With Friends Like These, Who Needs Enemies?

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Abstract

Why are political leaders often attacked by their ideological allies, even when this is electorally costly? The paper addresses this puzzle by presenting a model in which the the leader and his allies face a preference conflict, and voters are learning about their own policy preferences over time. Here, by dissenting against the incumbent (and thereby harming the party in the upcoming election), the allies can change his incentives to choose more or less extreme policies, which affects the amount of voter learning. This induces a trade-off between winning the current election and inducing the party leadership to pursue the allies? all-things-considered more-preferred policy. Optimally balancing this trade-off sometimes involves active dissent that damages the party in the short-run. In equilibrium dissent arises *precisely because* it is electorally costly, in order to induce a policy response by the leader.

“*(He) has no idea how to conduct himself as a leader.*”

— Peter Mandelson referring to UK Labour party leader Jeremy Corbyn

“*(He) shows a growing inability, and even unwillingness, to separate truth from lies*”

— John McCain referring to US President Donald Trump

“*(He) is worse than the Devil.*”

— Massimo D'Alema referring to Italian Prime Minister Matteo Renzi

These quotes, drawn from media interviews given by prominent politicians, show that political leaders often experience public attacks aimed at damaging their reputation. This is not surprising: this form of character assassination is part of the political and electoral game. What is instead surprising is that the authors of such public attacks are members of the leaders' *own* parties: a leader's fiercest critics are often his own co-partisans. Interestingly, we also observe similar behavior by a leader's allies outside the party: for instance, the media themselves sometimes denigrate political leaders with whom they are ideologically aligned. A prime example is the critical stance adopted by the right-leaning Evening Standard towards former UK Conservative prime minister Theresa May, whose cabinet it depicted as ‘stale’ and ‘enfeebled’ (Urwin 2017).

These and other examples thus show that political leaders are often publicly attacked by their own ideological allies, within or outside the party. Unsurprisingly, empirical evidence suggests that this form of ‘friendly fire’ typically damages a party's electoral performance. Voters dislike parties that appear divided, therefore such open manifestations of dissent tend to come with an electoral cost (Greene and Haber 2016; YouGov 2016; Kam 2009; Groeling 2010). When they choose to publicly attack the leader, his ideological allies are therefore also hurting themselves (by hurting their own party's electoral chances). Yet, in this paper I show that this form of dissent may emerge not *despite* the associated electoral cost, but rather precisely *because of it*.

Despite coming from the same side of the political spectrum, a leader and his allies often face an ideological conflict: they do not share exactly the same policy preferences. The leader makes policy choices, as function of both his ideological preferences and his re-election incentives. The allies have no way to influence the leader's ideological tastes, but they can try to manipulate the incentives coming from the electoral environment. Some policy choices are in fact inherently riskier than others. Depending on his electoral prospects, the leader will have incentives to either take risky choices, or avoid policy gambles. By publicly attacking the leader, and hence *endogenously manipulating* his electoral chances, the allies thus potentially change his policy choice. This generates a trade-off for the allies, between maximizing the probability that the leader wins the upcoming elections and inducing him to adopt a policy more in line with their own ideological preferences. If the gain from changing today's policy is sufficiently large, costly dissent emerges in equilibrium. Dissent thus serves the purpose of mitigating the ideological conflict between the leader and his allies, and should not be interpreted as a first step towards a party split. The contribution of this paper is to micro-found this intuition by presenting a theory of electorally costly dissent that investigates how, why, and when political actors may choose to damage the electoral prospects of their preferred party, in order to induce a policy response.

The model analyzes the strategic interaction between an incumbent, his ideological ally, a challenger and a representative voter. The ally chooses whether to publicly dissent against the incumbent. Dissent is electorally costly, in the sense that it generates an *endogenous valence shock* against the incumbent. Following the ally's move, the incumbent chooses which policy to implement. For the incumbent, some policy choices entail riskier experiments than others. In particular, I propose a new framework to think about policy experimentation, which allows me to think about experimentation in connection to ideology and thus investigate how the ideological conflict between the incumbent and his ally influences the likelihood of dissent. Within this framework, voters faces uncertainty over which policy, and therefore which candidate, is best for them. We can think about this uncertainty as pertaining to the consequences of the various policy choices: a voter may be unsure of how different policies map into outcomes, or face uncertainty about the impact that certain policies/outcomes will have on her own welfare. Faced with this uncertainty, the voters try to learn about their own preferences by observing how much they like or dislike the outcome of today's policy. However, this outcome is also a function of a random shock, which complicates their inference problem. In this world, the amount of learning depends on the exact location of the implemented policy along the left-right spectrum. In particular, the analysis shows that the more extreme the implemented policy is, the more a Bayesian voter learns upon observing the resulting outcome. Thus, extreme policies represent riskier experiments for the office holder: they increase the likelihood that the voter discovers her true preferences, which may or may not turn out to be aligned with the incumbent's ideological stances.

Within this setting, the incumbent has incentives to control information. His equilibrium policy choice thus maximizes the trade-off between implementing his preferred policy today and generating the optimal amount of information in order to be re-elected tomorrow. This, I show, is a function of the his ex-ante electoral strength. A leading incumbent, who is going to be re-elected even if the voters receive no new information, has incentives to implement moderate platforms that prevent information generation. A trailing one will instead want to engage in more extreme policies that allow voters to learn, in hopes of improving his electoral prospects. Finally, an incumbent who can never be re-elected (irrespective of what the voters learn) has no interest in controlling information, and simply follows his ideological preferences.

Turn now to the allies' choice. By dissenting, the allies generate a negative valence shock against the incumbent, thereby reducing his ex-ante electoral strength. This, in turns, creates incentives to implement more or less informative (i.e. extreme) policies. As such, dissent changes the incumbent's equilibrium policy choice, while also harming the party electorally. This generates a potential trade-off for the incumbent's allies, between ensuring that their preferred party wins the upcoming election and inducing the incumbent to implement a policy more in line with their own ideological preferences. Optimally balancing this trade-off sometimes involves active dissent that damages the party's electoral chances. Surprisingly, the analysis reveals that intra-party polarization plays an ambiguous role: reducing the intensity of the incumbent's ideological conflict with his allies may actually make dissent more likely to emerge. Further, improving the incumbent's electoral prospects will sometimes generate more dissent.

The results also highlight that the presence of an extreme ally to the incumbent party may be welfare improving for the voters. Voters benefit from informative policies being implemented as this increases the probability of making the correct electoral decision in the future. However, under some conditions, electoral accountability has the perverse consequence of inducing lower levels of policy experimentation relative to both the incumbent's ideological preferences and the voter's optimum. By dissenting the ally can incentivize the incumbent to implement extreme policies that allow the voters to learn, and thus mitigate this inefficiency.

Finally, the results have important implications for empirical research on the topic. Existing estimates of the electoral rewards of party unity, obtained by comparing treated and control units (parties that do and do not experience dissent), are inevitably biased. Furthermore, the bias may go in either direction. However, this does not imply that the model is not falsifiable. The model generates testable predictions regarding parties' electoral performance conditional on experiencing dissent: it should be positively correlated with variables such as the level of education, news media consumption and political engagement in the electorate. Focusing on the treated units, researchers can thus empirically identify the conditions under which dissent is expected to hurt parties the most.

Related Literature and Competing Explanations

This paper relates first and foremost to the formal literature on intra-party politics. Typically, the interaction between different factions is described as a bargaining game. For example, Mutlu-Eren (2015) considers how the threat of a split influences the party's behaviour in the legislature. Similarly, Hortala-Vallve and Mueller (2015) analyze how the threat of defection by the minority can induce the party leadership to democratize the candidates' selection process. In these papers, the threat is credible when the defecting faction is sufficiently likely to win the upcoming election if running alone after a split. Similar references to intra-party bargaining are also recurrent in the empirical literature (Sartori 1976, Belloni 1976, Budge et al. 2010).

Yet, this approach has some issues when we consider dissent that manifests as public attacks against the leader, rather than as formal defections. In a bargaining game dissent is used as a threat, to be executed *after* the incumbent has made his policy choice. However, at this point a public attack against the leader has no effect but to reduce the probability that the party wins the upcoming election. This strictly decreases both the leader's and the ally's expected payoff. As such, the threat can never be credible and we should never observe this form of dissent in equilibrium. Further, even beyond the issue of credibility, in a bargaining game the materialization of the threat typically lies off the equilibrium path. Hence, this is arguably not an appropriate framework to understand why political parties so often experience such public manifestations of dissent.

A second strand of literature considers politicians' incentives to pander to their constituencies (Carey and Shugart 1995, Buisseret and Prato 2018, Kirkland and Slapin 2019). Politicians may face a trade-off between the national party's electoral fortunes and their own success. In particular, this trade-off may emerge if a politician's local constituency is opposed to the national party line. Dissent may then serve the purpose of signaling the politician's misalignment with the leadership and alignment with the constituency.

This argument is intuitively appealing as well as empirically relevant. However, both anecdotal and systematic evidence seem to suggest that it does not fully capture the strategic incentives behind this phenomenon. First of all, according to this theory we should expect the individual dissenters to be in a relatively weak electoral position. However, Proksch and Slapin (2015) analyze data from the UK and Germany and show that, if anything, the opposite holds. Public attacks against the leadership are (weakly) more likely to come from members of parliament elected with a larger margin. Secondly, within this framework incentives to attack the party leadership should emerge under first past the post or open-list proportional electoral systems. In closed party-list systems, where the leader controls the list composition and as such the individual candidates electoral fate, incentives to dissent should be much weaker, if even present. Yet, Proksch and Slapin (2015) exploit the features of the German mixed-member proportional electoral system, and show that members elected with a party-list vote are as likely to dissent as those elected with a constituency vote. If we look across different countries, public manifestations of dissent emerge in majoritarian systems such as the UK, and proportional closed-list ones such as Italy. This calls for a theory that makes sense of this phenomenon even when the party members' individual electoral motives do not provide incentives (or worse, provide disincentives) to attack the leadership.

It also is important to note that the mechanism proposed here applies to dissent coming both from the leader's own co-partisans and from his ideological allies outside the party (such as media outlets, as discussed in the introduction). This is not the case for the explanation building on the trade-off between the dissenter's electoral incentives and the party's collective reputation.

Thus, I propose a substantially different type of model, in which dissent precedes rather than following the party leader's strategic choice (in contrast with a bargaining set-up), and emerges in order to induce a policy response (in contrast with a pandering set-up).

The core of the model is the assumption that different policies entail different levels of risk for the office holder, due to the fact that voters face uncertainty over her optimal choice and learn via experience. This connects the paper with the small but burgeoning research on learning and experimentation (Majumdar and Mukand 2004, Dewan and Hortala-Vallve 2018). I contribute to this literature by considering a setting in which the incumbent's incentives to gamble arise *endogenously* from his allies' strategic behaviour. Further, while most extant works assume that the electorate must learn about the incumbent's type, here the voters must discover their own policy preferences.

In this perspective, the paper is closely related to recent work by Callander (2011). The author considers a world in which players face uncertainty about how policies map into outcomes: they know the slope of the mapping function (the state of the world), but experiment to learn about the outcomes exact realization. In this paper I propose a different framework to think about policy experimentation, in which the nature of uncertainty is reversed: policy outcomes are always noisy, but the voter faces fundamental uncertainty over how the world works. This allows me to think about policy experimentation in connection to ideology, and generates the result that extreme policies, rather than small incremental changes as in Callander, produce more information. Additionally, Callander focuses on the *statically* optimal choice for a decision maker. He thus chooses to abstract from dynamic electoral considerations, by assuming either myopic players (Callander 2011) or exogenous re-election probabilities (Callander and Hummel 2014). Instead, the focus of this paper is precisely on the incumbent's *dynamic* incentives to control information.

The paper also relates to the literature on Bayesian Persuasion (see Austen-Smith 1998, Kamenica and Gentzkow 2011). In my model, as in the Bayesian Persuasion framework, the incumbent can engage in information control by manipulating the receiver's posterior distribution. In the Bayesian Persuasion framework the mechanism through which this happens is somewhat black-boxed. The key innovation of my paper is to explicitly model *how* this manipulation occurs, by looking at the impact that the *implemented policy* has on voter learning.

Finally, the normative findings speak to the literature on negative campaigning. This literature shows that, within the context of a signaling game, information about political candidates may be more credibly transmitted when each focuses on the opponent's flaws rather than on their own merit (Polborn and Yi 2006, Bhattacharya 2016). Thus, negative campaigns can be good for the voters. Here I uncover an analogous welfare improving effect of political attacks coming from a candidate's own allies, within the context of a policy experimentation framework.

The Model

I investigate the emergence of intra-party dissent within the context of a political agency model of elections. I therefore analyze the strategic interaction between an incumbent office holder (I), his ideological ally (A), a challenger (C), and a representative voter (V).[1] The most intuitive application is one where the ally represents a minority faction within the incumbent party. However, for the purpose of the model, the ally can be any actor whose ideological preferences are closer to the incumbent's than the challenger's (a media outlet, an interest group, a trade union etc.). The game begins with the incumbent's ally choosing whether to dissent. We can interpret the choice to dissent as taking the form of a 'character assassination', with the ally publicly attacking the incumbent. Alternatively, dissent can entail a public manifestation of disagreement, with the ally openly criticizing the party line. Dissent is *public* and therefore observed by both the incumbent and the voter. The second stage of the game is the policy-making one: the incumbent implements a policy $x_1 \in \mathbb{R}$. Upon observing the realization of her first-period payoff the voter then makes her

[1]The set-up can also be applied to understand dissent within the challenger party. The mechanism and insights are analogous to what described below, with the caveat that the challenger's ally would strategically manipulate his valence in order to induce a policy response by the office holder.

electoral decision, choosing whether to retain the incumbent I or replace him with the challenger C. Finally, the second period office holder selects a new policy x_2 to be implemented.

The key feature of the model is that the voter faces uncertainty over her own policy preferences: she is unsure of which policy is best for her.[2] We can interpret this uncertainty as pertaining to the possible consequences of the various policy choices. The voter may be unsure of how different policies map into outcomes (the unknown parameter being the slope of the mapping function), or face uncertainty about the impact that certain policies/outcomes will have on her own welfare. Formally, I assume that the voter's true bliss point x^v is unknown, and can take one of two values: $x^v \in \{\underline{\alpha}, \bar{\alpha}\}$.[3] For simplicity, but without loss of generality, I assume $\underline{\alpha} = -\bar{\alpha} < 0$. Since the focus of the paper is on the incumbent's incentives to take risks in policy making, the model features no asymmetry of information: x^v is ex-ante unknown to all players, that share a common prior assigning probability γ to the voter's ideal policy being a right-wing one ($\gamma = \text{prob}(x^v = \bar{\alpha})$).

Given this symmetric uncertainty, the only way that the voter can learn is via experience. She observes the consequences of the first period policy, that is how much she liked (or disliked) the implemented x_1, and tries to infer the true value of x^v by using Bayes' rule.[4]

Formally, the voter's payoff realization is a signal of the state of the world x^v:

$$U_t^v = -(x^v - x_t)^2 + \epsilon_t - \mathbb{I}\delta \tag{1}$$
$$\epsilon_t \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$$

As (1) highlights, the voter's payoff has three components. First, a policy component, function of her true bliss point and the implemented platform. Second, a period-specific random shock or noise term ϵ_t, that complicates her inference problem.[5] And finally, a valence component $-\mathbb{I}\delta$.

[2] We can think about this voter as representing the group of swing voters in the population.

[3] The results do not require a binary state space. For example the model's conclusions would be virtually unchanged if $x^v \sim U[\underline{\alpha}, \bar{\alpha}]$.

[4] Whether the other players also learn about x^v is inconsequential for the equilibrium results.

[5] The assumption that $\epsilon \sim U$ is not necessary for the results.

Here, valence is a function of dissent: I assume that $\delta > 0$, $\mathbb{I} = 1$ if the ally choose to dissent and the incumbent is re-elected, and $\mathbb{I} = 0$ otherwise. In other words, building on the observation that voters tend to dislike parties that appear divided, I assume that the ally's dissent *endogenously* generates a negative *valence shock* against the incumbent. In order to simplify the analysis and presentation of the results, I leave this shock back-boxed and discuss possible micro-foundations in a separate section.

Finally, the incumbent, his ally and the challenger are policy motivated, and their bliss points are common knowledge:[6]

$$U_t^i = -(x^i - x_t)^2 \quad \forall i \in \{I, A, C\} \tag{2}$$

As I will discuss in further details below, the assumption that politicians do not attach any value to holding office per se is not necessary for the results, and it is imposed solely for the purpose of isolating the impact of ideological conflict on dissent.

Without loss of generality, I will consider a right-wing incumbent running against a left-wing challenger: $x^C \leq 0 \leq x^I$. For simplicity, the candidates' bliss points are symmetric around 0: $x^I = -x^C \geq 0$.

While the incumbent and his ally face a preference conflict ($x^I \neq x^A$) they come from the same side of the ideological spectrum: the ally's preferences are always closer to the incumbent's than to the challenger's

$$|x^A - x^I| < |x^A - x^C| \tag{3}$$

To sum up, the interaction unfolds as follows:

1. Nature determines the value of $x^v \in \{\underline{\alpha}, \bar{\alpha}\}$, which remains unknown to all players
2. A chooses whether to dissent: $D \in \{0, 1\}$
3. I implements a policy $x_1 \in \mathbb{R}$

[6] The key insights of the paper would (under appropriate parameter choices) survive if the incumbent's preferences are his private information.

4. V's first-period payoff U_1^v realizes

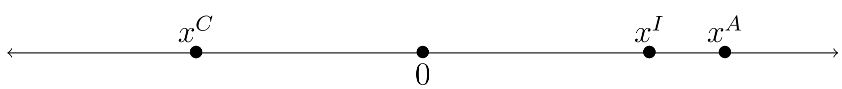
5. V chooses whether to re-elect I or replace him with C

6. The second-period office holder implements policy $x_2 \in \mathbb{R}$

7. Second-period payoffs realize and the game ends

The equilibrium concept is Perfect Bayesian Equilibrium. In order to avoid trivial results, I assume that when indifferent the ally chooses not to dissent. This is formally equivalent to assuming an infinitely small material cost of dissenting.

In the main body of the paper I will focus on an extreme ally ($x^A > x^I$).



In Appendix C, I show that within this framework dissent can emerge even when the ally is more moderate than the incumbent, and identify the conditions under which this occurs in equilibrium.

Equilibrium Analysis

As usual, we proceed by backwards induction. The second period is equivalent to a one-shot game: the office holder faces no electoral pressures, and will always implement his preferred platform. The voter therefore faces a selection problem, which is complicated by the fact that she is unsure of which policy is best for her. Further, recall that the voter's payoff from re-electing the incumbent has a policy component as well as an (endogenous) valence one. Her electoral choice will therefore be a function of two elements: the (posterior) belief that her own ideal policy is aligned with the incumbent's, and the presence or absence of dissent within the incumbent party. Specifically, denote μ the voter's posterior belief that her own bliss point takes a positive value ($\mu = prob(x^v = \bar{\alpha})$). In any PBE she chooses to re-elect the right-wing incumbent if and only if:

$$\mu > \frac{\mathbb{I}\delta + 4\bar{\alpha}x^I}{8\bar{\alpha}x^I} \tag{4}$$

Absent dissent and the resulting negative valence shock ($\mathbb{I} = 0$), the incumbent is always re-elected as long as the voter believes that her own preferences are more likely to be aligned with his than the challenger's ($\mu > \frac{1}{2}$). If instead the incumbent does experience dissent, the voter's posterior needs to be sufficiently high so as to offset the valence cost δ.

Learning and Experimentation

Consider now the voter's inference problem. The voter tries to learn about her true policy preferences (i.e., the value of x^v) by observing how much she liked or disliked the (consequences of) the platform implemented in the first period. Her inference problem is complicated by the presence of the random shock ϵ_t, that influences her payoff realization. This has a crucial impact on her learning process: the amount of information that she will be able to obtain depends on the exact location of the policy that was implemented. Specifically, the analysis shows that the voter learns more when extreme policies are enacted. This is a consequence of two factors. First, extreme policies are more likely to produce extreme (i.e., very good or very bad) outcomes, to which a Bayesian voter correctly attributes larger informative value. Secondly, as the implemented policy moves to the extreme, the distance in expected outcomes as a function of the true state (i.e., the voter's expected payoff under the two possible values of x^v) increases. As a consequence, the same outcome can convey more information to the voter if it results from a more extreme policy.

As the following Lemma highlights, this key feature of the learning process emerges in a stark way in a world in which ϵ_t is uniformly distributed:

Lemma 1: *The voter's learning satisfies the following properties:*

(i) Her posterior μ takes one of three values: $\mu \in \{0, \gamma, 1\}$;

(ii) The amount of learning is a function of the implemented policy x_1: the more extreme the policy, the higher the probability that $\mu \neq \gamma$;

(iii) There exists a policy x' such that if $|x_1| \geq |x'|$, then $\mu \neq \gamma$ with probability 1.

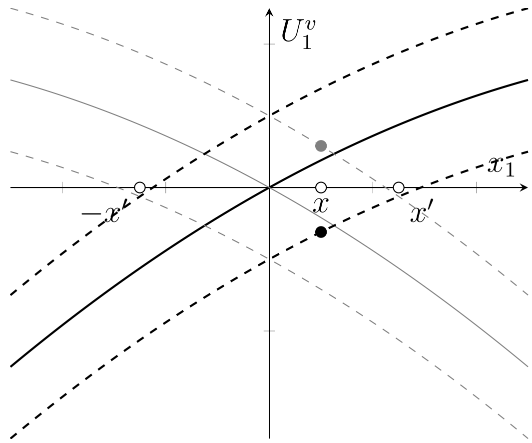


Figure 1: Voter's first-period payoff. The thick increasing (thin decreasing) curves represent the case in which $x_V = \overline{\alpha}$ ($x_V = \underline{\alpha}$). The solid curves represent the voter's expected payoff, while the dashed ones represent the maximum and minimum possible realizations given ϵ_t.

The voter either learns everything or nothing. Further, the probability that the voter discovers her true preferences increases as the implemented policy becomes more extreme. While a formal proof of this Lemma is presented in Appendix A, the underlying reasoning is easy to illustrate graphically.

In Figure 1, the solid lines represent the voter's *expected* first-period payoff as a function of the implemented policy x_1, for the two possible values of x^v.[7] The *realization* of the payoff will however also be a function of the shock ϵ_t. The dashed curves thus represent the maximum and minimum possible values of the payoff realization when we take the shock into account.[8]

As Figure 1 shows, the voter's payoff is, *in expectation*, always different under the two states of the world (for any $x_1 \neq 0$). However, the presence of the random shock creates a partial overlap in the support of the payoff *realization*. For any given policy $x_1 \in (-x', x')$, there exists a range of payoffs that may realize (i.e. be actually observed) whether the voter's true bliss point takes a positive or a negative value. Consider, for example, policy x as represented in the graph. Any payoff realization falling between the gray and black bullets may be observed with positive probability under both states of the world. Clearly, if the payoff realization falls outside this

[7]The thick increasing solid curve is $-(x_1 - \bar{\alpha})^2$ and the thin decreasing solid curve is $-(x_1 - \underline{\alpha})^2$.

[8]The thick increasing dashed curves are $-(x_1 - \bar{\alpha})^2 + \frac{1}{2\psi}$ and $-(x_1 - \bar{\alpha})^2 - \frac{1}{2\psi}$. Conversely, the thin decreasing dashed curves are $-(x_1 - \underline{\alpha})^2 + \frac{1}{2\psi}$ and $-(x_1 - \underline{\alpha})^2 - \frac{1}{2\psi}$.

range, it constitutes a fully informative signal. There is only one state of the world that could have generated that specific realization: the voter simply likes the policy too much, or too little, for this to be justified as a consequence of the shock. Thus, upon observing her payoff, the voter discovers her true preferences (i.e. the value of x^v). Conversely, any payoff realization that falls inside the range of overlap is completely uninformative. Since the shock is uniformly distributed, any such realization has exactly the same probability of being observed under the two states of the world. Thus, the voter learns nothing and must go back to her prior beliefs. As the implemented policy becomes more extreme, the gray and black bullets get closer and closer to each other. The range of overlap becomes smaller, and the voter is more likely to discover her true preferences.[9]

Let me highlight that this feature of the learning process (extreme policies are more informative), does not depend on the assumption that ϵ_t is uniformly distributed. Consider for example a world in which the shock is normally distributed with full support. The learning process would be much smoother: any outcome realization would be somewhat informative, but never fully so. However, it would still be the case that as the implemented policy becomes more extreme, the distance in the expected outcomes as a function of the state increases. This in turn increases each signal's informativeness. Extreme policies would therefore still generate more information. Nonetheless, in concluding this section it is also important to emphasize that the mechanism uncovered in this paper relies solely on the fact that the implemented policy influences the amount of information the voter receives. This is what allows a dissenting ally to influence the equilibrium policy. As such, the main insights of the paper would survive in a world in which more moderate (rather than more extreme) policies are more informative.

The Incumbent

Lemma 1 shows that, while the incumbent cannot control exactly which signal the voter will observe, he can determine the probability of such a signal being informative. In other words, the incumbent can manipulate the voter's posterior distribution. Since the voter's posterior determines

[9]Notice that the presence (or absence) of dissent and the resulting valence shock does not interfere with the voter's learning process: the valence shock simply shifts all the intercepts of the functions in Figure 1.

her electoral choice, the incumbent has an incentive to engage in information control: the first period equilibrium policy maximizes the incumbent's trade-off between implementing his bliss point today and generating the optimal amount of information in order to get re-elected tomorrow. The way that this trade-off is optimized, I show, depends on the incumbent's ex-ante electoral strength. Define a *leading* incumbent as one who is guaranteed re-election if the voter receives no new information (condition (4) is satisfied at $\mu = \gamma$). A *trailing* incumbent will instead be re-elected only if the voter updates in his favour ((4) fails at $\mu = \gamma$ but is satisfied at $\mu = 1$). Finally, a *certain loser* is replaced even if the voter observes favourable information ((4) fails at $\mu = 1$). The following Lemma holds:

Lemma 2: *In any PBE of the game*

- *A* ***certain loser*** *implements his bliss point*
  ($x_1^* = x^I$*)*
- *A* ***leading incumbent*** *implements a policy weakly more moderate than his bliss point*
  ($x_1^* \leq x^I$*)*
- *A* ***trailing incumbent*** *implements a policy weakly more extreme than his bliss point*
  ($x_1^* \geq x^I$*)*

Information revelation is risky. Even if information is more likely to favour the incumbent (i.e., $\gamma > \frac{1}{2}$), there is still a chance that the voter will instead learn that her own ideal policy is aligned with the challenger's (i.e., $x^v = \underline{\alpha}$). A leading incumbent has no reason to accept the risk since he is guaranteed re-election when the voter does not update. This incumbent therefore experiences *fear of failure*: he has incentives to prevent the voter from learning, and will always implement a policy that is (weakly) more moderate than his bliss point. On the contrary, a trailing incumbent can only be re-elected if the voter receives new information. No matter how small the probability of success, a trailing incumbent always wants to engage in policy experimentation, so as to generate as much information as possible and potentially improve its electoral prospects. Borrowing terminology from the IR literature (Downs and Rocke 1994), I say that this incumbent has incentives to *gamble for resurrection*, and therefore always implements a policy (weakly) more extreme than his bliss point. A certain loser trivially has no reason to engage in information control, and will always implement exactly his bliss point.[10] The exact policies adopted by a leading and a trailing incumbent are calculated in the Appendix, and are a function of x^I, γ, $|\alpha|$ and the variance of the noise ψ.

In the remainder of the paper I will assume that $x^I < x'$, where x' is the smallest (positive) policy that produces an informative signal with probability 1. The assumption is without loss of generality, and imposed in order to reduce the number of cases under consideration.

Dissent

Moving one step back, focus on the ally's strategic choice. First of all, I establish that in equilibrium dissent is always harmful for the party's expected electoral performance, even if the incumbent best responds by modifying his policy choice precisely to minimize this effect.

Lemma 3: *In equilibrium dissent always reduces the probability that the incumbent will be re-elected.*

Thus, by dissenting the ally reduces both his own and the incumbent's expected second period payoff. Nonetheless, he sometimes finds it optimal to dissent.

To understand this, recall that the incumbent's policy choice is determined by his willingness to take risks. This is, in turns, a function of his electoral strength (Lemma 2). Consider a leading incumbent. Absent dissent, he would implement a policy (weakly) more moderate than his bliss point, in order to reduce the probability that the voter updates her beliefs about her own true preferences. Suppose now that the ally chooses to dissent. If the resulting valence cost is sufficiently large, this turns the leading incumbent into a trailing one. This creates incentives for the incumbent to gamble on resurrection by engaging in extreme policies. Thus, electorally costly dissent would move the incumbent's equilibrium policy choice to the extreme, closer to the ally's

[10] The same would apply to an incumbent is always re-elected (for all μ). However, given the symmetry assumption, such a case never occurs.

own preferences. When the gain is sufficiently large relative to the cost of losing the upcoming election, the ally chooses to dissent in equilibrium. Proposition 1 identifies necessary and sufficient conditions for this to occur.[11]

Proposition 1: *There exist $\underline{\gamma}$, $\overline{\gamma}$, $\underline{x^A}$ and $\underline{x^I}$ such that the incumbent's extreme ally chooses to dissent if and only if:*

- *Absent dissent, the incumbent is leading, but his advantage is not too large*
  $\underline{\gamma} < \gamma < \overline{\gamma}$, *where* $\underline{\gamma} \geq \frac{1}{2}$
- *The electoral cost of dissent is sufficiently high that it turns the leading incumbent into a trailing one, but not so high that the incumbent loses for sure*
  $(2\gamma - 1)4\bar{\alpha}x^I \leq \delta < 4\bar{\alpha}x^I$
- *Both the incumbent and his ally are sufficiently extreme*
  $x^I > \underline{x^I}$ and $x^A > \underline{x^A} > x^I$

Intuition may suggest that dissent is more likely to materialize during periods of electoral crisis. The party is expected to perform poorly, and the ensuing internal turmoil degenerates into an open manifestation of conflict. The first result shows that, in the case of an extreme ally, the opposite is true. Suppose the incumbent is trailing even without experiencing dissent. Absent dissent, he will implement a policy that is weakly more extreme than his bliss point: he needs to generate information in order to be re-elected. Dissent either has no impact on his policy choice (if δ is so small that it does not affect the voter's electoral decision), or induces him to implement exactly his bliss point (if δ is sufficiently large to turn him into a sure loser). Hence, by dissenting the ally causes the incumbent to adopt a (weakly) more moderate policy, while also (weakly) reducing his own future expected payoff. Then, dissent is never observed in equilibrium. It is only when the incumbent is leading ($\gamma > \frac{1}{2}$) that the ally (potentially) gains from dissent by creating incentives to gamble for resurrection.

[11] The thresholds are a function of the other parameters in the model.

Consider now the conditions on the electoral cost of dissent, i.e. the size of the resulting valence shock δ. Intuitively, dissent never emerges when the cost is so large that it makes the incumbent lose for sure. In this scenario the expected loss would be maximized, while the gain for the extreme ally would be minimized. Recall that a sure loser has no reason to control information, and always implements exactly his bliss point. Thus, while dissent would be somewhat effective in modifying the equilibrium policy, it could not induce the incumbent to move beyond his bliss point. The policy gain would be too small for the incumbent's ally to be willing to pay the cost of condemning the party to electoral defeat. However, the analysis also reveals that for dissent to be observed, δ cannot be too small either. Recall that an incumbent is leading if the voter would choose to re-elect him upon receiving no new information. If δ is too small (relative to the prior γ), then the incumbent is still leading even after experiencing dissent. In this case, dissent has no effect on the voter's electoral choice and therefore no impact on the equilibrium policy. Trivially, the incumbent's ally has no reason to dissent in the first place. Thus, δ must be sufficiently large so as to turn a leading incumbent into a trailing one: dissent emerges precisely *because* it is electorally costly.

The first point in Proposition 1 also indicates that dissent requires γ to take an intermediate value. Recall that γ is the probability that the voter's true preferences are aligned with the incumbent's (i.e. $x^v = \overline{\alpha}$). As such, the higher γ, the less a leading incumbent fears information. When γ is too large dissent therefore has a very small impact on the equilibrium policy, and the ally has no reason to pay the associated electoral cost. Conversely, if γ is too small the probability that the party would win the election after experiencing dissent is too low (recall that a trailing incumbent is re-elected only if the voter updates in his favor). Dissent thus becomes too costly, and is never observed in equilibrium.

Finally, consider the impact of the ideological misalignment between the incumbent and his allies. Such misalignment represents the only source of conflict in the model. A naive observer may thus conclude that dissent should always be observed when the incumbent's and ally's ideological preferences are really far apart. Proposition 1 shows that this intuition needs to be qualified: when the incumbent is too moderate, his extremist ally will never choose to dissent against him. To

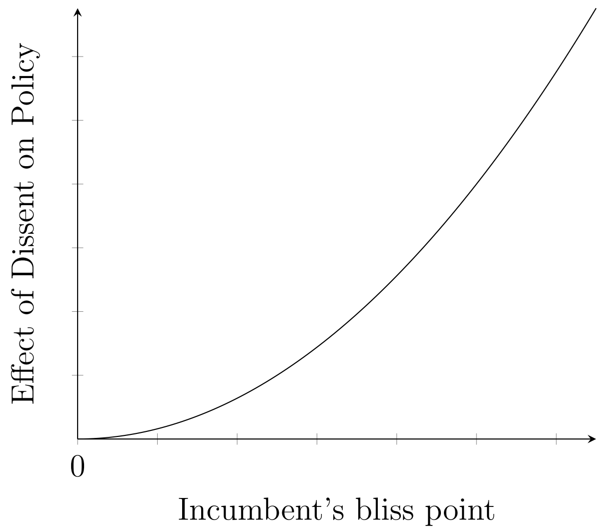


Figure 2: Effect of dissent on the equilibrium policy $((x_1^*|D=1) - (x_1^*|D=0))$

understand this result, recall that the ally dissents to generate incentives to gamble on resurrection. However, if the incumbent is too moderate, such incentives are too weak: gambling is too costly, and not very valuable. It is costly as it entails implementing extreme policies, potentially very far from the incumbent's bliss point. It is not very valuable since for a moderate incumbent the gain from winning the upcoming election is small (the distance from the opposition is small). Thus, as Figure 2 shows, the impact of dissent on the incumbent's choice is increasing in his bliss point. If the incumbent is too moderate dissent will have a very small effect on the equilibrium policy, which reduces the ally's incentives to dissent in the first place.

Proposition 2 further explores the ambiguous effect of intra-party polarization on the emergence of dissent by looking at the model's comparative statics:

Proposition 2:

- *The likelihood of observing dissent (weakly) increases as the ally becomes more extreme*
- *There exists a unique $\widehat{x^I}(x^A) > \underline{x^I}$ such that if $x^I < \widehat{x^I}(x^A)$, then the likelihood of observing dissent increases as the incumbent becomes more extreme*

When the ideological conflict within the party increases due to the incumbent's ally becoming more extreme, dissent always becomes more likely. The more extreme the ally is, the more he gains by moving the equilibrium policy closer to his bliss point. However, the same is not necessarily true when the ideological conflict deepens due to the incumbent becoming more moderate. As the incumbent becomes more extreme both a direct and indirect effects emerge. The direct effect is straightforward: the distance in the policy preferences of the incumbent and his ally decreases. This reduces the ally's incentives to dissent. As the above discussion highlights, the indirect effect instead goes in the opposite direction. If the incumbent's bliss point is sufficiently close to zero, this indirect effect dominates, and dissent is more likely to emerge as the ideological conflict decreases.

This result highlights the peculiar nature of dissent in this model. Far from representing the first step towards a party split, dissent here brings about unity. It serves the purpose of realigning the interests of the incumbent and his ally, thereby recomposing the existing ideological conflict. However, for dissent to be effective, such conflict cannot be too deep. From the point of view of the empirical researcher, this also suggests that analyses on the consequences of increasing intra-party polarization should adopt a specification that allows for this interaction (between change in polarization and change in bliss points).

Discussion

In this section I briefly investigate the robustness of the results, and discuss how the theory relates to alternative explanations.

Alternative assumptions. In the model, politicians do not attach any value to office per se. This assumption is imposed for presentation purposes, in order to isolate the impact of ideological disagreements on the likelihood of observing dissent. Office rents would in fact potentially introduce a second source of preference conflict between the incumbent and the ally. Suppose for example that the ally represents a minority faction within the party. Should the party win the upcoming election, the incumbent (leader of the majority faction) would arguably grab a larger share of the office rents relative to his ally. This potentially translates into different risk appetite in policy making, thereby increasing the conflict of (induced) preferences within the party. Hence (as long as the value of office is not too large) including office payoffs would strengthen the results, making dissent even easier to sustain in equilibrium.

A second assumption in the model is that the incumbent is essentially a policy dictator. His allies have no say over policy making, and dissent is therefore their only tool to influence the leader's choice. Are the results robust to relaxing this assumption? In Appendix B I address this question by analyzing an extension of the baseline model, in which (after the ally chooses whether to dissent) the first period policy is determined as the result of a bargaining process between the incumbent and the ally. The results show that dissent will still emerge in this setting. Indeed, under some conditions, the likelihood of observing dissent is even higher than in the baseline case in which the ally has no bargaining power: bargaining power and dissent will sometimes complement each other. Further, no matter how much bargaining power the ally has, there always exist parameter values under which he will choose to dissent: bargaining is not a perfect substitute for dissent.

Alternative explanations. In reviewing the relevant literature I have already discussed potential competing explanations advanced in previous works. Here, I focus on an alternative explanation that (to the best of my knowledge) has not been formally proposed in the literature but is, nonetheless, intuitively appealing. Namely, one possibility is that dissent emerges as the result of a struggle between competing factions within the party. For example, the dissenters may be trying to damage the leader so as to make it easier to depose him. Within this framework, dissent should emerge when the leader is expected to perform poorly in the upcoming elections. Yet, this is not always the case. For example, if we look at the Italian Democratic party under Matteo Renzi's leadership, dissent exploded against a leader who was expected to bring the party to electoral success. This is in line with the predictions of Proposition 1, according to which it is precisely when the incumbent is leading that the allies have incentives to attack him. Further, it is important to stress that if the dissenters' goal is to replace the dominant faction and take over the party, rather than simply depose the incumbent leader, then this argument complements the one proposed in this paper. For an extreme faction to take over, it has to believe that it has a chance of winning the (general) election. When the electorate is too moderate, this requires changing voters' policy preferences. That is, the faction has incentives to force the leader to experiment, just like in the model analyzed above. As such, the framework presented here could explain dissent for pure policy reasons (as in the current paper) or for both policy *and* instrumental reasons (taking over). In this perspective allowing for replacement does not alter (and if anything strengthens) the model's qualitative insights.

Welfare Analysis

What are the welfare implications of this theory? The voter values policy experimentation, as it increases the probability that she will make the correct electoral decision. However, the results presented above indicate that under some conditions the incumbent avoids policy gambles precisely in order to prevent the voter from learning about her true preferences. Electoral accountability therefore has the perverse consequence of inducing the incumbent to implement a platform that is more moderate than both his own bliss point and, potentially, what is optimal for the voter. By engaging in electorally costly dissent, the incumbent's extreme ally may mitigate this inefficiently. Within this setting, the presence of the incumbent's extreme ally can therefore be welfare improving for the voter. Proposition 3 identifies necessary conditions for this to be true.

Proposition 3: *In equilibrium the voter benefits from the presence of an extreme ally to the incumbent party if:*

- *The cost of dissent δ is sufficiently large that it turns the leading incumbent into a trailing one, but not so large that it always hurts the voter ex ante ($\underline{\delta} < \delta < \overline{\delta_w}$)*
- *The value of information is sufficiently high*
  - *The prior is sufficiently close to $\frac{1}{2}$ ($\frac{1}{2} < \gamma < \overline{\gamma_w}$)*
  - *Incumbent and challenger are moderately polarized ($\underline{x^I_w} < x^I < \overline{x^I_w}$)*
  - *Learning the true state has a sufficiently large impact on the voter's preferences ($\bar{\alpha} > \bar{\alpha}_w$)*
- *The incumbent's ally is sufficiently extreme ($x^A > \underline{x^A_w}$)*

The first two conditions are intuitive and ensure that, when it emerges, dissent is welfare improving for the voter. First, the voter must not dislike dissent so much that it always hurts her ex-ante. And second, obtaining new information must be sufficiently valuable that the voter benefits from moving the first period policy to the extreme. For this to be true, the voter's prior beliefs must be sufficiently uncertain (i.e., γ must be close to $\frac{1}{2}$), and the value of making the correct electoral decision must be large enough. The third condition seems more puzzling: as the ally becomes more extreme the ideological misalignment with the voter increases. However, recall that the ally's bliss point has no direct effect on the equilibrium policy choice, thus on the voter's welfare. The effect is only an indirect one, through the ally's willingness to dissent. Since the first conditions impose further restrictions on the parameters, for the incumbent's ally to be willing to dissent when such conditions are satisfied (and therefore dissent is beneficial for the voter) he must be sufficiently extreme.

Proposition 3 uncovers an alignment between the interests of a voter who cares about information and therefore has a taste for policy experimentation, and a political actor who has an ideological preference for extreme policies. This is reminiscent of the 'case for responsible parties', presented by Bernhardt, Duggan and Squintani (2009). Both papers highlight that the voter may benefit from the presence of extreme politicians, as ideological extremism may serve the purpose of mitigating accountability's perverse consequences.

The results also speak to the debate on the normative evaluation of party factions. As noted by Boucek (2009), negative perceptions of factionalism originated with Hume (1877) and are still predominant: factions 'exacerbate non-cooperative behaviour and so are antithetical to achievement of common goals' (Dewan and Squintani 2015, 861). A 'defence of factions' comes from the claim that organized and ideologically cohesive subgroups within political parties can instead play a role in fostering cooperation, by facilitating deliberation and pooling of valuable information (this argument is advanced initially by Bouceck 2008, investigated empirically by McAllister 1991, and proven formally by Dewan and Squintani 2015). This paper goes a step further, showing that factions can play a positive role even when, and precisely because, they engage in disruptive behavior.

Micro Founding the Electoral Cost of Dissent

A key assumption of the paper is that dissent is electorally costly. In the model I keep this cost black-boxed, by assuming that dissent generates an endogenous valence shock agains the incumbent and thus 'mechanically' reduces voters' appreciation of the party. While this assumption is arguably plausible even in this reduced form, it is important to discuss potential ways to micro-found it. Why exactly do voters dislike parties that experience dissent?

One possibility is that voters do not dislike divided parties per se, rather the observation of dissent causes them to negatively update their beliefs over the incumbent's honesty, competence, etc. Dissent may convey such information in two different ways.

First, if the incumbent's allies have access to *verifiable* information, by dissenting they can expose him as a liar, corrupt or incompetent. The specification and results of this micro-founded model would be exactly as presented above. Under the conditions identified in Proposition 1, the allies choose to dissent whenever they can reveal evidence that the incumbent is a bad type. If the conditions are not met, the allies keep quiet. The only difference with the model presented here is that, because verifiable information cannot be fabricated, dissent can never emerge if the incumbent is a good type.

Alternatively, we may assume that the incumbent's allies do not have access to such verifiable evidence. Nonetheless, they may have an informational advantage with respect to the voters. For example, the allies may scrutinize the incumbent's previous actions and performance, thereby obtaining additional information about his true competence (see Caillaud and Tirole 1999, Fox and Van Weelden 2010). As such, the allies can engage in a signaling game with the electorate. Dissent is electorally costly when, *in equilibrium*, it constitutes a negative signal of the incumbent's type. For such a (separating or semi-separating) equilibrium to be sustained, the gain from changing the incumbent's policy choice must be sufficiently large. The qualitative results would then be analogous to Proposition 1.[12]

In concluding this section let me emphasize that, while the assumption of electorally costly

[12] With some additional conditions required to sustain separation: the ally must care sufficiently about quality, and he cannot be too extreme.

dissent is motivated by both empirical evidence and the above theoretical reasoning, the mechanism identified in this paper relies only on the voters not being indifferent to dissent ($\delta \neq 0$). Indeed, it would survive in a world in which dissent produces a positive valence shock ($\delta < 0$), thus improving rather than damaging a party's electoral prospects. Clearly, under such an assumption the puzzle would be reversed: if dissent is electorally valuable, how do we explain cases in which we do not observe dissent? The mechanism identified in this paper would provide a potential answer. An extreme ally may choose not to improve its party's electoral prospects, in order to preserve the incumbent's incentives to gamble for resurrection.

Empirical Implications

Several scholars have recognized that party unity has a crucial impact on electoral performance, and have tried to obtain empirical estimates of this effect. The typical strategy has been to regress the probability of winning (or other measures of electoral success) at time t on a binary variable indicating whether the party experienced dissent at $t-1$ (Clark 2009, Kam 2009, Groeling 2010):

$$prob(W_i = 1) = \alpha + \beta_1 X_i + \beta_2 D_i + \epsilon_i \tag{5}$$

Where X_i is a vector of covariates, and β_2 is the coefficient of interest. Graphically, the quantity of interest is the average distance between the two curves in Figure 3, representing the probability of winning as a function of the party's ex-ante electoral strength (γ), with and without dissent.

The results of the model have two key implications. First of all, they show that it is impossible to isolate the *direct* effect of dissent. The incumbent best responds to dissent by modifying his policy choice precisely in order to mitigate this electoral cost. Thus, any estimate would at best reflect the *equilibrium* effect of dissent: the cost mediated through the incumbent's best response. Additionally, the model shows that this estimate would inevitably suffer from selection bias. Proposition 1 shows that whether parties experience dissent depends precisely on their ex-ante electoral strength (γ). Thus, it is impossible to observe both treated and control units for similar levels of ex-ante electoral strength. Figure 4 represents what the researcher can actually

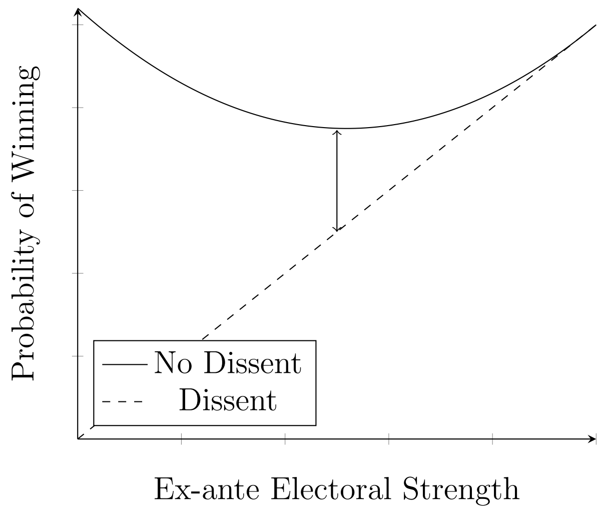


Figure 3: Probability of Winning as Function of Ex-ante Strength (γ).

observe: treated units at moderately high levels of electoral strength and untreated ones at γ close to $\frac{1}{2}$ or 1. Comparing parties that experience dissent with their untreated counterparts inevitably means comparing parties at different levels of electoral strength. Thus, we cannot recover an unbiased estimate of the (equilibrium) effect of dissent on parties' electoral performance.

Further, it is hard to know what the direction of the bias will be. In the example of Figure 4, the estimated electoral cost of dissent would likely be higher than the true one. However, under different parameter values, the dissenting region shifts. Consider for example Figure 5, obtained by increasing the ally's bliss point. Here, dissent emerges only at vey high values of γ, and the direction of the bias is no longer clear. Indeed, the estimate may even have the wrong sign. Thus, even if we are aware of the existence of the bias, it is hard to interpret the results of this type of analysis.

However, this discussion does not imply that the theory is not falsifiable. The model generates testable predictions regarding parties' electoral performance conditional on experiencing dissent. Following public manifestations of dissent, the party needs the voter to obtain new and favourable information in order to win. Thus, the larger the amount of information received by the voters and their ability to interpret such information, the higher the probability of winning conditional on experiencing dissent. We should then expect this conditional probability to be positively correlated with variables such as news media consumption, education or political engagement in the popula-

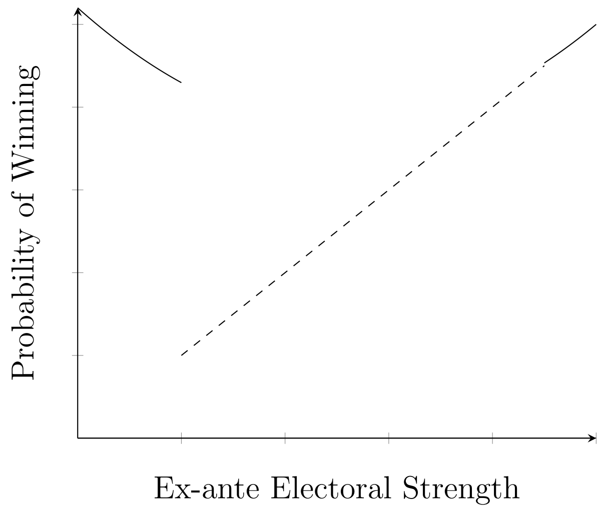


Figure 4: Probability of Winning as Function of Ex-ante Strength (γ) - Observable

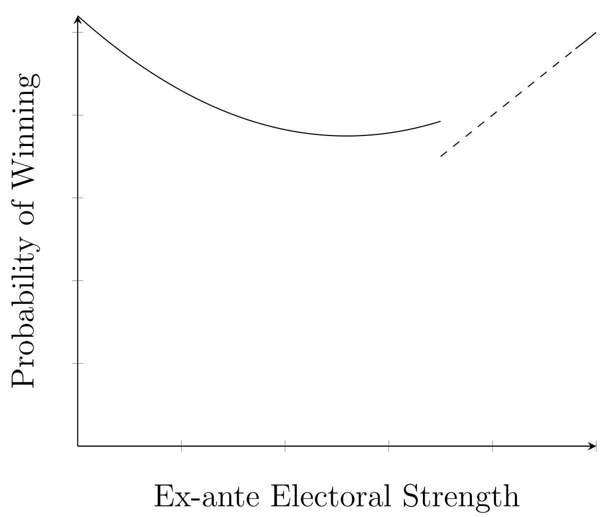


Figure 5: Probability of Winning as Function of Ex-ante Strength, second example.

tion. Additionally, fixing the information environment, the amount of voter learning depends on the incumbent's willingness to engage in policy experimentation. Recall that the more extreme the party leader is, the more he is willing to gamble. Thus, conditional on experiencing dissent, the party's electoral success should be increasing in the leader's ideological extremism. Focusing on the treated units, and thereby avoiding any selection bias, researchers can thus empirically investigate the conditions under which dissent is expected to hurt parties the most.

Conclusion

I have proposed a theory of electorally costly intra-party dissent, according to which public manifestations of dissent serve the purpose of mitigating the ideological conflict within the party, by inducing a policy response by the leadership. In equilibrium, dissent thus emerges precisely because it is electorally costly. The model's results help us qualify our intuitions about the conditions that are more likely to generate dissent. In particular, they highlight that improving the party's expected electoral performance may generate more dissent, and that an increase in intra-party ideological polarization plays an ambiguous role. Depending on whether the incumbent becomes more moderate or his allies more extreme, increased polarization may either decrease or increase the likelihood of the party experiencing public manifestations of dissent.

The theory also has relevant normative and empirical implications. From a normative stand point, it indicates that the presence of extremists within the incumbent party may be welfare improving for the voter, as it may mitigate the perverse consequences of electoral accountability. With regards to empirical research, the results show that existing estimates of the electoral rewards of party unity obtained by comparing treated and control units are inevitably biased.

In concluding this paper it is important to highlight that, while this work has focused on intra-party conflict, the mechanism it uncovers applies more generally. Trade-offs analogous to the ones explored here may for example characterize the strategic interaction between revolutionary groups and their domestic governments, or influence the relationship between the leaders of different countries. Indeed, the model can capture the dynamics of the interaction between political actors in any strategic situation that can be described as a principal agent model with two key features.

First, the principal faces uncertainty over the optimal retention decision and learns via experience, by observing the *outcome* of the agent's action. The principal's uncertainty can refer to her ideal policy (as in the model presented here) or to the agent's type, such as his ability or competence. The agent's action can thus represent a level of effort, a point in the policy space, the degree of reform, or even the amount of rent extraction. Second, a third player (either a friend or a foe of the agent) can take an action that, everything else constant, changes the probability that the agent is retained. In this setting, the ally/foe can therefore manipulate the agent's retention chances, in order to alter his incentives to take more or less risky actions. This generates the trade off between ensuring that the agent is (or is not) retained, and inducing him to take an action closer to the ally's own preferences. This framework can therefore be applied to several different settings, encompassing both developed and developing democracies, as well as authoritarian countries.

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Appendix A: Proofs

Lemma 1

Proof. The proof of Claims 1 and 2 below is necessary and sufficient to prove Lemma 1.

Claim 1: *Let* $x_t \geq 0$.

(i) A payoff realization $U_t^v \notin [-(x_t-\bar{\alpha})^2-\frac{1}{2\psi}, -(x_t-\underline{\alpha})^2+\frac{1}{2\psi}]$ *is fully informative. Upon observing* $U_t^v > -(x_t-\underline{\alpha})^2+\frac{1}{2\psi}$*, the voter forms posterior beliefs that* $x^v = \bar{\alpha}$ *with probability 1. Similarly, upon observing* $U_t^v < -(x_t-\bar{\alpha})^2-\frac{1}{2\psi}$ *the voter forms beliefs that* $x^v = \underline{\alpha}$ *with probability 1.*

(ii) A payoff realization $U_t^v \in [-(x_t-\bar{\alpha})^2-\frac{1}{2\psi}, -(x_t-\underline{\alpha})^2+\frac{1}{2\psi}]$*, is uninformative. Upon observing* U_t^v*, the voter confirms her prior belief that* $x^v = \bar{\alpha}$ *with probability* γ*.*

Symmetric results apply when $x_t < 0$*.*

Proof. The proof of part (i) is trivial given the boundedness of the distribution of ϵ, and is therefore omitted. Part (ii) follows straightforwardly from applying Bayes rule. Recall that the voter's payoff realization U_t^v is a function of the implemented policy (x_t) the voter's true bliss point (x^v) and the noise term (ϵ): $U_t^v = -(x^v - x_t)^2 + \epsilon$. Denote as $f(\cdot)$ the PDF of ϵ. Then,

$$prob(x^v = \bar{\alpha}|U_t^v) = \frac{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma}{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma + f(U_t^v + (x_t - \underline{\alpha})^2)(1-\gamma)} \tag{6}$$

Given the assumption that ϵ is uniformly distributed $f(U_t^v+(x_t-\bar{\alpha})^2) = f(U_t^v+(x_t-\underline{\alpha})^2)$ therefore the above simplifies to

$$prob(x^v = \bar{\alpha}|U_t^v) = \gamma \tag{7}$$

This concludes the proof of Claim 1. □

Claim 2: *Let L be a binary indicator, taking value 1 if the players learn the true value of* x^v *at the end of period 1, and 0 otherwise. There exists* $x' = \frac{1}{4\bar{\alpha}\psi}$ *such that*

- *For all* $|x_1| \geq |x'|$

$$Prob(L = 1|x_1) = 1 \tag{8}$$

- *For all* $x_1 \in [0, x')$

$$Prob(L = 1 | x' \geq x_1 \geq 0) = 4\bar{\alpha}\psi x_1 \tag{9}$$

- *For all* $x_1 \in (-x', 0]$

$$Prob(L = 1 | -x' \leq x_1 \leq 0) = -4\bar{\alpha}\psi x_1 \tag{10}$$

Proof. Let me first prove the existence of point x'. From Claim 1, x' is the point such that for any policy $|x| \geq |x'|$, the interval $[-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$ is empty. This requires

$$-(x_t - \underline{\alpha})^2 + \frac{1}{2\psi} + (x_t - \bar{\alpha})^2 + \frac{1}{2\psi} \leq 0 \tag{11}$$

Recall that $\bar{\alpha} = -\underline{\alpha}$, thus the above reduces to

$$x \geq \frac{1}{4\bar{\alpha}\psi} = x' \tag{12}$$

To complete the proof, assume $x_1 \in [0, x']$. The expected probability of the realized outcome being informative is

$$Prob(L = 1 | \gamma, 0 < x_1 < x') =$$

$$\gamma[Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \tfrac{1}{2\psi})] + (1 - \gamma)[Prob(-(x_t - \underline{\alpha})^2 + \epsilon_1 < -(x_t - \bar{\alpha})^2 - \tfrac{1}{2\psi})] \tag{13}$$

Given the symmetry $Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}) = Prob(-(x_t - \underline{\alpha})^2 + \epsilon_1 < -(x_t - \bar{\alpha})^2 - \frac{1}{2\psi})$, thus (8) simplifies to

$$Prob(L = 1 | x_1 > 0) = Prob(-(x_t - \bar{\alpha})^2 + \epsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi})) = 4\bar{\alpha}\psi x_1 \tag{14}$$

Similar calculations produce the result for $x_1 \in (-x', 0]$. This concludes the proof of Claim 2 □

This concludes the proof of Lemma 1 □

In what follows I will assume that $x^I < \frac{1}{4\bar{\alpha}\psi}$. This assumption is without loss of generality, and imposed in order to reduce the number of cases under consideration; results for the case in which

$x^I > \frac{1}{4\bar{\alpha}\psi}$ are available upon request.

Lemma 2

Proof. The proof of the first point is trivial: a certain loser is never re-elected, hence his policy choice does not influence his future payoff. He maximises his immediate utility by implementing his bliss point x^I. Leading and trailing incumbents will instead consider the expected informativeness of the policy, and how it influences their probability of re-election.

Consider first a trailing incumbent. The equilibrium policy solves the following maximisation problem:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -(\mathrm{x_1} - \mathrm{x^I})^2 - (1 - 4\bar{\alpha}\psi \mathrm{x_1}\gamma)(\mathrm{x^I} + \mathrm{x^I})^2 - 4\bar{\alpha}\psi \mathrm{x_1}\gamma(\mathrm{x^I} - \mathrm{x^I})^2 \\ \text{subject to} \quad & x_1 \leq \frac{1}{4\bar{\alpha}\psi} \end{aligned} \tag{15}$$

Hence:

$$x_1^* = \min\{x^I + 8\bar{\alpha}\psi(x^I)^2\gamma, \frac{1}{4\bar{\alpha}\psi}\} \tag{16}$$

The condition that $x_1 \leq \frac{1}{4\alpha\psi}$ derives from the fact that any policy weakly more extreme than $x' = \frac{1}{4\alpha\psi}$ is fully informative, therefore the leading incumbent would have no reason to move beyond x' (recall that we assume $x^I < x'$).

Consider now a leading incumbent. The equilibrium policy solves the following maximisation problem:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -(\mathrm{x_1} - \mathrm{x^I})^2 - 4\bar{\alpha}\psi \mathrm{x_1}(1-\gamma)(\mathrm{x^I} + \mathrm{x^I})^2 - (1 - 4\bar{\alpha}\psi \mathrm{x_1}(1-\gamma))(\mathrm{x^I} - \mathrm{x^I})^2 \\ \text{subject to} \quad & x_1 \geq 0 \end{aligned} \tag{17}$$

Hence $x_1^* = \max\{x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma), 0\}$. Recall that if the incumbent is leading then $\gamma > \frac{1}{2}$. Additionally, $x^I < \frac{1}{4\bar{\alpha}\psi}$ by assumption. Thus, $x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) > 0$ and

$$x_1^* = x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) \tag{18}$$

□

Lemma 3

Proof. Let x^d be the incumbent's policy choice after dissent, and x the policy that he would choose otherwise. Consider first of all a leading incumbent. We must distinguish between three cases: (i) $\delta < \underline{\delta}$ such that the incumbent's initial advantage is not outweighed (i.e. $\gamma > \frac{\delta + 4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi x^I}$). In this case dissent does not modify the incumbent's policy choice nor the voter's electoral decision. (ii) $\delta \geq \bar{\delta}$ such that the incumbent always loses after dissent (i.e $\delta \geq 4\bar{\alpha}x^I$). The claim follows straightforwardly. (iii) $\delta \in [\underline{\delta}; \bar{\delta})$, such that the incumbent wins if and only if the voter updates in his favour (i.e. dissent turns the leading incumbent into a trailing one). The following holds. Let $\pi(x_1)$ be the probability of the voter observing an informative signal at the end of period 1, as a function of the implemented policy. The probability of the incumbent being re-elected absent dissent is $1 - \pi(x)(1-\gamma)$. The probability of the incumbent being re-elected after dissent is instead $\pi(x^d)\gamma$. $1 - \pi(x)(1-\gamma) \geq \pi(x^d)\gamma$, since the LHS is at least $1 - (1-\gamma) = \gamma$ and the RHS is at most γ. Finally, consider a trailing incumbent. There are only two possibilities: (i) $\delta > \underline{\delta}$ such that after experiencing dissent the incumbent loses for sure. The claim follows trivially (ii) $\delta \leq \underline{\delta}$ such that the incumbent is still trailing even after experiencing dissent. Dissent has no impact on the policy choice, nor on the voter's electoral decision. □

Proposition 1

Proof. Let me first prove that dissent is never observed in equilibrium if the incumbent is trailing.[13] Absent dissent, a trailing incumbent implements policy $x_1^* = \min \in \{x^I + 8\bar{\alpha}\psi(x^I)^2\gamma, \frac{1}{4\bar{\alpha}\psi}\}$. Dissent has no impact on his policy choice (and therefore never emerges in equilibrium) if $\delta < 4\bar{\alpha}x^I$. Suppose instead that $\delta \geq 4\bar{\alpha}x^I$. Then, after experiencing dissent the incumbent is a sure loser:

[13]The reason why we must consider this case is that, even if dissent would always induce a trailing incumbent to moderate his policy choice, it may be the case that the ally's bliss point lies between the platforms that the incumbent would implement with and without dissent. It is therefore possible that $[x^A - (x_1^*|D = 1, \gamma < \frac{1}{2})]^2 < [x^A - (x_1^*|D = 0, \gamma < \frac{1}{2})]^2$ even if $(x_1^*|D = 1, \gamma < \frac{1}{2}) < (x_1^*|D = 0, \gamma < \frac{1}{2})$. We must therefore exclude that the ally's gain from dissenting against a trailing incumbent is larger than the cost.

even if the voter learns that $x^v = \bar{\alpha}$, she will still choose to replace the incumbent with his challenger. As a consequence, in the first period the incumbent would always implement exactly his bliss point x^I upon experiencing dissent.

Therefore, there are two possible cases that we must consider:

- $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$, and therefore $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = \frac{1}{4\bar{\alpha}\psi}$
- $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$, and therefore $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$

I will analyse each case separately.

Case 1: suppose $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$, and therefore $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = \frac{1}{4\bar{\alpha}\psi}$. Given the incumbent's best response, the ally chooses to dissent if and only if

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > -(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 \tag{19}$$

Which reduces to

$$\gamma < \frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} \tag{20}$$

Recall that we are considering a case in which $(x_1^*|D = 0) = \frac{1}{4\bar{\alpha}\psi}$, therefore $\gamma > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$. Thus, for dissent to emerge in equilibrium it must be the case that

$$\frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \tag{21}$$

Let $T = 4\bar{\alpha}\psi$. The above can be rearranged as

$$\frac{1 - 2x^A T(1 - x^I T) - (x^I T)^2}{2x^A} > \frac{1 - x^I T}{x^I} \tag{22}$$

Which reduces to

$$2x^A(1 - (x^I T)^2) < x^I(1 - (x^I T)^2) \tag{23}$$

Since $(x^I T)^2 = (4\bar{\alpha}\psi x^I)^2 < 1$, the above can never be satisfied when $x^A > x^I$.

Case 2: suppose $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$, and therefore $(x_1^*|D = 1) = x^I$ and $(x_1^*|D = 0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$. In this case, the ally chooses to dissent if and only if

$$\begin{aligned} -(x^I - x^A)^2 - (x^I + x^A)^2 > \\ -(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 - (1 - 4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(x^I + x^A)^2 \\ -4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - x^A)^2 \end{aligned} \tag{24}$$

Denoting $\Delta = 8\bar{\alpha}\psi\gamma(x^I)^2$, the above can be rearranged as

$$0 > -\Delta^2 - 2\Delta(x^I - x^A) + 16\bar{\alpha}\psi\gamma(x^I + \Delta)x^I x^A \tag{25}$$

Substituting $\Delta = 8\bar{\alpha}\psi\gamma(x^I)^2$ and dividing by $16\bar{\alpha}\psi\gamma(x^I)^2$, the above reduces to

$$x^A < \frac{x^I}{2} \tag{26}$$

Given $x^A > x^I$, the condition can never be satisfied.

Thus, dissent never emerges in equilibrium if $x^A > x^I$ and $\gamma < \frac{1}{2}$.

Consider now the conditions on the cost of dissent δ. The proof of the first condition ($\delta > 4\bar{\alpha}x^I(2\gamma - 1)$) is presented in the main body of the paper. Here, I present a formal proof of the second condition. Let $\delta \geq 4\bar{\alpha}x^I$. After experiencing dissent, the incumbent would turn into a sure loser. Therefore $(x_1^*|D = 1) = x^I$ Conversely, (from Lemma 2) if the leading incumbent experiences no dissent $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma)$

Thus, dissent strictly increases the ally's utility if and only if:

$$\begin{aligned} -(x^I - x^A)^2 - (x^I + x^A)^2 > \\ -(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma) - x^A)^2 - (1 - 4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(x^I - x^A)^2 \\ -4\bar{\alpha}\psi(1 - \gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1 - \gamma))(x^I + x^A)^2 \end{aligned} \tag{27}$$

This reduces to

$$x^A[1-2(4\bar{\alpha}\psi x^I(1-\gamma))(1-4\bar{\alpha}\psi x^I(1-\gamma))]+4\bar{\alpha}\psi(x^I)^2(1-\gamma)(1-4\bar{\alpha}\psi x^I(1-\gamma))<0 \tag{28}$$

The LHS is increasing in x^A and never satisfied at $x^A=0$. Hence, dissent by an extremist ally emerges only if $\delta<4\bar{\alpha}x^I$.

Finally, I must prove that there exist unique $\underline{\gamma}$, $\overline{\gamma}$, $\underline{x^I}$ and $\underline{x^A}$ such that dissent by an extremist ally emerges in equilibrium only if $\underline{\gamma}<\gamma<\overline{\gamma}$, $x^I>\underline{x^I}$ and $x^A>\underline{x^A}$.

Suppose that $4\bar{\alpha}x^I(2\gamma-1)<\delta<4\bar{\alpha}x^I$ and $\gamma>\frac{1}{2}$, and conjecture the existence of an equilibrium in which the ally chooses to dissent. We must consider two cases:

1. $x^I+8\bar{\alpha}\psi(x^I)^2\gamma\geq\frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1)=\frac{1}{4\bar{\alpha}\psi}$ and $(x_1^*|D=0)=x^I-8\bar{\alpha}\psi(1-\gamma)(x^I)^2$
2. $x^I+8\bar{\alpha}\psi(x^I)^2\gamma<\frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1)=x^I+8\bar{\alpha}\psi(x^I)^2\gamma$ and $(x_1^*|D=0)=x^I-8\bar{\alpha}\psi(1-\gamma)(x^I)^2$

I will analyse each of the two cases separately.

Case 1: $\gamma>\frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I}-1)$, therefore $(x_1^*|D=1)=\frac{1}{4\bar{\alpha}\psi}$ and $(x_1^*|D=0)=x^I-8\bar{\alpha}\psi(1-\gamma)(x^I)^2$. Given the anticipated best response of the incumbent, the ally chooses to dissent if and only if

$$\begin{aligned}-(\frac{1}{4\bar{\alpha}\psi}-x^A)^2-\gamma(x^I-x^A)^2-(1-\gamma)(x^I+x^A)^2 > \\ -(x^I-8\bar{\alpha}\psi(x^I)^2(1-\gamma)-x^A)^2-[1-4\bar{\alpha}\psi(1-\gamma)(x^I-8\bar{\alpha}\psi(x^I)^2(1-\gamma))](x^I-x^A)^2 \\ -4\bar{\alpha}\psi(1-\gamma)(x^I-8\bar{\alpha}\psi(x^I)^2(1-\gamma))(x^I+x^A)^2\end{aligned} \tag{29}$$

Let $I=4\bar{\alpha}\psi(x^I-8\bar{\alpha}\psi(x^I)^2(1-\gamma))$. We can rewrite the above condition as:

$$(1-\gamma)(1-I)((x^I-x^A)^2-(x^I+x^A)^2)>(\frac{1}{4\bar{\alpha}\psi}-x^A)^2-(\frac{I}{4\bar{\alpha}\psi}-x^A)^2 \tag{30}$$

Which is equivalent to

$$(1-\gamma)(1-I)(-4x^A x^I) > \frac{-x^A}{2\bar{\alpha}\psi}(1-I) + \frac{1}{(4\bar{\alpha}\psi)^2}(1+I)(1-I) \tag{31}$$

By substituting $I = 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma))$ and solving for γ we get the following condition:

$$\gamma > \frac{1 + (2x^A - x^I)(8\bar{\alpha}\psi x^I - 1)(4\bar{\alpha}\psi)}{2x^I(4\bar{\alpha}\psi)^2(2x^A - x^I)} \tag{32}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\gamma > \underline{\gamma} = max \quad \{\frac{1}{2}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1), \frac{1+(2x^A - x^I)(8\bar{\alpha}\psi x^I - 1)(4\bar{\alpha}\psi)}{2x^I(4\bar{\alpha}\psi)^2(2x^A - x^I)}\}$
2. $x^I > \frac{1}{8\bar{\alpha}\psi}$
3. $x^A > \frac{1}{8\bar{\alpha}\psi} + \frac{x^I}{2}$

Where the conditions on x^I and x^A ensure that the range $[\underline{\gamma}, 1]$ exists.

Case 2: $\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$, therefore $(x_1^*|D=1) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$ and $(x_1^*|D=0) = x^I - 8\bar{\alpha}\psi(1-\gamma)(x^I)^2$. Given the incumbent's best response, the ally chooses to dissent if and only if

$$\begin{aligned} &-(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 \\ &-(1 - 4\bar{\alpha}\psi(x^I + 8\alpha\psi(x^I)^2\gamma)\gamma)(x^I + x^A)^2 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)\gamma(x^I - x^A)^2 > \\ &-(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) - x^A)^2 - (1 - 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma))(1-\gamma))(x^I - x^A)^2 \\ &-4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma))(1-\gamma)(x^I + x^A)^2 \end{aligned} \tag{33}$$

Let $I = 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and $x^D = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$. We can rewrite the above condition as:

$$\begin{aligned} &-(x^D - x^A)^2 - (1-\gamma I)(x^I + x^A)^2 - \gamma I(x^I - x^A)^2 > \\ &-(x^D - x^A - 8\bar{\alpha}\psi(x^I)^2)^2 - (1 - (1-\gamma)(I - 4\bar{\alpha}\psi(8\bar{\alpha}\psi(x^I)^2)))(x^I - x^A)^2 \\ &-(1-\gamma)(I - 4\bar{\alpha}\psi(8\bar{\alpha}\psi(x^I)^2))(x^I + x^A)^2 \end{aligned} \tag{34}$$

By expanding, letting $x^D = \frac{I}{4\bar{\alpha}\psi}$ and dividing both sides by $4x^I$ we get:

$$\gamma I x^A > x^A - x^A((1-\gamma)(I - 2(4\bar{\alpha}\psi x^I)^2) + x^I(I - 4\bar{\alpha}\psi x^A - (4\bar{\alpha}\psi x^I)^2) \tag{35}$$

Which is equivalent to:

$$x^A + I(x^I - x^A) - 4\bar{\alpha}\psi x^I x^A + (4\bar{\alpha}\psi x^I)^2(2x^A(1-\gamma) - x^I) < 0 \tag{36}$$

By substituting $I = 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and solving for γ we get the following condition:

$$\gamma > \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)} \tag{37}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)}\} < \gamma < min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\} = \overline{\gamma}$
2. $\frac{\sqrt{3}-1}{8\bar{\alpha}\psi} < x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$
3. $x^A > \frac{x^I(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1)}{2(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1) - 1}$

Where the conditions on x^I and x^A ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists.

This concludes the proof of Proposition 1. □

The following corollary also holds, with respect to Case 2:

Corollary 1A: $\underline{\gamma} = \frac{1}{2} \implies \overline{\gamma} < 1$

Proof. Corollary 1A tells us that it can never be the case that (i) $\underline{\gamma} = \frac{1}{2}$ and (ii) $\overline{\gamma} = 1$. For (i) to be true we need:

$$\frac{1}{2} > \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)} \tag{38}$$

Which reduces to:

$$x^A[1 - 8\bar{\alpha}\psi x^I] + 4\bar{\alpha}\psi(x^I)^2 < 0 \quad (39)$$

Which clearly requires:

$$x^I > \frac{1}{8\bar{\alpha}\psi} \quad (40)$$

For (ii) to be true we need:

$$1 < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) \quad (41)$$

Which reduces to

$$x^I < \frac{1}{8\bar{\alpha}\psi} \quad (42)$$

Clearly the two conditions can never be simultaneously satisfied. □

Proposition 2

Proof. Denote $\Gamma(x^I, x^A, \bar{\alpha}\psi)$ the set of values of γ such that dissent is an equilibrium strategy iff $\gamma \in \Gamma$. From the proof of Proposition 1 is easy to verify that Γ is always weakly increasing in x^A. Analysing Cases 1 and 2, Γ is (weakly) increasing in x^I if and only if either one of the following sets of conditions is satisfied:

1. Case 1: $\underline{\gamma} = 1$. This requires $\frac{\sqrt{5}-1}{8\bar{\alpha}\psi} < x^I < \frac{1}{4\bar{\alpha}\psi}$ and $x^A > \frac{1+4\bar{\alpha}\psi x^I(1-4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)}$

2. Case 1: $\underline{\gamma} = \frac{1-4\bar{\alpha}\psi x^I}{2(4\bar{\alpha}\psi x^I)^2}$ which requires $x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$ and $x^A > \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2}$. It is easy to verify that when $\underline{\gamma} = \frac{1-4\bar{\alpha}\psi x^I}{2(4\bar{\alpha}\psi x^I)^2}$ in Case 1, irrespective of the bounds in Case 2 Γ will be weakly increasing in x^I

3. Case 2: $\underline{\gamma} = \frac{4\bar{\alpha}\psi x^I(2x^A-x^I)(4\bar{\alpha}\psi x^I-1)+x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A-x^I)}$ and $\overline{\gamma} = 1$, which requires $x^I < \frac{1}{8\bar{\alpha}\psi}$.

. Thus, necessary and sufficient condition for Γ to be increasing in x^I is that $x^I < \widehat{x^I}(x^A)$.

$x^A > \frac{1+4\bar{\alpha}\psi x^I(1-4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)} \implies \widehat{x^I}(x^A) = \frac{1}{4\bar{\alpha}\psi}$, $\frac{x^I(1-(4\bar{\alpha}\psi x^I)^2}{1-2(4\bar{\alpha}\psi x^I)^2} < x^A < \frac{1+4\bar{\alpha}\psi x^I(1-4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi(1-4\bar{\alpha}\psi x^I)} \implies \widehat{x^I}(x^A) = \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$, and $x^A < \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2}{1-2(4\bar{\alpha}\psi x^I)^2} \implies \widehat{x^I}(x^A) = \frac{1}{8\bar{\alpha}\psi}$ □

Proposition 3

Proof. In order to identify *sufficient* conditions for the voter to benefit from dissent, suppose that $\gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$. Then, $(x_1^*|D = 1) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$ and $(x_1^*|D = 0) = x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma)$. Dissent increases the voter's welfare if and only if the following condition is satisfied:

$$\begin{aligned}
-4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)\gamma\delta - \gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - \bar{\alpha})^2 \\
-(1-\gamma)(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma + \bar{\alpha})^2 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - \bar{\alpha})^2 \\
-(1 - 4\bar{\alpha}\psi(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(\gamma(x^I + \bar{\alpha})^2 + (1-\gamma)(x^I - \bar{\alpha})^2) > \\
-\gamma(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) - \bar{\alpha})^2 - (1-\gamma)(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma) + \bar{\alpha})^2 \\
-4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma))(x^I - \bar{\alpha})^2 \\
-(1 - 4\bar{\alpha}\psi(x^I - 8\bar{\alpha}\psi(x^I)^2(1-\gamma)))(\gamma(x^I - \bar{\alpha})^2 + (1-\gamma)(x^I + \bar{\alpha})^2)
\end{aligned} \tag{43}$$

Which reduces to:

$$\delta < \frac{(1-2\gamma)(1 - 8\bar{\alpha}\psi x^I + 2(4\bar{\alpha}\psi x^I)^2 + 16\bar{\alpha}\psi^2(x^I)^3) - 4\psi(x^I)^2 + 4(4\bar{\alpha}\psi x^I\gamma)^2}{\psi\gamma(1 + 8\bar{\alpha}\psi x^I\gamma)} = \overline{\delta_w} \tag{44}$$

If the above is satisfied, the voter benefits from dissent. However, for the ally to choose to dissent (given the incumbent's anticipated best response under $\frac{1}{2} < \gamma < \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$), we need (from the proof of Proposition 1):

1. $4\bar{\alpha}x^I(2\gamma - 1) \leq \delta < 4\bar{\alpha}x^I$
2. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{4\bar{\alpha}\psi x^I(2x^A - x^I)(4\bar{\alpha}\psi x^I - 1) + x^A}{32\bar{\alpha}^2\psi^2(x^I)^2(2x^A - x^I)}\} < \gamma < min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\} = \overline{\gamma}$
3. $\frac{\sqrt{3}-1}{8\bar{\alpha}\psi} < x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$
4. $x^A > \frac{x^I(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1)}{2(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I + 1) - 1}$

Thus, for the voter to benefit the presence of the incumbent's extreme ally, both (46) and conditions 1 to 4 above must be satisfied. This requires $\overline{\delta_w} > 4\bar{\alpha}x^I(2\gamma-1)$, which reduces to:

$$(1-2\gamma)(1+4\bar{\alpha}\psi x^I(8\bar{\alpha}\psi x^I - 2 + 4\psi(x^I)^2 + 8\bar{\alpha}\psi x^I\gamma^2 + \gamma)) - 4\psi(x^I)^2 + 4(4\bar{\alpha}\psi x^I\gamma)^2 > 0 \quad (45)$$

The LHS is decreasing in γ, therefore the above establishes an upper bound $\widehat{\gamma_w}$. For the condition to be possible to satisfy in equilibrium we need $\widehat{\gamma_w} > \underline{\gamma}$. From the proof of Case 2 we can verify that $\underline{\gamma} = \frac{1}{2}$ when $x^I > \frac{1}{8\bar{\alpha}\psi}$ and $x^A > \frac{4\bar{\alpha}\psi(x^I)^2}{8\bar{\alpha}\psi x^I - 1}$. Additionally, given Corollary 1A $\underline{\gamma} = \frac{1}{2} \implies \overline{\gamma} = \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)$. Thus, the voter benefits from the presence of the incumbent's extreme ally if:

1. $4\bar{\alpha}x^I(2\gamma-1) \leq \delta < \overline{\delta_w}$
2. $\frac{1}{2} < \gamma < min\{\widehat{\gamma_w}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\} = \overline{\gamma_w}$
3. $\underline{x_w^I} = \frac{1}{8\bar{\alpha}\psi} < x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$
4. $x^A > max\{\frac{4\bar{\alpha}\psi(x^I)^2}{8\bar{\alpha}\psi x^I - 1}, \frac{x^I(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I+1)}{2(4\bar{\alpha}\psi x^I)(4\bar{\alpha}\psi x^I+1)-1}\} = \underline{x_w^A}$

$\widehat{\gamma_w} > \frac{1}{2}$ if and only if the following is satisfied:

$$-4\psi(x^I)^2 + 4(2\bar{\alpha}\psi x^I)^2 > 0 \quad (46)$$

Which reduces to

$$\alpha > \frac{1}{2\sqrt{\psi}} = \bar{\alpha}_w \quad (47)$$

This concludes the proof.

□

Appendix B: What if the Ally Has Bargaining Power?

In this section I will consider the case in which the incumbent's ally has bargaining power over the first period policy making. I will thus assume that in the first period the incumbent maximises a weighted average of his own and the ally's utility:

$$U_1^W = \beta[-(x_1 - x^A)^2 + U_2^A(x_1, x_2, x^A)] + (1-\beta)[-(x_1 - x^I)^2 + U_2^I(x_1, x_2, x^I)] \tag{48}$$

This is equivalent to analysing a game in which, after the ally chooses wether to dissent, it engages in a bargaining stage with the incumbent to determine the policy to be implemented in the first period. Therefore, the parameter β represents, in this reduced form, the ally's bargaining power in the first period. As in the baseline model, I assume $x^C = -x^I \leq 0$ and $x^I < \frac{1}{4\alpha\psi}$. Additionally, I assume that in the second period the ally has no bargaining power. In this section I will establish two results. First (Proposition 1A), I will show that when the ally has bargaining power dissent can emerge at all values of x^I. Recall that in the baseline model dissent never emerges when I is too moderate (since incentives to gamble are too weak). Thus, this result indicates that under some conditions dissent and bargaining power complement each other: dissent is more likely to emerge if the ally also has formal power over policy making. The intuition is straightforward: if the ally is given formal authority over policy making, it can effectively 'compensate' for an excessively moderate incumbent, so that dissent has a sufficiently large impact on the equilibrium policy . Secondly (Corollary 2A) I will show that, under some conditions, dissent and bargaining power are not perfect substitutes: dissent can emerge at all values of β.

First, we must determine the equilibrium policy choice of the incumbent, proceeding as in the proof of Lemma 2. Consider first a trailing incumbent. The following holds:

- Let $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}a\psi}$, then $x_1^* = \beta x^A + (1-\beta)x^I$
- Let $\beta x^A + (1-\beta)x^I < \frac{1}{4\bar{\alpha}\psi}$, then $x_1^* = \quad \min \quad \{\frac{1}{4\bar{\alpha}\psi}; [\beta x^A + (1-\beta)x^I][1 + 8\bar{\alpha}\psi x^I \gamma]\}$

Consider now a leading incumbent:

- Let $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}\psi}$.
  Then $x_1^* = \beta x^A + (1-\beta)x^I$ if $\gamma > \frac{1+4\bar{\alpha}\psi[(\beta x^A+(1-\beta)x^I)(4\alpha\psi x^I-1)]}{(4\bar{\alpha}\psi)^2[x^I(\beta x^A+(1-\beta)x^I)]}$, and $x_1^* = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi x^I(1-\gamma)]$ otherwise[14]

[14]When $\beta x^A + (1-\beta)x^I \geq \frac{1}{4\bar{\alpha}\psi}$ the leading incumbent's overall utility as a function of the first period policy

- Let $\beta x^A + (1-\beta)x^I < \frac{1}{4\bar{\alpha}\psi}$, then $x_1^* = [\beta x^A + (1-\beta)x^I][1 - 8\alpha\psi x^I(1-\gamma)]$

Proposition 1A: *For all $x^I \geq 0$, there exist non-measure zero sets $\Gamma(x^I)$ and $B(x^I)$ such that if $\gamma \in \Gamma(x^I)$ and $\beta \in B(x^I)$ then dissent by an extreme ally occurs in equilibrium*

Proof. I proceed as in the proof of Proposition 1. Suppose that $4\bar{\alpha}x^I(2\gamma - 1) < \delta < 4\bar{\alpha}x^I$ and $\gamma > \frac{1}{2}$, and conjecture the existence of an equilibrium in which the ally chooses to dissent. We must consider three cases:

1. $(x_1^*|D=1) = \beta x^A + (1-\beta)x^I$ and $(x_1^*|D=0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\bar{\alpha}}\psi(1-\gamma)x^I]$

2. $(x_1^*|D=1) = \frac{1}{4\bar{\alpha}\psi}$ and $(x_1^*|D=0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\bar{\alpha}}\psi(1-\gamma)x^I]$

3. $(x_1^*|D=1) = [\beta x^A + (1-\beta)x^I][1 + 8\bar{\alpha}\psi\gamma x^I]$ and $(x_1^*|D=0) = [\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi(1-\gamma)x^I]$

I will analyse each of the three cases separately.

Case 1: $(x_1^*|D=1) = \beta x^A + (1-\beta)x^I$**,** $(x_1^*|D=0) = (\beta x^A + (1-\beta)x^I)(1 - 8\alpha\psi x^I(1-\gamma))$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{49}$$

$$\beta \geq \frac{1 - 4\bar{\alpha}\psi x^I}{4\alpha\psi(x^A - x^I)} \tag{50}$$

$$\gamma < \frac{1 + 4\bar{\alpha}\psi((\beta x^A + (1-\beta)x^I)(4\bar{\alpha}\psi x^I - 1))}{(4\bar{\alpha}\psi)^2 x^I(\beta x^A + (1-\beta)x^I)} \tag{51}$$

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has two maxima: one at $\beta x^A + (1-\beta)x^I$ and a second at $[\beta x^A + (1-\beta)x^I][1 - 8\bar{\alpha}\psi x^I(1-\gamma)]$. The condition on γ identifies which one of the two is the global maximum.

Additionally, the equilibrium condition for the ally is

$$\begin{aligned} & -(\beta x^A + (1-\beta)x^I - x^A)^2 - \gamma(x^I - x^A)^2 \\ -(1-\gamma)(x^I + x^A)^2 > & -[(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma)) - x^A]^2 \\ & -[1 - 4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I - x^A)^2 \\ & -[4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I + x^A)^2 \end{aligned} \tag{52}$$

Let $x^D = \beta x^A + (1-\beta)x^I$ and $x^D - \Delta = (\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$ where $\Delta = (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)$. The above reduces to

$$-\Delta^2 + 2\Delta(x^D - x^A) + 4x^I x^A(1-\gamma) - 16\bar{\alpha}\psi(1-\gamma)x^I x^A(x^D - \Delta) < 0 \tag{53}$$

Substituting $\Delta = (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)$ and dividing for $4x^I(1-\gamma)$ gives

$$\begin{aligned} -x^I(4\bar{\alpha}\psi)^2(1-\gamma)(\beta x^A + (1-\beta)x^I)^2 + 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(x^D - x^A) \\ + x^A - 4\bar{\alpha}\psi x^A(x^D - (\beta x^A + (1-\beta)x^I)8\bar{\alpha}\psi x^I(1-\gamma)) < 0 \end{aligned} \tag{54}$$

Substituting $x^D = \beta x^A + (1-\beta)x^I$ and solving for γ gives us condition:

$$\gamma > 1 + \frac{x^A - 4\bar{\alpha}\psi[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}{(4\bar{\alpha}\psi)^2 x^I[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]} \tag{55}$$

Thus, the conjectured equilibrium exist if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, 1 + \frac{x^A - 4\bar{\alpha}\psi[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}{(4\bar{\alpha}\psi)^2 x^I[\beta x^A + (1-\beta)x^I][2x^A - \beta x^A - (1-\beta)x^I]}\} < \gamma < \frac{1 + 4\bar{\alpha}\psi((\beta x^A + (1-\beta)x^I)(4\bar{\alpha}\psi x^I - 1))}{(4\bar{\alpha}\psi)^2 x^I(\beta x^A + (1-\beta)x^I)} = \overline{\gamma}$
2. $\underline{\beta} = \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \leq \beta < min \quad \{1, \frac{1 + 4\bar{\alpha}\psi x^I(2\bar{\alpha}\psi x^I - 1)}{4\bar{\alpha}\psi(x^A - x^I)(1 - 2\bar{\alpha}\psi x^I)}\} = \overline{\beta}$
3. $x^A > \frac{1}{4\bar{\alpha}\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x^A ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 2: $(x_1^*|D=1) = \frac{1}{4\bar{\alpha}\psi}$, $(x_1^*|D=0) = (\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma)$ The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{56}$$

$$\beta < \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \tag{57}$$

$$\gamma > \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)} - 1) \tag{58}$$

Additionally, the equilibrium condition for the ally is

$$\begin{aligned} -(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 > \\ -[(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma)) - x^A]^2 \\ -[1 - 4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I - x^A)^2 \\ -[4\bar{\alpha}\psi(1-\gamma)(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))](x^I + x^A)^2 \end{aligned} \tag{59}$$

Let $I = 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$. The above can be rewritten as:

$$\begin{aligned} -(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 > \\ -(\frac{I}{4\bar{\alpha}\psi} - x^A)^2 - (1 - I(1-\gamma))(x^I - x^A)^2 - I(1-\gamma)(x^I + x^A)^2 \end{aligned} \tag{60}$$

Which reduces to

$$(1-I)(\frac{x^A}{2\bar{\alpha}\psi} - 4x^I x^A(1-\gamma) - \frac{1+I}{(4\bar{\alpha}\psi)^2}) > 0 \tag{61}$$

By substituting $I = 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1-\gamma))$ and solving for γ we get condition:

$$1 + \frac{-1 + 4\bar{\alpha}\psi(2x^A - x^I - \beta(x^A - x^I)}{-2(4\bar{\alpha}\psi)^2 x^I(2x^A - x^I - \beta(x^A - x^I)} < \gamma < 1 \tag{62}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, 1 + \frac{-1+4\bar{\alpha}\psi(2x^A - x^I - \beta(x^A - x^I)}{-2(4\bar{\alpha}\psi)^2 x^I(2x^A - x^I - \beta(x^A - x^I)}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)} - 1)\} < \gamma < 1 = \overline{\gamma}$

2. $\underline{\beta} = \max \quad \{0, \frac{1-4\bar{\alpha}\psi x^I - 2(4\bar{\alpha}\psi x^I)^2}{4\bar{\alpha}\psi(x^A - x^I)(8\bar{\alpha}\psi x^I + 1)}\} < \beta < \overline{\beta} = min \quad \{\frac{1-4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)}, \frac{4\bar{\alpha}\psi(2x^A - x^I) - 1}{4\bar{\alpha}\psi(x^A - x^I)}\}$

3. $x^A > \underline{x^A} = max \quad \{\frac{1+4\bar{\alpha}\psi x^I}{8\bar{\alpha}\psi}, \frac{1+4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(1+8\bar{\alpha}\psi x^I)}\}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x^A ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 3: $(x_1^*|D = 1) = (\beta x^A + (1 - \beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)$**,** $(x_1^*|D = 0) = (\beta x^A + (1 - \beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma)$ The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2} \tag{63}$$

$$\beta < \frac{1 - 4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)} \tag{64}$$

$$\gamma < \frac{1}{8\bar{\alpha}\psi x^I}\left(\frac{1}{4\bar{\alpha}\psi(\beta x^A + (1 - \beta)x^I)} - 1\right) \tag{65}$$

Additionally, the equilibrium condition for the ally is

$$\begin{aligned} &-[(\beta x^A + (1 - \beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)) \\ &-x^A]^2 - [1 - 4\bar{\alpha}\psi\gamma(\beta x^A + (1 - \beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)](x^I + x^A)^2 \\ &-[4\alpha\psi\gamma(\beta x^A + (1 - \beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)](x^I - x^A)^2 > \\ &-[(\beta x^A + (1 - \beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma)) - x^A]^2 \\ &-[1 - 4\bar{\alpha}\psi(1 - \gamma)(\beta x^A + (1 - \beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma))](x^I - x^A)^2 \\ &-[4\bar{\alpha}\psi(1 - \gamma)(\beta x^A + (1 - \beta)x^I)(1 - 8\bar{\alpha}\psi x^I(1 - \gamma))](x^I + x^A)^2 \end{aligned} \tag{66}$$

Let $x^D = (\beta x^A + (1 - \beta)x^I)(1 + 8\bar{\alpha}\psi x^I\gamma)$. We can rewrite the above as:

$$\begin{aligned} &-(x^D - x^A)^2 - (1 - 4\bar{\alpha}\psi x^D\gamma)(x^I + x^A)^2 - 4\bar{\alpha}\psi x^D\gamma)(x^I - x^A)^2 > \\ &-(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1 - \beta)x^I) - x^A)^2 \\ &-(1 - 4\bar{\alpha}\psi(1 - \gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1 - \beta)x^I))(x^I - x^A)^2 \\ &-4\bar{\alpha}\psi(1 - \gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1 - \beta)x^I)(x^I + x^A)^2 \end{aligned} \tag{67}$$

Which reduces to

$$-4x^Ix^A + 16\bar{\alpha}\psi x^D x^I x^A \gamma > \tag{68}$$
$$-(8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I))^2 + 16\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I)(x^D - x^A)$$
$$-16\bar{\alpha}\psi x^I x^A(1-\gamma)(x^D - 8\bar{\alpha}\psi x^I(\beta x^A + (1-\beta)x^I))$$

By substituting $x^D = (\beta x^A + (1-\beta)x^I)(1 + 8\bar{\alpha}\psi(x^I)^2\gamma)$ and solving for γ we obtain condition:

$$\gamma > \frac{x^A + 4\bar{\alpha}\psi(\beta x^A + (1-\beta)x^I)(1 - 4\bar{\alpha}\psi x^I)(\beta x^A + (1-\beta)x^I - 2x^A)}{2x^I(4\bar{\alpha}\psi)^2(\beta x^A + (1-\beta)x^I)(-\beta x^A - (1-\beta)x^I + 2x^A)} \tag{69}$$

Thus the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \quad \{\frac{1}{2}, \frac{x^A+4\bar{\alpha}\psi(\beta x^A+(1-\beta)x^I)(1-4\bar{\alpha}\psi x^I)(\beta x^A+(1-\beta)x^I-2x^A)}{2x^I(4\bar{\alpha}\psi)^2(\beta x^A+(1-\beta)x^I)(-\beta x^A-(1-\beta)x^I+2x^A)}\} < \gamma <$ $min \quad \{1, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi(\beta x^A+(1-\beta)x^I)} - 1)\} = \overline{\gamma}$
2. $\underline{\beta} = max \quad \{0, 1-\frac{1}{2}\sqrt{\frac{x^I(4\bar{\alpha}\psi x^A)^2+4\bar{\alpha}\psi(x^A)^2-x^A}{\bar{\alpha}\psi(x^A-x^I)^2(1+4\bar{\alpha}\psi x^I)}}\} < \beta < min \quad \{\frac{1+2x^I(4\bar{\alpha}\psi)^2(x^A-x^I)-\sqrt{1+4(4x^Ax^I\bar{\alpha}\psi)^2(4\bar{\alpha}\psi)^2}}{32\bar{\alpha}^2\psi^2x^I(x^A-x^I)},$ $\frac{1-4\bar{\alpha}\psi x^I(1+4\bar{\alpha}\psi x^I)}{4\bar{\alpha}\psi(x^A-x^I)(1+4\bar{\alpha}\psi x^I)}\} = \overline{\beta}$
3. $x^A > max \quad \{\frac{1}{4\bar{\alpha}\psi(1+4\bar{\alpha}\psi x^I)}, \frac{x^I(1-(4\bar{\alpha}\psi x^I)^2)}{1-2(4\bar{\alpha}\psi x^I)^2}, \frac{1+4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(1+8\bar{\alpha}\psi x^I)}\}$
4. $x^I < \frac{\sqrt{5}-1}{8\bar{\alpha}\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The conditions on x^A and x^I ensure that the range $[\underline{\beta}, \overline{\beta}]$ exists. □

Corollary 2A: *Suppose that* $\frac{1}{8\bar{\alpha}\psi} < x^I$ *and* $\frac{1}{4\bar{\alpha}\psi} < x^A < \frac{1}{4\bar{\alpha}\psi(1-2\bar{\alpha}\psi x^I)}$. *Then, for all* $\beta \in [0,1)$, *there exists a non-measure zero set* $\Gamma(\beta)$ *such that if* $\gamma \in \Gamma(\beta)$, *then dissent by an extreme ally occurs in equilibrium*

Proof. From an analysis of the cases above we can verify that sufficient conditions for the claim (for all $\beta \in [0,1)$, there exists a non-measure zero set $\Gamma(\beta)$) to hold are:

- The binding upper bound $\overline{\beta}$ in case 1 is $= 1$

- The binding lower bound $\underline{\beta}$ in case 2 is $= 0$
- The binding upper bound $\overline{\beta}$ in case 2 is $= \frac{1-4\bar{\alpha}\psi x^I}{4\bar{\alpha}\psi(x^A - x^I)}$ (which is also the lower bound from case 1)

For the three conditions to be satisfied we need $\frac{1}{4\alpha\psi} < x^A < \frac{1}{4\alpha\psi(1-2\alpha\psi x^I)}$ and $x^I > \frac{1}{8\alpha\psi}$ □

Appendix C: Dissent by a Moderate Ally

In this section I consider an ally whose bliss point is to the left of the incumbent: $0 < x^A < x^I$. In line with the rest of the paper, I maintain the assumption that $x^I < \frac{1}{4\bar{\alpha}\psi}$.

Proposition 2A: *There exist $\underline{\gamma}_m$, $\overline{\gamma}_m$, $\overline{x^A}_m$ and $\overline{x^I}_m$ such that the incumbent's moderate ally chooses to dissent in equilibrium if and only if:*

1. *The party is trailing, but its disadvantage is not too large*
   ($\underline{\gamma}_m < \gamma < \overline{\gamma}_m$, where $\overline{\gamma}_m \leq \frac{1}{2}$)
2. *Electoral cost of dissent sufficiently large to turn trailing incumbent into sure loser ($\delta \geq \overline{\delta}$)*
3. *Both the incumbent and his ally are sufficiently moderate*
   ($x^I < \overline{x^I}_m$ and $x^A < \overline{x^A}_m$)

Proof. The proof of the first point (incumbent must be trailing) is omitted, since it is obtained by applying the same logic used in proving Proposition 1 (i.e. it is easy to verify given the calculations in the proof of Proposition 1 that dissent never emerges if $x^A < x^I$ and $\gamma > \frac{1}{2}$). To prove the remainder of the proposition I must analyse all possible pairs of equilibrium policies. From above we know that in any equilibrium in which the ally chooses to dissent $(x_1^*|D=1) = x^I$, and that dissent never occurs in equilibrium if $x^I \geq \frac{1}{4\bar{\alpha}\psi}$. Therefore, we must consider two cases:

- $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1) = x^I$ and $(x_1^*|D=0) = \frac{1}{4\bar{\alpha}\psi}$
- $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1) = x^I$ and $(x_1^*|D=0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$

I will analyse each case separately, conjecturing the existence of an equilibrium in which the ally chooses to dissent.

Case 1: $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma \geq \frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1) = x^I$ and $(x_1^*|D=0) = \frac{1}{4\bar{\alpha}\psi}$. Given the incumbent's best response, the ally dissents if and only if

$$-(x^I - x^A)^2 - (x^I + x^A)^2 > -(\frac{1}{4\bar{\alpha}\psi} - x^A)^2 - \gamma(x^I - x^A)^2 - (1-\gamma)(x^I + x^A)^2 \tag{70}$$

Which reduces to

$$\gamma < \frac{-8\bar{\alpha}\psi x^A(1 - 4\bar{\alpha}\psi x^I) - (4\bar{\alpha}\psi x^I)^2 + 1}{(8\bar{\alpha}\psi)^2 x^I x^A} \tag{71}$$

Thus, the conjectured equilibrium exists if and only if the following holds: **1)** $\frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1) < \gamma < min \quad \{\frac{1}{2}, \frac{-8\bar{\alpha}\psi x^A(1-4\bar{\alpha}\psi x^I)-(4\bar{\alpha}\psi x^I)^2+1}{(8\bar{\alpha}\psi)^2 x^I x^A}\}$, **2)** $\frac{\sqrt{5}-1}{8\bar{\alpha}\psi} < x^I < \frac{1}{4\bar{\alpha}\psi}$ and **3)** $x^A < \frac{x^I}{2}$. The conditions on x^I and x^A ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists.

Case 2: $x^I + 8\bar{\alpha}\psi(x^I)^2\gamma < \frac{1}{4\bar{\alpha}\psi}$, therefore $(x_1^*|D=1) = x^I$ and $(x_1^*|D=0) = x^I + 8\bar{\alpha}\psi(x^I)^2\gamma$. In this case, the ally chooses to dissent if and only if

$$\begin{aligned} -(x^I - x^A)^2 - (x^I + x^A)^2 > \\ -(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma - x^A)^2 - (1 - 4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma))(x^I + x^A)^2 \\ -4\bar{\alpha}\psi\gamma(x^I + 8\bar{\alpha}\psi(x^I)^2\gamma)(x^I - x^A)^2 \end{aligned} \tag{72}$$

Which reduces to

$$x^A < \frac{x^I}{2} \tag{73}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied **1)** $0 < \gamma < min \quad \{\frac{1}{2}, \frac{1}{8\bar{\alpha}\psi x^I}(\frac{1}{4\bar{\alpha}\psi x^I} - 1)\}$, **2)** $x^A < \frac{x^I}{2}$ and **3)** $x^I < \frac{1}{4\bar{\alpha}\psi}$.

This concludes the proof. □