^I) - 1}{4\bar{\alpha}\psi(x^A - x^I)}\}$

3. $x^A > \underline{x^A} = max \quad \{\frac{1 + 4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi}, \frac{1 + 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(1 + 8\bar{\alpha}\psi x^I)}\}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x^A ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 3: $(x_1^*|D=1) = (\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)$, $(x_1^*|D=0) = (\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{76}$$

$$\beta < \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \tag{77}$$

$$\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)} - 1) \tag{78}$$

Additionally, the equilibrium condition for the faction is

$$-[(\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi x^I \gamma)) \tag{79}$$

$$-x^A]^2 - [1 - 4\bar{\alpha}\psi\gamma(\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi x^I \gamma)](x^I + x^A)^2$$

$$-[4\alpha\psi\gamma(\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi x^I \gamma)](x^I - x^A)^2 >$$

$$-[(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma)) - x^A]^2$$

$$-[1 - 4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I - x^A)^2$$

$$-[4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I + x^A)^2$$

Let $x^D = (\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi x^I \gamma)$. We can rewrite the above as:

$$-(x^D - x^A)^2 - (1 - 4\bar{\alpha}\psi x^D \gamma)(x^I + x^A)^2 - 4\bar{\alpha}\psi x^D \gamma)(x^I - x^A)^2 > \tag{80}$$

$$-(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I) - x^A)^2$$

$$-(1 - 4\bar{\alpha}\psi(1-\gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I))(x^I - x^A)^2$$

$$-4\bar{\alpha}\psi(1-\gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I)(x^I + x^A)^2$$

Which reduces to

$$-4x^I x^A + 16\bar{\alpha}\psi x^D x^I x^A \gamma > \tag{81}$$

$$-(8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I))^2 + 16\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I)(x^D - x^A)$$

$$-16\bar{\alpha}\psi x^I x^A(1-\gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I))$$

By substituting $x^D = (\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and solving for γ we obtain condition:

$$\gamma > \frac{x^A + 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 4\bar{\alpha}\psi x^I)(\beta x^A + (1-\beta)x^I - 2x^A)}{2x^I(4\bar{\alpha}\psi)^2(\beta x^A + (1-\beta)x^I)(-\beta x^A - (1-\beta)x^I + 2x^A)} \tag{82}$$

Thus the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{x^A + 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 4\bar{\alpha}\psi x^I)(\beta x^A + (1-\beta)x^I - 2x^A)}{2x^I(4\bar{\alpha}\psi)^2(\beta x^A + (1-\beta)x^I)(-\beta x^A - (1-\beta)x^I + 2x^A)}\} < \gamma <$

$min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A+(1-\beta)x^I)} - 1)\} = \bar{\gamma}$

2. $\underline{\beta} = max \quad \{0, 1-\frac{1}{2}\sqrt{\frac{x^I(4\bar{\alpha}\psi x^A)^2+4\bar{\alpha}\psi(x^A)^2-x^A}{\bar{\alpha}\psi(x^A-x^I)^2(1+4\bar{\alpha}\psi x^I)}}\} < \beta < min \quad \{\frac{1+2x^I(4\bar{\alpha}\psi)^2(x^A-x^I)-\sqrt{1+4(4x^A x^I \bar{\alpha}\psi)^2(4\bar{\alpha}\psi)^2}}{32\bar{\alpha}^2\psi^2 x^I(x^A-x^I)},$

   $\frac{1-4\bar{\alpha}\psi x^I(1+4\bar{\alpha}\psi x^I)}{4\bar{\alpha}\psi(x^A-x^I)(1+4\bar{\alpha}\psi x^I)}\} = \bar{\beta}$

3. $x^A > max \quad \{\frac{1}{4\bar{\alpha}\psi(1+4\bar{\alpha}\psi x^I)}, \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2}, \frac{1+4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(1+8\bar{\alpha}\psi x^I)}\}$

4. $x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \bar{\gamma}]$ exists. The conditions on x^A and x^I ensure that the range $[\underline{\beta}, \bar{\beta}]$ exists. $\qquad \square$

Corollary 1: *Suppose that $\frac{1}{8\bar{\alpha}\psi} < x^I$ and $\frac{1}{4\bar{\alpha}\psi} < x^A < \frac{1}{4\bar{\alpha}\psi(1-2\bar{\alpha}\psi x^I)}$. Then, for all $\beta \in [0,1)$, there exists a non-measure zero set $\Gamma(\beta)$ such that if $\gamma \in \Gamma(\beta)$, then dissent by an extreme ally occurs in equilibrium*

Proof. From an analysis of the cases above we can verify that sufficient conditions for the claim (for all $\beta \in [0,1)$, there exists a non-measure zero set $\Gamma(\beta)$) to hold are:

- The binding upper bound $\bar{\beta}$ in case 1 is $= 1$

- The binding lower bound $\underline{\beta}$ in case 2 is $= 0$

- The binding upper bound $\bar{\beta}$ in case 2 is $= \frac{1-4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A-x^I)}$ (which is also the lower bound from case 1)

For the three conditions to be satisfied we need:

- $\frac{1}{4\bar{\alpha}\psi} < x^A < \frac{1}{4\bar{\alpha}\psi(1-2\bar{\alpha}\psi x^I)}$

- $x^I > \frac{1}{8\bar{\alpha}\psi}$

$\qquad \square$

Appendix B: Dissent by a Moderate Ally

In this section I consider an ally whose bliss point is to the left of the incumbent: $0 < x^A < x^I$. In line with the rest of the paper, I maintain the assumption that $x^I < \frac{1}{4\bar{\alpha}\psi}$.

Proposition 1A: *There exist $\underline{\gamma}_m$, $\overline{\gamma}_m$, $\overline{x^A}_m$ and $\overline{x^I}_m$ such that the incumbent's moderate ally chooses to dissent in equilibrium if and only if:*

1. *The party is trailing, but its disadvantage is not too large*

   $(\underline{\gamma}_m < \gamma < \overline{\gamma}_m$, where $\overline{\gamma}_m \leq \frac{1}{2})$

2. *Electoral cost of dissent sufficiently large to turn trailing incumbent into sure loser $(\delta \geq \bar{\delta})$*

3. *Both the incumbent and his ally are sufficiently moderate*

   $(x^I < \overline{x^I}_m$ and $x^A < \overline{x^A}_m)$

Proof. The proof of the first point (incumbent must be trailing) is omitted, since it is obtained by applying the same logic used in proving Proposition 1 (i.e. it is easy to verify given the calculations in the proof of Proposition 1 that dissent never emerges if $x^A < x^I$ and $\gamma > \frac{1}{2}$). To prove the remainder of the proposition I must analyse all possible pairs of equilibrium policies. From above we know that in any equilibrium in which the ally chooses to dissent $(x_1^* | D = 1) = x^I$, and that dissent never occurs in equilibrium if $x^I \geq \frac{1}{4\bar{\alpha}\psi}$. Therefore, we must consider two cases:

- $(x_1^* | D = 1) = x^I$ and $(x_1^* | D = 0) = \frac{1}{4\bar{\alpha}\psi}$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$

- $(x_1^* | D = 1) = x^I$ and $(x_1^* | D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$, when $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$

I will analyse each case separately, conjecturing the existence of an equilibrium in which the ally chooses to dissent.

Case 1: $(x_1^* | D = 1) = x^I$ and $(x_1^* | D = 0) = \frac{1}{4\bar{\alpha}\psi}$

The equilibrium conditions for the incumbent are

$$\gamma \leq \frac{1}{2} \tag{83}$$

$$\gamma > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \tag{84}$$

Additionally, the equilibrium condition for the faction is

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > -(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 \tag{85}$$

Which reduces to

$$\gamma < \frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} \tag{86}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) < \gamma < min \quad \{\frac{1}{2}, \frac{-8\bar{\alpha}\psi x^A(1-4\bar{\alpha}\psi x^I)-(4\bar{\alpha}\psi x^I)^2+1}{(8\bar{\alpha}\psi)^2 x^I x^A}\}$

2. $\frac{\sqrt{5}-1}{8\bar{\alpha}\psi} < x^I < \frac{1}{4\bar{\alpha}\psi}$

3. $x^A < \frac{x^I}{2}$

The conditions on x^I and x^A ensure that the range $[\underline{\gamma}, \bar{\gamma}]$ exists.

Case 2: $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$

The equilibrium conditions for the incumbent are

$$\gamma \leq \frac{1}{2} \tag{87}$$

$$\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \tag{88}$$

$$\tag{89}$$

Which requires

$$x^I < \frac{1}{4\bar{\alpha}\psi} \tag{90}$$

Additionally, the equilibrium condition for the faction is

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > \tag{91}$$

$$-(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 - (1 - 4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(x^I + x^A)^2$$

$$-4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - x^A)^2$$

Which reduces to

$$x^A < \frac{x^I}{2} \tag{92}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $0 < \gamma < min \quad \{\frac{1}{2}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\}$

2. $x^A < \frac{x^I}{2}$

3. $x^I < \frac{1}{4\bar{\alpha}\psi}$

This concludes the proof. □