With Friends Like These, Who Needs Enemies?

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Abstract

Why do politicians publicly attack the leaders of their own party, even when they have no opportunistic reasons to do so and such attacks are electorally costly? The paper addresses this question by presenting a model in which the leader faces a preference conflict with dissenting members of his party, and voters are learning about their own policy preferences over time. Here, by publicly attacking the leader (and thereby harming the party in the upcoming election), the dissenters can change his incentives to choose more or less extreme policies, which affects the amount of voter learning. This induces a trade-off between winning the current election and inducing the party leadership to pursue the dissenters' all-things-considered more-preferred policy. Optimally balancing this trade-off sometimes involves public dissent that damages the party in the short-run. In equilibrium open dissent arises precisely because it is electorally costly, in order to induce a policy response by the leader.

"He has no idea how to conduct himself as a leader."

– Peter Mandelson (Labour) on UK Labour leader Jeremy Corbyn[1]

"He shows a growing inability, and even unwillingness, to separate truth from lies."

– John McCain (Republican) on US Republican President Donald Trump[2]

"He is worse than the Devil."

– Massimo D'Alema (PD) on Italy's Prime Minister Matteo Renzi (PD)[3]

During his 1966 electoral campaign for the California gubernatorial elections, Ronald Reagan issued a call for unity in his party: "Thou shalt not speak ill of a fellow Republican". This warning would famously come to be known as the "eleventh commandment", reflecting the awareness that public manifestations of internal conflict are typically damaging for political parties. Indeed, both survey evidence and experimental results show that, everything else being equal, voters tend to dislike parties that appear divided (e.g., YouGov (2016); Greene and Haber (2015); Groeling (2010)). Yet, despite these costs, politicians routinely break Reagan's eleventh commandment, so much so that a party leader's fiercest critics are often his own copartisans, as the above quotes document (see also Groeling (2010)).

What, then, motivates politicians to engage in this form of friendly fire against the leaders of their own parties? Existing theories posit that politicians may attack their copartisans in order to increase their electoral chances (Carey and Shugart, 1995; Kirkland and Slapin, 2018; Invernizzi, 2019; Buisseret and Prato, 2020). In these frameworks, politicians face a tradeoff between the party's collective electoral performance and their individual success, and may therefore choose to publicly dissent *despite* the associated electoral cost for the party.

Here, I complement these approaches and show that politicians may openly attack their party leader even when they face no such tradeoff. Even absent competing electoral concerns (or a conflict

[1] $https://www.theguardian.com/politics/2017/feb/21/peter-mandelson-i-try-to-undermine-jeremy-corbyn-every-day$

[2] https : //www.washingtonpost.com/news/the fix/wp/2017/02/17/john mccain just systematicallydismantleddonaldtrumpsentireworldview/?utmterm = .15a944873049

[3] https://www.repubblica.it/politica/2016/06/15/news/dalema$_renzi - 142045544$/

for the control of the party), I show that electorally costly dissent may emerge as a tool to recompose internal divisions, realigning the policy tastes of the leader with those of opposing factions within the party. In my framework, this form public open dissent emerges precisely *because* it is electorally costly.

The argument is as follows. A leader may often face an ideological conflict with members of his party: even though they come from the same side of the political spectrum, they may not share exactly the same policy preferences (Janda, 1980). The leader makes policy choices as a function of both his ideological tastes and his electoral incentives. While the leader's misaligned copartisans cannot change his ideological preferences, they can try to *indirectly* influence his policy choices by strategically manipulating his electoral incentives. For the party leader, some policy choices are in fact inherently riskier than others (Dewan and Hortala-Vallve, 2019). For example, bold policy experiments may fail, and thus damage the leader's electoral chances, or succeed, and boost his prospects. Safer policy choices instead have less uncertain outcomes, and thus allow the incumbent to avoid risks. Depending on his electoral prospects, the leader will either have incentives to take risky choices or avoid policy gambles. By publicly attacking him, and thereby hurting his electoral prospects, the leader's copartisans can therefore influence his policy choice. This generates a potential tradeoff between maximizing the party's electoral prospects and inducing the leader to adopt a policy more in line with the dissenters' own ideological preferences. If the gain from changing today's policy is sufficiently large, electorally costly public dissent emerges in equilibrium. Thus, rather than representing a mere manifestation of conflict or a first step towards a party split, public dissent here serves the purpose of mitigating the ideological disagreements within the party.

The contribution of this paper is threefold. First, I microfound this argument within the context of an electoral accountability model, where I introduce a new framework to analyze policy gambles along an ideological spectrum. Second, I characterize conditions under which public dissent is more likely to emerge and analyze its impact on voter welfare. Finally, I describe the implications of the theory for empirical research on party unity in elections. While the model is not meant to capture all instances of dissent (nor to exactly fit any specific case), it uncovers a set of strategic incentives that have so far remained unexplored, i.e., incentives to publicly dissent in order to influence the party leader's willingness to engage in risky policies. Complementing existing theories, this paper thus hopes to enrich our understanding of intra-party dynamics and patterns of dissent in the real world.

In the model, an incumbent implements a policy along the left-right spectrum. Upon observing his policy choice and the resulting outcome, a representative voter chooses whether to reelect him or replace him with a challenger from the opposing party. In addition, before the incumbent decides which policy to implement, an ideologically misaligned member of his party chooses whether to publicly dissent against him. Open dissent is electorally costly: it generates an endogenous valence shock that disfavors the party in the upcoming election.

In this setting the voter faces a selection problem, which is complicated by the fact that she is unsure of which policy is optimal for her. We can think about this uncertainty as pertaining to the consequences of the various policy choices: a voter may be unsure of how different policies map into outcomes, or face uncertainty about the impact that certain policies/outcomes will have on her own welfare. Faced with this uncertainty, the voter tries to learn about her own optimal platform by observing how much she likes or dislikes the outcome of today's policy (similar to the literature on micropartisanship, e.g., Fiorina (1982), Achen (1992), Gerber and Green (1998)).

In this framework, the amount of voter learning depends on the exact location of the implemented policy along the left-right spectrum. In particular, I show that the more extreme the implemented policy is, the more a Bayesian voter learns upon observing the resulting outcome. Thus, extreme policies represent riskier experiments for the office holder: they increase the likelihood that the voter discovers her true preferences, which may or may not turn out to be aligned with the incumbent's ideological stances.

As a consequence, the incumbent has incentives to control information. His equilibrium policy choice thus maximizes the tradeoff between implementing his preferred policy today and generating the optimal amount of information to be reelected tomorrow. This, I show, is a function of his ex-ante electoral strength. A leading incumbent, who is going to be re-elected if the voter receives no new information, has incentives to implement moderate platforms that prevent voter learning. A trailing one instead wants to engage in more extreme policies that facilitate information generation, in hopes of improving his electoral prospects. Finally, an incumbent that has no chances of winning the upcoming election has no reason to engage in information control, and simply follows his ideological preferences.

Turn now to the incumbent's misaligned copartisan. By publicly dissenting and attacking the incumbent, they generate a negative valence shock against him and thereby reduce his ex-ante electoral strength. This, in turns, creates incentives to implement more or less informative (i.e., extreme) policies. As such, public dissent changes the incumbent's equilibrium policy choice, while also harming the party's electoral chances and thus the dissenter's own future expected payoff. This generates a potential trade-off for the incumbent's copartisans. Optimally balancing this trade-off sometimes involves public dissent that damages the party's electoral prospects but induces the incumbent to implement a policy more in line with the dissenters' ideological preferences.

Surprisingly, the analysis reveals that intraparty polarization plays an ambiguous role. Reducing the intensity of the ideological conflict within the party may actually make public dissent *more* likely to materialize. This result emphasizes the peculiar nature of dissent in this model. Far from representing the first step towards a party split, public dissent here brings about unity, by realigning the interests of the incumbent and his copartisans. However, for dissent to be effective, the ideological conflict within the party cannot be too deep. Further, I show that public dissent may be more likely to emerge when the incumbent is not a policy dictator, i.e., if the implemented policy is the result of a bargaining process within the party. Thus, electorally costly dissent may complement, rather than simply substitute for, other less disruptive tools to influence policy. Finally, even if the incumbent's copartisans participate in the division of the spoils of office, I show that increasing the value of office rents may actually make electorally costly dissent more likely. This is because increasing office rents increases the incumbent's incentives to control information, and thus increases the impact of costly dissent on his policy choice. This in turn strengthens the dissenter's incentives to attack him.

The results also highlight that the presence of an extreme faction in the incumbent party may improve voter welfare. Voters benefit from informative policies being implemented as this increases the probability of making the correct electoral decision in the future. However, under some conditions, reelection incentives induce lower levels of policy experimentation relative to both the incumbent's ideological preferences and the voters' optimum. By publicly attacking him, the dissenter can incentivize the incumbent to take risks by implementing extreme policies that allow the voters to learn, and thus mitigate this inefficiency.

Additionally, the theory presented here has important implications for empirical research on party unity in elections. My setup suggests that there are two ways to think about the electoral cost of intra-party dissent. One way is to directly consider how perceptions of internal divisions influence voters' evaluation of political parties, i.e., a model primitive (specifically, the size of the valence shock). The surveys and experiments mentioned earlier adopt this approach (YouGov (2016); Greene and Haber (2015); Groeling (2010)). An alternative approach is instead to look at the party's realized electoral performance and how it is impacted by the occurrence of public dissent, i.e., an equilibrium outcome. In other words, this second approach compares the electoral success of parties that do and do not experience dissent. The results of my model highlight that this approach is likely to yield biased estimates. Furthermore, the bias may go in either direction. However, this does not imply that we cannot empirically investigate the equilibrium consequences of intraparty conflict. The model generates testable predictions regarding parties' electoral performance *conditional on experiencing public dissent*: it should be positively correlated with variables such as the electorate's level of education, news media consumption and political engagement. Focusing on the treated units, researchers can thus empirically identify the conditions under which open dissent is expected to hurt parties the most.

In this project, I consider public dissent coming from actors *within* the leader's party. In concluding the paper, I briefly discuss how my mechanism may also describe the strategic incentives of the incumbent's ideological allies *outside* the party, such as coalition partners, media outlets, external donors or even special interest groups. When such actors come from the leader's own ideological camp but nonetheless face a preference conflict with him, they may experience a tradeoff analogous to the one described above. They may thus choose to publicly attack the leader, criticize him, or even reduce their electoral contributions in order to influence his policy choice, even if (and indeed precisely because) by doing so they hurt their preferred party's electoral prospects.

Relation to Existing Theories and Competing Explanations

An important body of work within the literature on intra-party politcs considers the problem that individual politicians face when they are subject to two principals: the party and their own constituents (Carey and Shugart, 1995; Kirkland and Slapin, 2018; Buisseret and Prato, 2020).[4] If a politician's local constituency is opposed to the national party line, this politician faces a trade-off between the national party's electoral fortunes and his own success. Within this framework, dissent may serve the purpose of signaling the politician's misalignment with the leadership and alignment with the constituency.[5]

This paper complements the existing literature, by presenting a theory that may help us understand why electorally costly dissent emerges even when the party members' individual electoral motives do not provide incentives (or even provide disincentives) to attack the leadership. For example, according to the 'dual principals' framework, incentives to publicly dissent should emerge under first past the post or open-list proportional electoral systems. In closed-list systems, where the leader controls the list composition and as such the individual candidates' electoral fate, incentives to publicly attack him should be much weaker, if even present.[6] Yet, Proksch and Slapin (2015) exploit the German mixed-member proportional electoral system and show that members elected with a party-list vote are as likely to dissent as those elected with a constituency vote. If we look across different countries, public manifestations of dissent emerge in majoritarian systems such as the UK and proportional closed-list ones such as Italy. Similarly, if open dissent is motivated by

[4]Other scholars study the optimal level of internal cohesion from a party's perspective (e.g., Matakos et al. (2018).

[5]Snyder and Ting (2002) also look at party discipline within the context of a signaling model, but focus on the (expressive) ideological cost of joining a party whose position is misaligned with the individual politician's.

[6]Invernizzi (2019) considers a different setting, where competing factions attack each other to obtain rewards from a party leader who stands above factions.

competing (individual and collective) electoral concerns, we should expect the individual dissenters to be in a relatively weak electoral position. However, Proksch and Slapin (2015) analyze data from the UK and Germany and show that, if anything, the opposite holds. Public dissent is (weakly) more likely to come from members of parliament elected with a larger margin.

In addition, most of the literature on dissent in legislative politics assumes that the party leadership controls the agenda (Slapin et al., 2018). The leader makes a proposal and then legislators choose whether to dissent by expressing their disagreement on the specific issue on the table or by voting against the proposal. In my model, instead, dissent takes the form of a public attack against the leader aimed at generating a negative valence shock, and is thus not necessarily tied to a specific policy issue or dimension (as illustrated in the quotes at the beginning of this paper). Furthermore, here the order of play is reversed: public dissent comes first and the leader responds by potentially altering his policy choice. Thus, my theory may allow us to understand forms of dissent that may emerge at any time and on any issue, and that may therefore not fit the conventional arguments developed in relation to legislative dissent.

An alternative set of arguments considers electorally costly dissent as merely a tool to weaken the current party leader, and make it easier to depose him/her. My model does not allow for leadership turnover since my goal is to understand why and when electorally costly public dissent may emerge even *absent* such opportunistic considerations. However, in a separate section below I discuss how such dynamics may be incorporated into my framework, and may indeed complement the mechanism I propose.

The core of the model presented here is the voter's retrospective learning process. Voters face uncertainty about their optimal policy, and learn by observing the outcome of platforms implemented in the past. In this respect, the paper is closely related to recent work by Callander (2011). The author considers a world in which voters know whether right-wing or left-wing policies tend to generate better outcomes, but experiment to learn about the exact consequences of each policy program. In this paper I propose a different framework to think about policy experimentation, in which the nature of uncertainty is reversed. Voters aim to learn whether liberal or conservative platforms are optimal in expectation, even though the exact consequences of each policy are somewhat unpredictable. This allows me to think about policy experimentation in connection to ideology, and generates the result that extreme policies, rather than small incremental changes as in Callander, produce more information. Additionally, Callander focuses on the *statically* optimal choice for a decision maker. He thus chooses to abstract from dynamic electoral considerations, by assuming either myopic players (Callander, 2011) or exogenous re-election probabilities (Callander and Hummel, 2014). Instead, the focus of this paper is precisely on the incumbent's *dynamic* incentives to control information.

The paper also relates to the literature on Bayesian Persuasion (Austen-Smith, 1998; Kamenica and Gentzkow, 2011). In my model, as in the Bayesian Persuasion framework, the incumbent can engage in information control by manipulating the receiver's posterior distribution. In the Bayesian Persuasion framework the mechanism through which this happens is somewhat black-boxed. The key innovation of my paper is to explicitly model *how* this manipulation occurs, by looking at the impact that the *implemented policy* has on voter learning.

Finally, the normative findings speak to the literature on negative campaigning. This literature shows that, within the context of a signaling game, information about political candidates may be more credibly transmitted when each candidate focuses on the opponent's flaws rather than their own merit (Polborn and Yi, 2006; Boucek, 2009). Thus, negative campaigns can be good for the voters. Here I uncover an analogous welfare improving effect of political attacks coming from a candidate's own allies, within the context of a policy experimentation framework.

The Model

Players and actions. Consider a two-period political agency model. The players are an incumbent office holder I, a misaligned member of his party M, a challenger C, and a representative voter V. Here, M can represent an individual politician or also a faction within the party. I and M come from the same side of the political spectrum (i.e., are both right-wing or left-wing), but do not share exactly the same preferences. At the beginning of the game, the incumbent's misaligned copartisan either *publicly* expresses his dissent against him or remains silent, $a_d \in \{0,1\}$. The incumbent implements a policy $x_1 \in \mathbb{R}$. The voter observes the dissenter's choice, the incumbent's policy choice and the resulting outcome (i.e., the realization of her own policy payoff), and decides whether to retain the incumbent or replace him with the challenger. Whoever is elected in the second period implements a new policy $x_2 \in \mathbb{R}$.

Payoffs and information. The voter's per-period *policy* utility is

$$U_t^V = -(x_t - x_V)^2 + \varepsilon_t,$$

where x_t is the policy implemented at time t, ε_t is a shock i.i.d. in each period from a uniform distribution over $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$, and x_V is an unknown state of the world, indicating the voter's optimal policy. Here, this policy can take one of two values, that for simplicity but without much loss of generality I assume to be symmetric around 0: $x_V \in \{-\alpha, \alpha\}$. Thus, if $x_V = \alpha$, the voter's true optimal policy is a right-wing one. Otherwise, if $x_V = -\alpha$, the voter's optimum is a left-wing platform. We can interpret uncertainty over x_V as referring to the possible consequences of the various policy choices (i.e., the function mapping policies into outcomes). Alternatively, we can think about a voter who sees policy as an experience goods, and therefore does not know how different policies will impact her welfare until such policies are implemented. Since the focus of the model is on the incumbent's incentives to take risks in policymaking, I shut down any asymmetry of information: all players share common prior beliefs that $p\left(x_V = \alpha\right) = \gamma$.

In addition to her expected policy utility, the voter's evaluation of the incumbent is a function of the dissenter's choice: public dissent generates an endogenous valence shock against the incumbent.

Formally, the voter re-elects the incumbent if and only if

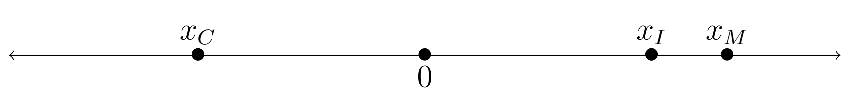
$$E[U_2^V(I)] - \alpha_d \delta \geq E[U_2^V(C)],$$

where $E[U_2^v(I)]$ is the voter's expected second-period policy utility from re-electing the incumbent, and $E[U_2^v(C)]$ is the policy utility she expects from the challenger. Recall that $\alpha_d = 1$ if M publicly dissents and attacks the incumbent, and $a_d = 0$ otherwise. For purposes of presentation, I leave the valence shock δ black boxed, simply assuming that open dissent mechanically reduces the party's appeal to the voters. We could microfound this electoral cost in a richer game where observing dissent causes voters to negatively update their evaluation of the incumbent. As I discuss further in a separate section, this would not alter the results presented below.

Finally, politicians are policy motivated. In each period, politician $j \in \{M, I, C\}$ gets utility

$$U_t^j = -(x_j - x_t)^2,$$

where $x_C < 0 < x_I$, i.e., I consider a right-wing incumbent running against a left-wing challenger. For simplicity, the candidates' bliss points are symmetric around 0, $x_C = -x_I$. Finally recall that, although $x_I, x_M > 0$, I assume $x_I \neq x_M$: the incumbent and his misaligned copartisan come from the same side of the spectrum, but have different ideological stances. In the main body, I focus on an extreme dissenter, $x_M > x_I$.



In Appendix C, I instead consider a dissenter whose ideological preferences are more moderate than the incumbent's, $0 < x_M < x_I$, and identify conditions under which public dissent emerges in equilibrium.

Notice that, here, the politicians' bliss points are common knowledge: the voter faces no uncertainty about the policy stances of the two parties. In a separate section, I briefly discuss the results' robustness if we allow the incumbent to have private information about his ideological preferences.

Timing. To sum up interaction unfolds as follows:

1. Nature determines the value of $x_V \in \{\underline{\alpha}, \alpha\}$, which remains unknown to all players,
2. M chooses whether to publicly dissent: $a_d \in \{0, 1\}$,
3. I implements a policy $x_1 \in \mathbb{R}$,
4. V observes the policy choice x_1 and her own utility realization U_1^V, and chooses whether to re-elect I or replace him with C,
5. The second-period office holder implements policy $x_2 \in \mathbb{R}$,
6. Second-period payoffs realize and the game ends.

The equilibrium concept is Perfect Bayesian Equilibrium.

Equilibrium Analysis

As usual, we proceed by backwards induction. In the second period the office holder faces no electoral pressures, and will always implement his preferred platform. Thus, the voter faces a selection problem. Her electoral choice is therefore a function of two elements: the (posterior) belief that her own ideal policy is aligned with the incumbent's, and whether the incumbent experiences public dissent, i.e., the *endogenous* valence shock. Specifically, denote μ the voter's posterior belief that her own bliss point takes a positive value ($\mu = prob(x_V = \alpha)$). Then, we have that the voter prefers to reelect the incumbent if and only if

$$-\mu(x_I - \alpha)^2 - (1-\mu)(x_I + \alpha)^2 - a_d\,\delta \geq -\mu(x_C - \alpha)^2 - (1-\mu)(x_C + \alpha)^2.$$

Lemma 1 follows straightforwardly, recalling that $x_C = -x_I$:

Lemma 1. *The voter reelects the right-wing incumbent if and only if:*

$$\mu > \frac{a_d\,\delta + 4\alpha x_I}{8\alpha x_I} \equiv \widehat{\mu}(a_d) \tag{1}$$

Absent public dissent and the resulting negative valence shock ($a_d = 0$), the incumbent is reelected as long as the voter believes that her own preferences are more likely to be aligned with his than the challenger's ($\widehat{\mu}(0) = \frac{1}{2}$). If instead the incumbent does experience open dissent ($a_d = 1$), the voter's posterior needs to be sufficiently high so as to offset the valence cost δ ($\widehat{\mu}(1) > \frac{1}{2}$). Moving one step backward, we must therefore analyze how the posterior μ is formed.

Voter Learning and Policy Extremism

Consider the voter's inference problem. Here, the voter observes the consequences of the first-period implemented platform (i.e., her realized policy utility U_1^V), and updates her beliefs about the location of her optimal policy x_V. The higher her realized policy utility, the more confident the voter becomes that the implemented policy is aligned with her optimal platform.[7] However, her inference problem is complicated by the fact that this realized policy utility is also a function of a random shock: $U_1^V = -(x_V - x_1)^2 + \varepsilon_1$.

This, I show, has a crucial impact on the voter's learning process: the *amount* of learning will depend on the exact location of the implemented policy x_1. In particular, the voter learns more when more extreme policies are enacted. This is a consequence of two factors. First, extreme policies are more likely to produce extreme (i.e., very good or very bad) outcomes, to which a Bayesian voter correctly attributes larger informative value. Secondly, as the implemented policy gets more extreme, the distance in expected outcomes as a function of the true state (i.e., the voter's expected payoff under the two possible values of x_V) increases. As a consequence, the same outcome conveys more information to the voter if it results from a more extreme policy. These key features of the

[7] That is, that the implemented policy has the same sign as x_V.

learning process emerges starkly under a uniformly distributed shock ε_t:

Lemma 2. *The voter's learning satisfies the following properties:*

i. Her posterior μ takes one of three values: $\mu \in \{0, \gamma, 1\}$;

ii. The amount of learning is a function of the implemented policy x_1, as more extreme policies increase the probability that $\mu \neq \gamma$;

iii. There exists a policy x' such that if $|x_1| \geq |x'|$, then $\mu \neq \gamma$ with probability 1.

Lemma 2 tells us that the voter either learns everything or nothing. Further, the *probability* that the voter discovers the true value of x_V increases as the implemented policy becomes more extreme. Appendix A contains a formal proof of these results, but the underlying reasoning is easy to illustrate graphically.

In Figure 1, the solid lines represent the voter's *expected* first-period policy payoff as a function of the implemented policy x_1, for the two possible values of x_V.[8] The *realization* of the payoff, hwowever, is also a function of the shock ε_t. The dashed curves thus represent the maximum and minimum possible values of the payoff realization after accounting for the shock.[9]

As Figure 1 shows, the voter's payoff is, *in expectation*, always different under the two states of the world (for any $x_1 \neq 0$). However, the presence of the random shock creates a partial overlap in the support of the payoff *realization*. For any given policy $x_1 \in (-x', x')$, there exists a range of payoffs that may realize (i.e. be actually observed) whether the voter's true bliss point takes a positive or a negative value. Consider, for example, policy x as represented in the graph. Any payoff realization falling between the gray and black bullets may be observed with positive probability under both states of the world. Clearly, if the payoff realization falls outside this range, it constitutes a fully informative signal. There is only one state of the world that could have generated that specific realization: the voter simply likes the policy too much, or too little, for this

[8]The thick increasing solid curve is $-(x_1 - \alpha)^2$ and the thin decreasing solid curve is $-(x_1 + \alpha)^2$.

[9]The thick increasing dashed curves are $-(x_1 - \alpha)^2 + \frac{1}{2\psi}$ and $-(x_1 - \alpha)^2 - \frac{1}{2\psi}$. Conversely, the thin decreasing dashed curves are $-(x_1 + \alpha)^2 + \frac{1}{2\psi}$ and $-(x_1 + \alpha)^2 - \frac{1}{2\psi}$.

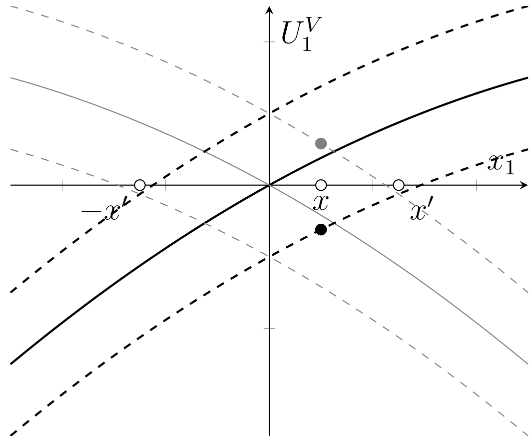


Figure 1: Voter's first-period payoff. The thick increasing (thin decreasing) curves represent the case in which $x_V = \alpha$ ($x_V = -\alpha$). The solid curves represent the voter's expected payoff, while the dashed ones represent the maximum and minimum possible realizations given ε_t.

to be justified as a consequence of the shock. Thus, upon observing her payoff, the voter discovers her true preferences (i.e. the value of x_V). Conversely, any payoff realization that falls inside the range of overlap is completely uninformative. Since the shock is uniformly distributed, any such realization has exactly the same probability of being observed under the two states of the world. Thus, the voter learns nothing and uses her prior beliefs. As the implemented policy becomes more extreme, the gray and black bullets get closer and closer to each other. The range of overlap becomes smaller, and the voter is more likely to discover her true preferences.[10]

Let me highlight that this feature of the learning process (extreme policies are more informative), does not depend on the assumption that ε_t is uniformly distributed. Consider for example a world in which the shock is normally distributed. The learning process is smoother, as any outcome realization is somewhat informative, but never fully so. Yet, it remains true that as the implemented policy becomes more extreme, the distance in the expected outcomes as a function of the state increases. This in turn increases each signal's informativeness. Extreme policies therefore still generate more information. Nonetheless, it is also important to emphasize that the mechanism uncovered in this paper relies solely on the fact that the implemented policy influences the amount

[10]Notice that public dissent does not interfere with the voter's learning process: even if the voter were to observe her overall utility (as opposed to the policy utility U_1^V), the valence shock would simply shift all the intercepts of the functions in Figure 1.

of information the voter receives. This is what allows a dissenter to influence the equilibrium policy. As such, the main insights of the paper (in relation to the mechanism and strategic incentives uncovered) would survive in a world in which more moderate, rather than more extreme, policies are more informative.

The incumbent's problem

Lemma 2 shows that the implemented policy influences the incumbent's utility via two channels: a static one, via his own ideological preferences, and a dynamic one, via the voter's learning and retention decision. By choosing to implement more or less extreme policies, the incumbent can in fact either facilitate or hinder the voter learning process. More formally, he can manipulate the variance in the voter's posterior distribution. Since this posterior determines the voter's electoral choice, the incumbent has incentives to engage in information control.

The incumbent's first-period equilibrium policy choice therefore optimizes the trade-off between implementing his own preferred policy today, and generating the optimal amount of information to be reelected tomorrow. The nature of this trade-off, I show, depends on the incumbent's ex-ante electoral strength. In what follows, I adopt the following definition:

Definition 1. *We say that an incumbent is:*

- *leading if* $\gamma > \widehat{\mu}(a_d)$*;*
- *trailing if* $1 > \widehat{\mu}(a_d) > \gamma$*;*
- *a certain loser if* $\widehat{\mu}(a_d) > 1$*.*

Recall that $\widehat{\mu}(a_d)$ is the retention threshold as a function of dissent, as defined in equation (1). γ is the prior probability that the voter's optimal policy is a right-wing one, $x_V = \alpha$. Then, a leading incumbent is guaranteed reelection when the voter learns nothing new. A trailing incumbent, instead, needs the voter to update in his favor in order to be retained. Finally, a certain loser is always ousted, regardless of whether (and what) the voter learns about her optimal platform.

Then, we have:

Lemma 3. *In equilibrium*

- *a leading incumbent implements a policy (weakly) more moderate than his bliss point,* $x_1^* \leq x_I$*;*
- *a trailing incumbent implements a policy (weakly) more extreme than his bliss point,* $x_1^* \geq x_I$*;*
- *a certain loser implements his bliss point,* $x_1^* = x_I$*.*

First, consider a leading incumbent. When the voter learns nothing new about her optimal policy, this incumbent is guaranteed reelection. If instead the voter observes an informative signal, the incumbent is reelected when the voter learns that her own optimal policy is a right-wing one, $x_V = \alpha$, and ousted otherwise. Thus, this incumbent can never gain from generating information. Indeed, his equilibrium policy solves:

$$-2(x_I - x_1) - 4x_I^2 Prob(L = 1|x_1)(1 - \gamma) = 0,$$

where $Prob(L = 1|x_1)$ is the probability that the voter observes an informative outcome realization, linearly increasing as x_1 moves away from 0 (see the proof of Lemma 2 in the Appendix). Even if the true state is very likely to be in his favor (that is, even if γ is arbitrarily close to 1), the incumbent's retention concerns create incentives to prevent voter learning. Because more moderate policies are less informative, we have that in equilibrium $x_1^* \leq x_I$.

The opposite holds for a trailing incumbent, who is always ousted if the voter learns nothing new. This incumbent has nothing to lose from generating information, and thus always wants to facilitate voter learning, even if it is likely to backfire (that is, even if γ is arbitrarily close to 0). His equilibrium policy solves:

$$-2(x_I - x_1) - 4x_I^2(1 - Prob(L = 1|x_1)\gamma) = 0.$$

Thus, we have that in equilibrium he always implements a policy (weakly) more extreme, and thus more informative, than his static optimum, $x_1^* \geq x_I$.

Finally, if the incumbent knows that he will always be ousted regardless of if and what the voter learns, he has no incentives to control information. In equilibrium, this certain loser simply follows his ideological preferences and implements $x_1^* = x_I$.

The equilibrium policy choices of trailing and leading incumbents are a function of γ, α, ψ and the incumbent's bliss point x_I, and are fully characterized in the Online Appendix. In the remainder of the paper, I maintain that $x_I < x'$, where x' is the smallest (positive) policy that produces an informative signal with probability 1 (Lemma 2). This assumption is imposed to reduce the number of cases under consideration, but does not alter the qualitative results presented below.

Public Dissent

Moving one step backward, we can finally analyze the incumbent's misaligned copartisan's (M) choice whether to publicly dissent against him. To avoid trivialities, I assume that when indifferent M chooses not to publicly dissent.

Lemma 3 emphasizes that the incumbent's equilibrium choice is a function of his ex-ante electoral prospects, which determine his willingness to engage in more or less risky policies. By openly dissenting and generating an endogenous valence shock against the incumbent, his copartisans can therefore influence his incentives to control information and thus his policy choice. This generates a potential tradeoff between maximizing the incumbent's electoral prospects (and thus his copartisan's own future expected payoff), and inducing him to implement a policy closer to the copartisan's own preferences. Here, I identify conditions under which optimizing this trade-off involves public and electorally costly dissent in equilibrium.

The first two results follow straightforwardly from Lemma 3:

Lemma 4. *Public dissent emerges in equilibrium only if, absent dissent, the incumbent is leading, i.e.,* $\gamma > \frac{1}{2}$.

Naive intuition may suggest that public dissent materializes during periods of electoral crisis: the leader is expected to perform poorly, and the ensuing internal turmoil degenerates into an open manifestation of conflict. Lemma 4 tells us that, in the case of an extreme dissenter, the opposite is true. First, suppose that $\gamma < \frac{1}{2}$. Then, absent dissent the incumbent is trailing and implements a policy more extreme than his ideological preferences, $x_1^* \geq x_I$. Here, dissent either has no impact on his policy choice (if δ is so small that it does not influence the voter's retention strategy), or it induces him to implement exactly his bliss point (if δ is so large that it turns the incumbent into a certain loser). Thus, public dissent causes the incumbent to implement a (weakly) more moderate policy *and* (weakly) reduces his electoral prospects. The incumbent's extreme copartisan therefore never chooses to publicly attack in equilibrium. Instead, when the incumbent is leading (i.e., $\gamma > \frac{1}{2}$), his misaligned copartisan can potentially gain from dissent by creating incentives to experiment with extreme policies.

Next, consider the electoral cost of public dissent δ:

Lemma 5. *Public dissent emerges in equilibrium only if its electoral cost is sufficiently high to turn the leading incumbent into a trailing one, but not so high that the incumbent becomes a certain loser, i.e.,* $4\alpha x_I(2\gamma - 1) < \delta < 4\alpha x_I$.

Intuitively, public dissent never emerges if the resulting the valence shock δ is so large that it makes the incumbent lose for sure, i.e., if $\delta > 4\alpha x_I$. From Lemma 4, we know that the misaligned copartisan may only ever choose to dissent against a leading incumbent. Recall that when the incumbent is leading, absent dissent we would have $x_1^* < x_I$. Now assume instead that this leading incumbent experiences public dissent, and this generates a valence shock $\delta > 4\alpha x_I$. The incumbent anticipates that he will lose the upcoming election regardless of what the voter learns, and implements his static optimum, $x_1^* = x_I$. Then, public dissent is effective in moving the equilibrium policy to the extreme towards the dissenter's preferred platform, but it does not induce the incumbent to move beyond his static optimum. This policy gain is too small to compensate the dissenter for the high cost of condemning the party to electoral failure, and public dissent never emerges in equilibrium.

At the same time, however, open dissent is never observed if δ is too small. In particular, if $\delta < 4\alpha x_I(2\gamma - 1)$, the dissenter's choice has no effect on the voter's electoral strategy: even if he experiences open dissent, the incumbent remains electorally leading. Consequently, public dissent has no impact on the equilibrium policy and the misaligned copartisan has no reason to express dissent in the first place. Thus, δ must be sufficiently large so as to turn a leading incumbent into a trailing one: open dissent emerges precisely *because* it is electorally costly.

Lemmas 4 and 5 have characterized the parameter values under which public dissent moves the incumbent's equilibrium choice to the extreme, away from the incumbent's bliss point and towards the dissenter's own preferred policy. This is necessary for open dissent to emerge in equilibrium, but not sufficient. Proposition 1 completes the analysis by identifying conditions under which the gain from moving the equilibrium policy today outweighs the cost from reducing the incumbent's electoral prospects tomorrow:

Proposition 1. *Suppose that $\gamma > \frac{1}{2}$ and $4\alpha x_I(2\gamma - 1) < \delta < 4\alpha x_I$. Then, there exist unique $\underline{x_I}$, $\underline{x_M}(x_I)$ and $\underline{\gamma}(x_I)$ such that that public dissent emerges in equilibrium if and only if*

- *The incumbent's lead is sufficiently large, $\gamma > \underline{\gamma} \geq \frac{1}{2}$, and*
- *Both the incumbent and his misaligned copartisan are sufficiently extreme, $x_I > \underline{x_I}$ and $x_M > \underline{x_M}$.*

The first result is intuitive. In equilibrium, an incumbent that experiences open dissent wins reelection if and only if the voter learns that her own ideal policy is a right-wing one, $x_V = \alpha$. When γ is low this event is very unlikely, and dissent is too costly for the incumbent's copartisan. Recall that as γ increases the voter's ex-ante preferences move to the right. Substantively, this indicates that open dissent emerges only if the incumbent's electoral lead is sufficiently large.

Finally, consider the impact of the ideological misalignment between the incumbent and his copartisan. Such misalignment represents the only source of conflict in the model. A naive observer may therefore conclude that public dissent should always be observed when the incumbent's and dissenter's ideological preferences are far apart. Proposition 1 shows that this intuition needs to be

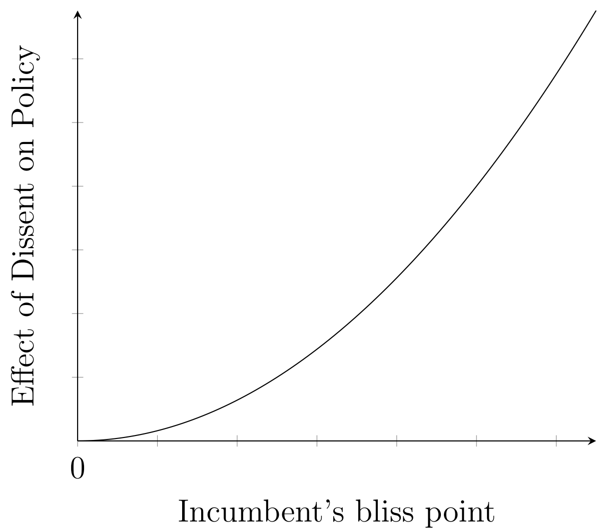


Figure 2: Effect of public dissent on the equilibrium policy (i.e., $x_1^*(a_d = 1) - x_1^*(a_d = 0)$)

qualified: when the incumbent is too moderate, his extreme copartisan will never openly dissent against him. To understand this result, recall that public dissent aims at generating incentives for the incumbent to gamble with extreme policies. However, if the incumbent is too moderate, such incentives are too weak: gambling is too costly, and not very valuable. It is costly as it entails implementing extreme policies, potentially very far from the incumbent's bliss point. It is not very valuable since for a moderate incumbent the gain from winning the upcoming election is small (the distance from the opposition is small). Thus, as Figure 2 shows, the impact of public dissent on the incumbent's choice is increasing in his bliss point. If the incumbent is too moderate public dissent will have a very small effect on the equilibrium policy, which weakens his extreme copartisan's incentives to dissent in the first place.

Corollary 1 further explores the ambiguous effect of intra-party polarization on the emergence of public dissent:

Corollary 1. *The misaligned copartisan's incentives to publicly dissent against the incumbent*

- *increase as the copartisan becomes more extreme;*

- *are non-monotonic in the incumbent's bliss point, increasing then decreasing as the incumbent becomes more extreme.*

The more extreme the dissenter is, the more he gains by moving the equilibrium policy closer to his bliss point. Thus, when the ideological conflict within the party increases due to the incumbent's copartisan becoming more extreme, public dissent always becomes more likely. The same is not necessarily true when the ideological conflict deepens due to the incumbent becoming more moderate. As the incumbent becomes more extreme, both a direct and indirect effects emerge. The direct effect is straightforward: the distance between incumbent's and his copartisan's policy preferences decreases. This reduces the copartisan's incentives to openly dissent. As highlighted above, the indirect effect instead goes in the opposite direction: public dissent has a weaker effect on policy when the incumbent is more moderate. If the incumbent's bliss point is sufficiently close to zero, this indirect effect dominates, and open dissent is more likely to emerge as the ideological conflict weakens.

This result emphasizes the peculiar nature of dissent in this model. Far from representing the first step towards a party split, open dissent here brings about unity. It serves the purpose of realigning the interests of the incumbent and his copartisans, thereby recomposing the existing ideological conflict. However, for dissent to be effective, such conflict cannot be too deep.

Public Dissent and Intraparty Bargaining

So far I have assumed that the incumbent is essentially a policy dictator. His misaligned copartisans have no formal bargaining power, and public dissent is the only tool to influence the equilibrium policy. Would electorally costly dissent ever emerge if the incumbent's misaligned copartisans have less disruptive ways to influence the policymaking process? To address this question I analyze an amended version of the model where, in the first period, the incumbent maximises a weighted average of his own and his copartisan's utility:

$$U_1^W = (1-\beta)U^I + \beta U^M \tag{2}$$

This is equivalent to considering (in a reduced form) a game in which *after* the copartisan chooses whether to publicly attack the incumbent, the two engage in in a bargaining game over the policy choice. β thus represents the misaligned copartisan's influence over policy making in the first period.[11]

It is straightforward to see why bargaining power and public dissent are, to a certain extent, substitutes. Open dissent is a tool to influence the incumbent's equilibrium policy choice. When the copartisan has some formal control over policymaking ($\beta > 0$), the incentives to pay the electoral cost of public dissent are weaker. Indeed, in the limiting case in which the extreme copartisan is given full authority over policy ($\beta = 1$), he will never choose to dissent against himself.

However, the analysis also uncovers a second and more subtle effect. Bargaining power and public dissent will sometimes complement rather than substitute each other, so that dissent is more likely to be observed compared to the case in which the incumbent is a policy dictator. Recall that, in the no-bargaining baseline, open dissent emerges in equilibrium only if the incumbent is sufficiently extreme; if the incumbent is too moderate, his incentives to gamble with extreme policies are too weak. Public dissent will then have a small effect on his policy choice, to small for the incumbent's misaligned copartisan to be willing to hurt the party's electoral chances. However, if the incumbent's extreme copartisan is given formal authority over policy making, it can effectively substitute for an excessively moderate incumbent. Then, dissent can emerge in equilibrium for every (positive) value of the incumbent's bliss point:

Proposition 2. *For all $x_I > 0$, there exist non-measure zero sets $\Gamma(x_I)$ and $B(x_I)$ such that if $\gamma \in \Gamma(x_I)$ and $\beta \in B(x_I)$ then public dissent occurs in equilibrium.*

Thus, endowing the misaligned copartisan with some formal policymaking authority does not always eliminate, and indeed can increase, his incentives to publicly attack the incumbent.

[11] Assuming that the copartisan has bargaining power only over the first-period policy is a way to obtain a meaningful comparison with the baseline model. Suppose that the second-period policy is also determined via a bargaining process. Recall that open dissent occurs in equilibrium only if the incumbent is leading. In the baseline model this requires $\gamma > \frac{1}{2}$ (since incumbent and challenger are assumed to be symmetric). If the extreme copartisan has formal bargaining power over the second-period policy, the condition becomes $\gamma > \frac{(\beta x_M + (1-\beta)x_I + \alpha)^2 - (x_I - \alpha)^2}{4\alpha(x_I + \beta x_M + (1-\beta)x_I)} > \frac{1}{2}$. Therefore, when comparing the bargaining extension to the baseline model, I would not only be altering the β parameter, but also imposing further conditions on γ, which would make the comparison less meaningful.

Finally, we can show that even if β is arbitrarily close to 1, so that the copartisan granted almost full discretion over the first period policy, public dissent will still emerge under some conditions:

Corollary 2. *Suppose that* $\frac{1}{8\alpha\psi} < x_I$ *and* $\frac{1}{4\alpha\psi} < x_M < \frac{1}{4\alpha\psi(1-2\alpha\psi x_I)}$*. Then, for all* $\beta \in [0, 1)$*, there exists a non-measure zero set* $\Gamma(\beta)$ *such that if* $\gamma \in \Gamma(\beta)$ *public dissent occurs in equilibrium.*

Public Dissent and Office Rents

So far, I assumed that both the incumbent and his copartisan are purely policy-motivated. However, electoral success is also often valuable in itself, as it grants the party access to the spoils of office. If the incumbent's misaligned copartisan participates in the division of the spoils, will this reduce his incentives to engage in electorally costly dissent?

I consider an extended version of the baseline model where, in addition to caring about policy, both the incumbent and the copartisan value office rents ξ. If the party wins the second period election, office rents are shared between the incumbent and his copartisan. Specifically, the incumbent obtains a share ρ of the rents, and his copartisan the remaining $1 - \rho > 0$. For *any* value of ρ, we have that:

Proposition 3. *There exists a unique* $\widehat{x_M}$ *s.t. if* $x_M > \widehat{x_M}$*, then the misaligned copartisan's incentives to dissent are stronger under higher office rents. Otherwise, if* $x_M < \widehat{x_M}$*, then the incentives to dissent decrease as office rents increase.*

Increasing office rents makes electoral success more valuable for both the copartisan and the incumbent. This has two competing effects on the copartisan's incentives to engage in electorally costly dissent. First, it has a direct effect. The copartisan obtains a larger benefit from the party winning the upcoming election, which reduces his incentives to dissent. Second, it has an indirect effect. Making electoral success more valuable increases the incumbent's incentives to control information, which in turns increases the impact of public dissent on his equilibrium policy choice. This increases the copartisan's incentives to dissent. If the incumbent's misaligned copartisan is sufficiently extreme, the second effect dominates and increasing office rents generates more dissent.

Public Dissent and Voter Welfare

The results above focus on the positive implications of my model, describing the conditions under which dissent is more likely to emerge. Here, it is important to pause on the theory's normative implications. In this setting, the presence of an extreme faction within the incumbent party may be beneficial for the voter. The voter values policy experimentation, as it increases the probability that she will make the correct electoral decision. However, a leading incumbent's reelection incentives induce him to implement moderate policies precisely with the aim of preventing voter learning. Thus, electoral accountability may induce the officeholder to implement a policy that is more moderate than both his own ideological preferences and what is optimal for the voter. By engaging in electorally costly dissent, and thus creating incentives for the incumbent to take risks in policymaking, his extreme copartisan can mitigate this inefficiency. If the information gain is sufficiently large, voter welfare increases even despite the cost δ. In the online Appendix B, I identify sufficient conditions for this to hold true.

These welfare results thus uncover an alignment between the interests of a voter who cares about information and therefore has a taste for policy experimentation, and a political faction who has an ideological preference for extreme policies (similar to Bernhardt, Duggan and Squintani (2009)). As noted by Boucek (2009), negative perceptions of factionalism originated with Hume (1877) and are still predominant: factions 'exacerbate non-cooperative behaviour and so are antithetical to achievement of common goals' (Dewan and Squintani, 2016, p. 861). A 'defence of factions' comes from the claim that organized and ideologically cohesive subgroups within political parties can instead play a role in fostering cooperation, by facilitating deliberation and pooling of valuable information. This argument is advanced initially by Boucek (2009), investigated empirically by McAllister (1991), and proven formally by Dewan and Squintani (2016). This paper goes a step further, showing that factions can play a positive role even when, and precisely because, they engage in disruptive behavior.

Discussion and Robustness

Microfounding the Valence Shock

Having presented the model's results, let me pause to discuss a crucial feature of the setup. A key assumption of the model is that open dissent generates a valence cost for the party. For purposes of presentation, I leave this cost black-boxed, assuming that public dissent 'mechanically' reduces the party's appeal to voters. In other words, voters dislike divided parties per se.

While this assumption is perhaps plausible even in this reduced form, it is important to discuss potential ways to micro-found it. One possibility is that public dissent causes voters to negatively update their beliefs over the incumbent's valence. For example, the dissenters can expose the incumbent as a liar, corrupt or incompetent. The specification and results of this micro-founded model would then be exactly as presented above. Under the conditions identified in Proposition 1, the misaligned copartisans choose to publicly attack the incumbent whenever they can reveal evidence that he is a bad type, even if they do not care about competence at all. If the conditions are not met, the copartisans keep quiet.

Alternatively, we may assume that the incumbent's copartisans do not have access to such verifiable evidence. Nonetheless, they may have an informational advantage with respect to the voters. For example, his copartisans may be able scrutinize the incumbent's previous actions and performance, thereby obtaining additional information about his true competence (see Caillaud and Tirole (1999), Fox and Van Weelden (2010)). As such, they can engage in a signaling game with the electorate. Public dissent is then electorally costly when, *in equilibrium*, it constitutes a negative signal of the incumbent's type. The conditions identified in Proposition 1 are then necessary for such a (separating) equilibrium to be sustained,[12] and thus for electorally costly dissent to emerge in this setting.

In concluding this subsection let me emphasize that, while the assumption of electorally costly dissent is motivated by both empirical evidence and the above theoretical reasoning, the mechanism

[12] Under the assumption that the incumbent's copartisans do not care so much about competence that they would always prefer the challenger to a bad-type incumbent.

identified in this paper relies only on the voters not being indifferent to dissent ($\delta \neq 0$). Indeed, it would survive in a world in which public dissent produces a positive valence shock ($\delta < 0$), thus improving rather than damaging a party's electoral prospects. Clearly, under such an assumption the puzzle would be reversed: if open dissent is electorally valuable, how do we explain cases without dissent? The mechanism identified in this paper provides a potential answer. The incumbent's copartisans may choose not to improve his electoral prospects if, by doing so, they induce him to implement a policy that they dislike.

Asymmetric Information

In this paper, there is no asymmetry of information between voters and politicians. In particular, voters face no uncertainty about parties' ideological stances, and can thus anticipate what kind of policies the incumbent would implement if reelected, and which ones instead the challenger would pursue. While fully relaxing this assumption is beyond the scope of this project, it is important to highlight that the strategic incentives uncovered in this paper would (under some conditions) survive in such an asymmetric information setting.

To see this, suppose that the incumbent may be moderate or extreme, with his type being his private information. Then, a leading incumbent has two potential reasons to adopt a moderate policy: to signal to the voter that he is a moderate type, and to prevent the voter from learning about her own preferences. Suppose that δ is sufficiently large that, conditional on experiencing dissent, the incumbent is reelected if and only if the voter observes an informative policy outcome and learns that her own bliss point is a right-wing one (regardless of her beliefs about the incumbent's type).[13] Then, public dissent would induce the incumbent to implement an extreme policy in order to generate informative outcomes, even if this may lead the voter to believe that he is an extreme type. Thus, as in the model analyzed here, electorally costly dissent moves the equilibrium policy to the extreme (regardless of the incumbent's true type). The tradeoff uncovered in this paper would therefore continue to emerge. As in this symmetric information setting, the incumbent's extreme

[13] Let x^M denote the moderate-type incumbent's bliss point, and x^E the extreme types' preferred point. The condition requires $-\gamma(\alpha - x^M)^2 - (1-\gamma)(\underline{\alpha} - x^M)^2 - \delta < -\gamma(\alpha - x_C)^2 - (1-\gamma)(\underline{\alpha} - x_C)^2$ and $-(\alpha - x^E)^2 - \delta > -(\alpha - x_C)^2$.

copartisan will sometimes find it optimal to publicly attack him in order to move the equilibrium policy today, even if this imposes an electoral cost.

Replacing the Leader

One possibility that my model does not explicitly consider is that electorally costly dissent may weaken the party leader and thus make it easier to replace him. Then, above and beyond their desire to influence policy, the dissenters may want to publicly attack the leader if they have hopes of taking his place. In this paper I do not allow for leadership turnover, since my goal is to understand why and when electorally costly dissent may emerge even *absent* such opportunistic considerations (e.g., even when the misaligned copartisans have no chances of replacing the leader). However, it is important to discuss how such dynamics may be incorporated into my framework, and how they may indeed complement the mechanism I propose.

Suppose that, at the end of the first period and before the second-period election, a leadership contest opens within the party. Then, a necessary condition for the extreme faction to take over (e.g., for its candidates to succeed in primary elections), is that its policy stances are sufficiently appealing to the voter, so that the party would still have a chance of winning the general election. If the electorate is ex-ante too moderate, this requires changing voters' policy preferences. Therefore, a leading incumbent would want to avoid policy gambles in order to win the upcoming election (as in the current model) *and* maintain a strong hold on the party. Similarly, the extreme faction has ideological as well as opportunistic incentives to engage in electorally costly dissent, in order to force the party leader to experiment with extreme policies. In this sense, public dissent would then *endogenously* make the party leader more vulnerable to being ousted by the opposing faction.

The framework presented here may therefore capture public dissent that emerges for purely policy reasons (as in the current paper) or for both policy *and* opportunistic ones. In this perspective, allowing for replacement need not alter the model's qualitative insights, and may indeed strengthen them.

Empirical implications

That internal conflict may be electorally costly for political parties is fairly intuitive. The setup of my model suggests that there are two ways to think about this cost empirically. One way is to directly consider the valence shock δ, i.e., *a model primitive*. The surveys and experiments mentioned in the introduction (YouGov (2016) Greene and Haber (2015); Groeling (2010)) adopt this approach, trying to directly assess how perceptions of internal divisions influence voters' evaluations (fixing the *actual* level of dissent within the party and its policy choices).

An alternative approach is instead to look at the party's realized electoral performance, and how it is impacted by the occurrence of open dissent, i.e., *an equilibrium outcome*.[14] The typical strategy regresses the probability of winning (or other measures of electoral success) at time t on a binary variable indicating whether the party experienced open dissent at $t-1$ (e.g., Clark (2009), Kam (2009)):

$$prob(W_i = 1) = \alpha + \beta_1 X_i + \beta_2 D_i + \epsilon_i, \tag{3}$$

where X_i is a vector of covariates, and β_2 is the coefficient of interest. Graphically, the quantity of interest is the average distance between the two curves in Figure 3, representing the probability of winning as a function of the party's ex-ante electoral strength (γ), with and without dissent.

The results of the model have two key implications. First, they show that this strategy does not allow us to isolate the *direct* effect of open dissent. The incumbent best responds to dissent by modifying his policy choice precisely to mitigate this electoral cost. Thus, any estimate would at best reflect the *equilibrium* effect of public dissent: the cost mediated through the incumbent's best response.

Second, the model results suggest that this estimate would likely suffer from selection bias. Proposition 1 shows that whether parties experience open dissent depends precisely on their ex-ante electoral strength (γ). Thus, (fixing the incumbent's and dissenter's ideological preferences)

[14] In the model, this is $Prob(L = 1|x_1^*(a_d = 1))\gamma - [1 - Prob(L = 1|x_1^*(a_d = 0))(1-\gamma)]$. In equilibrium, this difference is always negative, i.e., dissent always reduces the probability that the incumbent will be reelected.

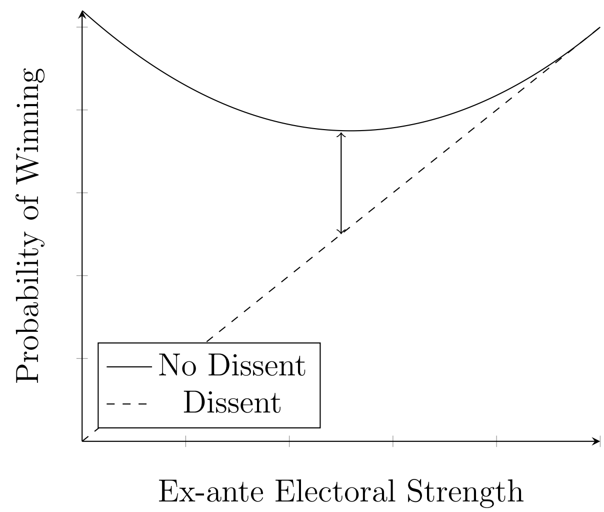


Figure 3: Probability of Winning as Function of Ex-ante Strength (γ).

we cannot observe both treated and control units for similar levels of ex-ante electoral strength. Figure 4 represents what the researcher can actually observe: treated units at sufficiently high levels of electoral strength and untreated ones at γ close to $\frac{1}{2}$. Comparing parties that experience open dissent against their untreated counterparts thus means comparing parties at different levels of electoral strength. As such, recovering an unbiased estimate of the (equilibrium) effect of public dissent on parties' electoral performance proves challenging.

Further, it is hard to know what the direction of the bias will be. In the example of Figure 4, the estimated electoral cost of public dissent would likely be higher than the true one. However, under different parameter values, the dissenting region shifts. Consider for example Figure 5, obtained by increasing the dissenter's bliss point. Here, open dissent emerges only at very high values of γ, and the direction of the bias is no longer clear. Indeed, the estimate may even have the wrong sign. Thus, even if we are aware of the existence of the bias, it is hard to interpret the results of this type of analysis of aggregate outcomes.

However, the above discussion does not imply that we cannot empirically investigate the equilibrium effect of dissent. The model in fact generates testable predictions regarding parties' electoral performance *conditional* on experiencing public dissent. Following public manifestations of dissent, the party needs the voter to obtain new and favourable information in order to win. Thus, the

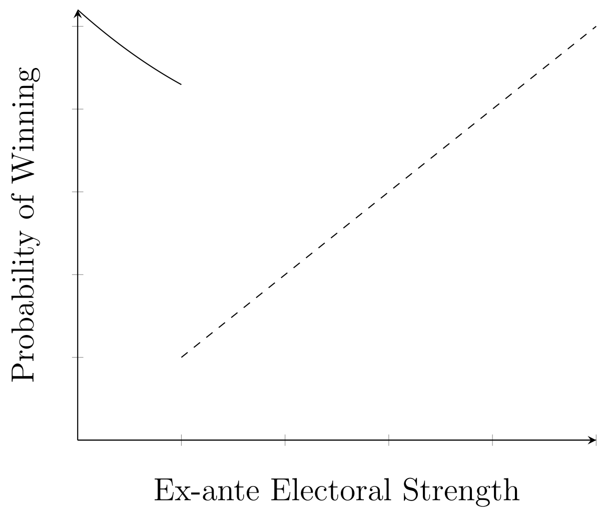


Figure 4: Probability of Winning as Function of Ex-ante Strength (γ) - Observable

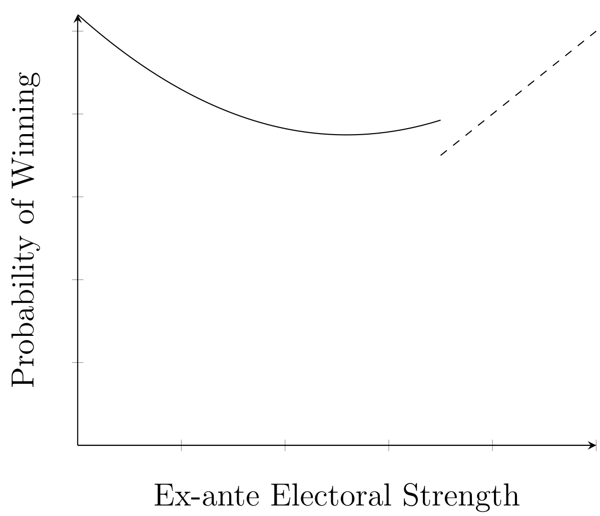


Figure 5: Probability of Winning as Function of Ex-ante Strength, second example.

larger the amount of information received by the voters and their ability to interpret such information, the higher the probability of winning conditional on experiencing dissent. We should then expect this conditional probability to be positively correlated with variables such as news media consumption, education or political engagement in the population. Additionally, fixing the information environment, the amount of voter learning depends on the incumbent's willingness to engage in policy experimentation. Recall that a more extreme party leader is more willing to gamble. Thus, conditional on experiencing open dissent, the party's electoral success should be increasing in the leader's ideological extremism. Focusing on the treated units, and thereby sidestepping issues of selection, researchers can thus empirically investigate the conditions under which public dissent is expected to hurt parties the most.

Conclusion

I have proposed a theory of electorally costly intraparty dissent, according to which public manifestations of dissent may serve the purpose of mitigating the ideological conflict within the party by inducing a policy response by the leadership. In equilibrium, public dissent thus emerges precisely because it is electorally costly. The model's results can help us qualify our intuitions about the conditions that are more likely to generate open dissent. In particular, they highlight that improving the party's expected electoral performance may generate more dissent, and that an increase in intraparty ideological polarization plays an ambiguous role. Depending on whether the incumbent becomes more moderate or the dissenters more extreme, increased polarization may either decrease or increase the likelihood of the party experiencing public manifestations of dissent.

The theory also has relevant normative and empirical implications. From a normative stand point, it indicates that the presence of extremists within the incumbent party may be welfare improving for the voter, as it may mitigate perverse consequences of electoral accountability. With regards to empirical research, the results show that existing estimates of the electoral rewards of party unity obtained by comparing treated and control units may suffer from selection bias.

In concluding this paper it is important to highlight that, while this project has focused on the interaction between a leader and his copartisans, the mechanism it uncovers applies more generally. For example, media outlets often denigrate political leaders from their own ideological camp. The right-leaning Evening Standard has often openly attacked UK Conservative prime minister Theresa May, depicting her cabinet as 'stale' and 'enfeebled'.[15] Similarly The Guardian, historically left-leaning, has described UK Labour leader Corbyn as 'dismal, lifeless, spineless'.[16] Within my framework, this behavior may serve the purpose of indirectly influencing the leader's strategic choices, by altering his incentives to take risky gambles. Similarly, incentives to manipulate political leaders' endogenous risk preferences may play a role in special interests' decision whether and how much to contribute to their electoral campaigns.

More generally, my model may provide a richer understanding of the interaction between political actors in any strategic situation that can be described as a principal agent model with two key features. First, there is some (common) uncertainty on what is the principal's optimal decision, and the amount of information that is generated is a function of the agent's action. The principal's uncertainty can refer to her ideal policy, as in the model presented here, or to the agent's type, e.g., his ability or competence. Second, the agent's ideological ally (i.e. an actor whose payoff is higher when the agent is retained than when he is replaced) can take an action that, everything else constant, changes the probability that the agent is retained. Exploring the model's insights beyond the realm of party politics (e.g., in relation to the interaction between revolutionary groups and their domestic governments, ornbetween the leaders of different countries) is perhaps a particularly interesting direction for future research.

[15]https://www.standard.co.uk/comment/comment/rosamund-urwin-the-next-generation-do-government-leaders-have-a-political-shelf-life-a3619341.html

[16]https://www.theguardian.com/commentisfree/2016/jun/25/jeremy-corbyn-referedum-campaign

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Appendix A: Proofs

Lemma 2. *The voter's learning satisfies the following properties:*

i. Her posterior μ takes one of three values: $\mu \in \{0, \gamma, 1\}$;

ii. The amount of learning is a function of the implemented policy x_1, as more extreme policies increase the probability that $\mu \neq \gamma$;

iii. There exists a policy x' such that if $|x_1| \geq |x'|$, then $\mu \neq \gamma$ with probability 1.

Proof. The proof of Claims 1 and 2 below is necessary and sufficient to prove Lemma 1.

Claim 1: *Let $x_t \geq 0$.*

(i) A payoff realization $U_t^v \notin [-(x_t-\alpha)^2 - \frac{1}{2\psi}, -(x_t+\alpha)^2 + \frac{1}{2\psi}]$ is fully informative. Upon observing $U_t^v > -(x_t+\alpha)^2 + \frac{1}{2\psi}$, the voter forms posterior beliefs that $x_V = \alpha$ with probability 1. Similarly, upon observing $U_t^v < -(x_t-\alpha)^2 - \frac{1}{2\psi}$ the voter forms beliefs that $x_V = \underline{\alpha}$ with probability 1.

(ii) A payoff realization $U_t^v \in [-(x_t-\alpha)^2 - \frac{1}{2\psi}, -(x_t+\alpha)^2 + \frac{1}{2\psi}]$, is uninformative. Upon observing U_t^v, the voter confirms her prior belief that $x_V = \alpha$ with probability γ.

Symmetric results apply when $x_t < 0$.

Proof. The proof of part (i) is trivial given the boundedness of the distribution of ϵ, and is therefore omitted. Part (ii) follows straightforwardly from applying Bayes rule. Recall that the voter's payoff realization U_t^v is a function of the implemented policy (x_t) the voter's true bliss point (x_V) and the noise term (ϵ): $U_t^v = -(x_V - x_t)^2 + \epsilon$. Denote as $f(\cdot)$ the PDF of ϵ. Then,

$$prob(x_V = \alpha | U_t^v) = \frac{f\left(U_t^v + (x_t-\alpha)^2\right)\gamma}{f\left(U_t^v + (x_t-\alpha)^2\right)\gamma + f\left(U_t^v + (x_t+\alpha)^2\right)(1-\gamma)}. \tag{4}$$

Given the assumption that ϵ is uniformly distributed $f(U_t^v + (x_t-\alpha)^2) = f(U_t^v + (x_t+\alpha)^2)$ therefore the above simplifies to

$$prob(x_V = \alpha | U_t^v) = \gamma. \tag{5}$$

This concludes the proof of Claim 1. □

Claim 2: *Let L be a binary indicator, taking value 1 if the players learn the true value of x_V at the end of period 1, and 0 otherwise. There exists $x' = \frac{1}{4\alpha\psi}$ such that*

- *For all $|x_1| \geq |x'|$*

$$Prob(L = 1|x_1) = 1 \tag{6}$$

- *For all $x_1 \in [0, x')$*

$$Prob(L = 1|x' \geq x_1 \geq 0) = 4\alpha\psi x_1 \tag{7}$$

- *For all $x_1 \in (-x', 0]$*

$$Prob(L = 1| - x' \leq x_1 \leq 0) = -4\alpha\psi x_1 \tag{8}$$

Proof. Let me first prove the existence of point x'. From Claim 1, x' is the point such that for any policy $|x| \geq |x'|$, the interval $[-(x_t - \alpha)^2 - \frac{1}{2\psi}, -(x_t + \alpha)^2 + \frac{1}{2\psi}]$ is empty. This requires

$$-(x_t + \alpha)^2 + \frac{1}{2\psi} + (x_t - \alpha)^2 + \frac{1}{2\psi} \leq 0. \tag{9}$$

Recall that $\alpha = +\alpha$, thus the above reduces to

$$x \geq \frac{1}{4\alpha\psi} = x' \tag{10}$$

To complete the proof, assume $x_1 \in [0, x']$. The expected probability of the realized outcome being informative is

$$Prob(L = 1|\gamma, 0 < x_1 < x') =$$

$$\gamma \left[Prob\left(-(x_t - \alpha)^2 + \epsilon_1 > -(x_t + \alpha)^2 + \tfrac{1}{2\psi}\right)\right] + (1 - \gamma)\left[Prob\left(-(x_t + \alpha)^2 + \epsilon_1 < -(x_t - \alpha)^2 - \tfrac{1}{2\psi}\right)\right]. \tag{11}$$

Given the symmetry $Prob\left(-(x_t - \alpha)^2 + \epsilon_1 > -(x_t + \alpha)^2 + \frac{1}{2\psi}\right) = Prob\left(-(x_t + \alpha)^2 + \epsilon_1 < -(x_t - \alpha)^2 - \frac{1}{2\psi}\right)$ thus 11 simplifies to

$$Prob(L = 1|x_1 > 0) = Prob\left(-(x_t - \alpha)^2 + \epsilon_1 > -(x_t + \alpha)^2 + \frac{1}{2\psi}\right) = 4\alpha\psi x_1. \tag{12}$$

Similar calculations produce the result for $x_1 \in (-x', 0]$. This concludes the proof of Claim 2 $\square$

This concludes the proof of Lemma 1 $\square$

In what follows I will assume that $x_I < \frac{1}{4\alpha\psi}$. This assumption is without loss of generality, and imposed in order to reduce the number of cases under consideration; results for the case in which $x_I > \frac{1}{4\alpha\psi}$ are available upon request.

Lemma 3. *In equilibrium*

- *a leading incumbent implements a policy (weakly) more moderate than his bliss point,* $x_1^* \leq x_I$*;*
- *a trailing incumbent implements a policy (weakly) more extreme than his bliss point,* $x_1^* \geq x_I$*;*
- *a certain loser implements his bliss point,* $x_1^* = x_I$*.*

Proof. Denote $\mathbb{P}(x_1)$ the probability that the incumbent is reelected, as a function of his first-period policy choice. Then, the incumbent's equilibrium choice maximizes

$$-(x_1 - x_I)^2 - (1 - \mathbb{P}(x_1))\, 4x_I^2, \tag{13}$$

where $\mathbb{P}$ satisfies

- $\mathbb{P} = 1 - (1-\gamma)Prob(L = 1|x_1)$ for a leading incumbent;
- $\mathbb{P} = Prob(L = 1|x_1)\gamma$ for a trailing incumbent;
- $\mathbb{P} = 0$ for a certain loser.

From Lemma 1, we have that

$$Prob(L = 1|x_1) = \min \in \{4\alpha\psi|x_1|, 1\}. \tag{14}$$

Notice that, for any pair of policies x' and x'' s.t. $x' = -x''$, we have that $Prob(L = 1|x') = Prob(L = 1|x'')$. This implies that, in equilibrium, a right-wing incumbent always implements policy (weakly) to the right of zero.

Lemma 2 follows straightforwardly: $\mathbb{P}$ is weakly decreasing as x_1 increases away from 0 for a leading incumbent, weakly increasing for a trailing one, and not a function of x_1 for a certain loser. More specifically, solving the maximization problem we obtain that, when the incumbent is leading we have

$$x_1^* = \max\{x_I - 8\alpha\psi x_I^2(1-\gamma), 0\}. \tag{15}$$

In contrast, when the incumbent is trailing we have

$$x_1^* = \min\{x_I + 8\alpha\psi x_I^2\gamma, \frac{1}{4\alpha\psi}\}. \tag{16}$$

Trivially, when the incumbent is a certain loser $x_1^* = x_I$ □

Lemma 4. *Public dissent emerges in equilibrium only if, absent dissent, the incumbent is leading, i.e.,* $\gamma > \frac{1}{2}$.

Proof. In what follows, I denote x_d^* the incumbent's equilibrium policy choice if he experiences dissent, and x_{nd}^* his choice if he does not. Suppose $\gamma < \frac{1}{2}$, i.e., the incumbent is ex-ante trailing. First, assume $\delta < 4_I$. Then, dissent would have no impact on the incumbent's retention prospects: whether or not he experiences dissent, the incumbent is reelected if and only if an informative and favorable policy outcome is generated. As a consequence, dissent would have no impact on the incumbent's policy choice ($x_d^* = x_{nd}^*$) and therefore never emerges in equilibrium. Suppose instead that $\delta \geq 4_I$. Then, after experiencing dissent the incumbent is a sure loser: even if the voter learns that $x_V = \alpha$, she will still choose to replace the incumbent with his challenger. As a consequence, $x_d^* = x_I$. Suppose instead the incumbent does not experience, dissent. Here, there are two possible cases that we must consider:

1. $x_I + 8\alpha\psi x_I^2\gamma \geq \frac{1}{4\alpha\psi}$, and therefore $x_{nd}^* = \frac{1}{4\alpha\psi}$

2. $x_I + 8\alpha\psi x_I^2\gamma < \frac{1}{4\alpha\psi}$, and therefore $x_{nd}^* = x_I + 8\alpha\psi x_I^2\gamma$

I will analyse each case separately.

Case 1: $x_d^* = x_I$ **and** $x_{nd}^* = \frac{1}{4\alpha\psi}$

Given the incumbent's best response, his misaligned copartisan chooses to publicly attack him if and only if

$$-(x_I - x_M)^2 - (x_I + x_M)^2 > -(\frac{1}{4\alpha\psi} - x_M)^2 - \gamma(x_I - x_M)^2 - (1-\gamma)(x_I + x_M)^2, \tag{17}$$

which reduces to

$$\gamma < \frac{-8\alpha\psi x_M(1 - 4\alpha\psi x_I) - (4\alpha\psi x_I)^2 + 1}{(8\alpha\psi)^2 x_I x_M}. \tag{18}$$

Recall that we are considering a case in which $x_{nd}^* = \frac{1}{4\alpha\psi}$, therefore $\gamma > \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi x_I} - 1)$. Thus, for dissent to emerge in equilibrium it must be the case that

$$\frac{-8\alpha\psi x_M(1 - 4\alpha\psi x_I) - (4\alpha\psi x_I)^2 + 1}{(8\alpha\psi)^2 x_I x_M} > \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi x_I} - 1). \tag{19}$$

Let $T = 4\alpha\psi$. The above can be rearranged as

$$\frac{1 - 2x_M T(1 - x_I T) - (x_I T)^2}{2x_M} > \frac{1 - x_I T}{x_I}, \tag{20}$$

which reduces to

$$2x_M(1 - (x_I T)^2) < x_I(1 - (x_I T)^2). \tag{21}$$

Since $(x_I T)^2 = (4\alpha\psi x_I)^2 < 1$ (by assumption), the above can never be satisfied when $x_M > x_I$.

Case 2: $x_d^* = x_I$ **and** $x_{nd}^* = x_I + 8\alpha\psi x_I^2\gamma$

In this case, the dissenter attacks if and only if

$$\begin{aligned} -(x_I - x_M)^2 - (x_I + x_M)^2 > \\ -(x_I + 8\alpha\psi x_I^2\gamma - x_M)^2 - \left(1 - 4\alpha\psi\gamma(x_I + 8\alpha\psi x_I^2\gamma)\right)(x_I + x_M)^2 \\ -4\alpha\psi\gamma(x_I + 8\alpha\psi x_I^2\gamma)(x_I - x_M)^2 \end{aligned} \tag{22}$$

Denoting $\Delta = 8\alpha\psi\gamma(x_I)^2$, the above can be rearranged as

$$0 > -\Delta^2 - 2\Delta(x_I - x_M) + 16\alpha\psi\gamma(x_I + \Delta)x_I x_M. \tag{23}$$

Substituting $\Delta = 8\alpha\psi\gamma x_I^2$ and dividing by $16\alpha\psi\gamma(x_I)^2$, the above reduces to

$$x_M < \frac{x_I}{2}. \tag{24}$$

Given $x_M > x_I$, the condition can never be satisfied.

Thus, dissent never emerges in equilibrium if $x_M > x_I$ and $\gamma < \frac{1}{2}$.

Lemma 5. *Public dissent emerges in equilibrium only if its electoral cost is sufficiently high to turn the leading incumbent into a trailing one, but not so high that the incumbent becomes a certain loser, i.e.,* $4\alpha x_I(2\gamma - 1) < \delta < 4\alpha x_I$.

Consider now the conditions on the cost of dissent δ. First, suppose $\delta < 4_I(2\gamma - 1)$. Then, whether or not he experiences dissent, the incumbent is electorally leading: he is retained in office with probability $1 - (1 - \gamma)Prob(L = 1)$. Thus, dissent has no impact on his equilibrium policy choice ($x_d^* = x_{nd}^*$) and never emerges in equilibrium. Next, suppose $\delta \geq 4_I$. After experiencing dissent, the incumbent would turn into a sure loser, therefore $x_d^* = x_I$. Conversely, if the leading incumbent experiences no dissent $x^*_{nd} = \max\{0, x_I - 8\alpha\psi x_I^2(1-\gamma)\}$. Recall that $x_I < \frac{1}{4\alpha\psi}$, therefore $x^*_{nd} = x_I - 8\alpha\psi x_I^2(1-\gamma)$.

Thus, public dissent strictly increases the copartisan's utility if and only if:

$$\begin{aligned} -(x_I - x_M)^2 - (x_I + x_M)^2 > \\ -(x_I - 8\alpha\psi x_I^2(1-\gamma) - x_M)^2 - \left[1 - 4\alpha\psi(1-\gamma)(x_I - 8\alpha\psi x_I^2(1-\gamma))\right](x_I - x_M)^2 \\ -4\alpha\psi(1-\gamma)(x_I - 8\alpha\psi x_I^2(1-\gamma))(x_I + x_M)^2. \end{aligned} \tag{25}$$

This reduces to

$$x_M\left[1 - 2(4\alpha\psi x_I(1-\gamma))(1 - 4\alpha\psi x_I(1-\gamma))\right] + 4\alpha\psi x_I^2(1-\gamma)(1 - 4\alpha\psi x_I(1-\gamma)) < 0. \tag{26}$$

The LHS is increasing in x_M and never satisfied at $x_M = 0$. Hence, dissent by an extremist can never emerge when $\delta \geq 4_I$.

Proposition 1. *Suppose that $\gamma > \frac{1}{2}$ and $4\alpha x_I(2\gamma - 1) < \delta < 4\alpha x_I$. Then, there exist unique $\underline{x_I}$, $\underline{x_M}(x_I)$ and $\underline{\gamma}(x_I)$ such that that public dissent emerges in equilibrium if and only if*

- *The incumbent's lead is sufficiently large, $\gamma > \underline{\gamma} \geq \frac{1}{2}$, and*
- *Both the incumbent and his misaligned copartisan are sufficiently extreme, $x_I > \underline{x_I}$ and $x_M > \underline{x_M}$.*

Suppose $\delta \in (4\alpha\psi x_I(2\gamma - 1), 4\alpha x_I)$ and $\gamma > \frac{1}{2}$. Then, dissent would tearn the leading incumbent into a trailing one and thus change his equilibrium policy choice. Thus, dissent emerges in equilibrium if and only if

$$-(x_d^* - x_M)^2 - (1 - 4\alpha\psi x_d^*\gamma)(x_I + x_M)^2 - 4\alpha\psi x_d^*\gamma(x_I - x_M)^2 > \tag{27}$$
$$-(x_{nd}^* - x_M)^2 - (1 - 4\alpha\psi x_{nd}^*(1-\gamma))(x_I - x_M)^2 - 4\alpha\psi x_{nd}^*(1-\gamma)(x_I + x_M)^2.$$

Recall that, since $x_I < \frac{1}{4\bar{\alpha}\psi}$, we have $x_{nd}^* = x_I - 8\alpha\psi x_I^2(1-\gamma)$. In contrast, from Lemma 2 we know that the policy following dissent may take one of two values:

1. If $x_I > \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$, then $x_d^* = \frac{1}{4\alpha\psi}$
2. If instead $x_I < \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$, then $x_d^* = x_I + 8\alpha\psi x_I^2\gamma$

I will analyse each of the two cases separately.

Case 1: $x_I > \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$

Given the anticipated best response of the incumbent, the dissenter chooses to attack if and only if

$$-(\frac{1}{4\alpha\psi} - x_M)^2 - \gamma(x_I - x_M)^2 - (1-\gamma)(x_I + x_M)^2 > \tag{28}$$
$$-(x_I - 8\alpha\psi x_I^2(1-\gamma) - x_M)^2 - [1 - 4\alpha\psi(1-\gamma)(x_I - 8\alpha\psi(x_I)^2(1-\gamma))](x_I - x_M)^2$$
$$-4\alpha\psi(1-\gamma)(x_I - 8\alpha\psi x_I^2(1-\gamma))(x_I + x_M)^2.$$

Let $I = 4\alpha\psi(x_I - 8\alpha\psi(x_I)^2(1-\gamma))$. We can rewrite the above condition as:

$$(1-\gamma)(1-I)((x_I - x_M)^2 - (x_I + x_M)^2) > (\frac{1}{4\alpha\psi} - x_M)^2 - (\frac{I}{4\alpha\psi} - x_M)^2, \tag{29}$$

which is equivalent to

$$(1-\gamma)(1-I)(-4x_M x_I) > \frac{-x_M}{2\alpha\psi}(1-I). + \frac{1}{(4\alpha\psi)^2}(1+I)(1-I) \tag{30}$$

By substituting $I = 4\alpha\psi(x_I - 8\alpha\psi(x_I)^2(1-\gamma))$ and solving for γ we get the following condition:

$$\gamma > \frac{1 + (2x_M - x_I)(8\alpha\psi x_I - 1)(4\alpha\psi)}{2x_I(4\alpha\psi)^2(2x_M - x_I)} = \underline{\gamma}_1. \tag{31}$$

Given $\gamma < 1$, this in turn requires

$$x_M > \frac{1}{8\alpha\psi} + \frac{x_I}{2}. \tag{32}$$

Case 2: $x_I < \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$

Given the incumbent's best response, the dissenter chooses to attack if and only if

$$\begin{aligned} &-(x_I + 8\alpha\psi x_I^2\gamma - x_M)^2 \\ -(1 - 4\alpha\psi(x_I + 8\alpha\psi x_I^2\gamma)\gamma)(x_I + x_M)^2 &- 4\alpha\psi(x_I + 8\alpha\psi x_I^2\gamma)\gamma(x_I - x_M)^2 > \\ -(x_I - 8\alpha\psi x_I^2(1-\gamma) - x_M)^2 &- (1 - 4\alpha\psi(x_I - 8\alpha\psi x_I^2(1-\gamma))(1-\gamma))(x_I - x_M)^2 \\ &-4\alpha\psi(x_I - 8\alpha\psi x_I^2(1-\gamma))(1-\gamma)(x_I + x_M)^2. \end{aligned} \tag{33}$$

Let $I = 4\alpha\psi(x_I + 8\alpha\psi(x_I)^2\gamma)$ and $x^D = x_I + 8\alpha\psi x_I^2\gamma$. We can rewrite the above condition as:

$$\begin{aligned} -(x^D - x_M)^2 - (1-\gamma I)(x_I + x_M)^2 - \gamma I(x_I - x_M)^2 > \\ -(x^D - x_M - 8\alpha\psi(x_I)^2)^2 - (1 - (1-\gamma)(I - 4\alpha\psi(8\alpha\psi x_I^2)))(x_I - x_M)^2 \\ -(1-\gamma)(I - 4\alpha\psi(8\alpha\psi x_I^2))(x_I + x_M)^2. \end{aligned} \tag{34}$$

By expanding, and dividing both sides by $4x_I$ we get:

$$\gamma I x_M > x_M - x_M((1-\gamma)(I - 2(4\alpha\psi x_I)^2) + x_I(I - 4\alpha\psi x_M - (4\alpha\psi x_I)^2), \tag{35}$$

which is equivalent to:

$$x_M + I(x_I - x_M) - 4\alpha\psi x_I x_M + (4\alpha\psi x_I)^2(2x_M(1-\gamma) - x_I) < 0. \tag{36}$$

By substituting $I = 4\alpha\psi(x_I + 8\alpha\psi x_I^2\gamma)$ and solving for γ we get the following condition:

$$\gamma > \frac{4\alpha\psi x_I(2x_M - x_I)(4\alpha\psi x_I - 1) + x_M}{32\alpha^2\psi^2 x_I^2(2x_M - x_I)} = \underline{\gamma}_2. \tag{37}$$

Given $\gamma < 1$, this in turns requires

$$x_M > \frac{x_I(4\alpha\psi x_I)(4\alpha\psi x_I + 1)}{2(4\alpha\psi x_I)(4\alpha\psi x_I + 1) - 1}, \tag{38}$$

and

$$x_I > \frac{\sqrt{3} - 1}{8\bar{\alpha}\psi}. \tag{39}$$

Combining all of the above, we obtain that there exist unique $\underline{x_I}$, $\underline{x_M}(x_I)$ and $\underline{\gamma})(x_I)$ s.t. dissent emerges in equilibrium if and only if $x_I > \underline{x_I}$, $x_M > \underline{x_M}$ and $\gamma > \underline{\gamma}$. Specifically, $\underline{x_I} = \frac{\sqrt{3}-1}{8\bar{\alpha}\psi}$, $\underline{x_M}$ solves either 32 or 38, depending on the value of x_I, and $\underline{\gamma}(x_I)$ is either $max \in \{\frac{1}{2}, \underline{\gamma}_1\}$ or $max \in \{\frac{1}{2}, \underline{\gamma}_2\}$, depending on the value of x_I.[17]

□

Corollary 1. *The misaligned copartisan's incentives to publicly dissent against the incumbent*

- *increase as the copartisan becomes more extreme;*
- *are non-monotonic in the incumbent's bliss point, increasing then decreasing as the incumbent becomes more extreme.*

[17] $\underline{\gamma}_1$ and $\underline{\gamma}_2$ are defined in 31 and 37 respectively.

Proof. From the proof of Proposition 1 we can verify that $\underline{\gamma}$ is always weakly decreasing in x_M. Further, from inspection of 31 and 37, we can see that $\underline{\gamma}$ is (weakly) decreasing in x_I in Case 2, i.e., when $x_I < \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$, but (weakly) increasing in x_I in Case 1, i.e., when $x_I > \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi\gamma}$. □

Proposition 2. *For all $x_I > 0$, there exist non-measure zero sets $\Gamma(x_I)$ and $B(x_I)$ such that if $\gamma \in \Gamma(x_I)$ and $\beta \in B(x_I)$ then public dissent occurs in equilibrium.*

Proof. First, we must determine the equilibrium policy choice of the incumbent, proceeding as in the proof of Lemma 2.

Consider first a trailing incumbent. The following holds:

- Suppose $\beta x_M + (1-\beta)x_I \geq \frac{1}{4\psi}$. Then $x_1^* = \beta x_M + (1-\beta)x_I$
- Suppose $\beta x_M + (1-\beta)x_I < \frac{1}{4\alpha\psi}$. Then $x_1^* = \min \{\frac{1}{4\alpha\psi}, [\beta x_M + (1-\beta)x_I][1 + 8\alpha\psi x_I \gamma]\}$

Consider now a leading incumbent:

- Suppose $\beta x_M + (1-\beta)x_I \geq \frac{1}{4\alpha\psi}$. Then $x_1^* = \beta x_M + (1-\beta)x_I$ if $\gamma > \frac{1+4\alpha\psi[(\beta x_M+(1-\beta)x_I)(4\alpha\psi x_I-1)]}{(4\alpha\psi)^2[x_I(\beta x_M+(1-\beta)x_I)]}$, and $x_1^* = [\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi x_I(1-\gamma)]$ otherwise[18]
- Suppose $\beta x_M + (1-\beta)x_I < \frac{1}{4\alpha\psi}$. Then $x_1^* = [\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi x_I(1-\gamma)]$

Next, to identify conditions under which dissent emerges in equilibrium, I proceed as in the proof of Proposition 1. Suppose that $4_I(2\gamma - 1) < \delta < 4_I$ and $\gamma > \frac{1}{2}$, and conjecture the existence of an equilibrium in which the dissenter chooses to attack. We must consider three cases:

1. $x_d^* = \beta x_M + (1-\beta)x_I$ and $x_{nd}^* = [\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi(1-\gamma)x_I]$
2. $x_d^* = \frac{1}{4\alpha\psi}$ and $x_{nd}^* = [\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi(1-\gamma)x_I]$
3. $x_d^* = [\beta x_M + (1-\beta)x_I][1 + 8\alpha\psi\gamma x_I]$ and $x_{nd}^* = [\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi(1-\gamma)x_I]$

I will analyse each of the three cases separately.

[18]When $\beta x_M + (1-\beta)x_I \geq \frac{1}{4\alpha\psi}$ the leading incumbent's overall utility as a function of the first period policy has two maxima: one at $\beta x_M + (1-\beta)x_I$ and a second at $[\beta x_M + (1-\beta)x_I][1 - 8\alpha\psi x_I(1-\gamma)]$. The condition on γ identifies which one of the two is the global maximum.

Case 1: $x_d^* = \beta x_M + (1-\beta)x_I$, $x_{nd}^* = (\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2}, \tag{40}$$

$$\beta \geq \frac{1 - 4\alpha\psi x_I}{4\alpha\psi(x_M - x_I)}, \tag{41}$$

$$and \quad \gamma < \frac{1 + 4\alpha\psi((\beta x_M + (1-\beta)x_I)(4\alpha\psi x_I - 1))}{(4\alpha\psi)^2 x_I(\beta x_M + (1-\beta)x_I)}. \tag{42}$$

Additionally, the equilibrium condition for the dissenter is

$$\begin{aligned} -(\beta x_M + (1-\beta)x_I - x_M)^2 - \gamma(x_I - x_M)^2 \\ -(1-\gamma)(x_I + x_M)^2 > \\ -[(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma)) - x_M]^2 \\ -[1 - 4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I - x_M)^2 \\ -[4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I + x_M)^2. \end{aligned} \tag{43}$$

Let $x^D = \beta x_M + (1-\beta)x_I$ and $x^D - \Delta = (\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))$ where $\Delta = (\beta x_M + (1-\beta)x_I)8\alpha\psi x_I(1-\gamma)$. The above reduces to

$$-\Delta^2 + 2\Delta(x^D - x_M) + 4x_I x_M(1-\gamma) - 16\alpha\psi(1-\gamma)x_I x_M(x^D - \Delta) < 0. \tag{44}$$

Substituting $\Delta = (\beta x_M + (1-\beta)x_I)8\alpha\psi x_I(1-\gamma)$ and dividing for $4x_I(1-\gamma)$ gives

$$\begin{aligned} -x_I(4\alpha\psi)^2(1-\gamma)(\beta x_M + (1-\beta)x_I)^2 + 4\alpha\psi(\beta x_M + (1-\beta)x_I)(x^D - x_M) \\ +x_M - 4\alpha\psi x_M(x^D - (\beta x_M + (1-\beta)x_I)8\alpha\psi x_I(1-\gamma)) < 0. \end{aligned} \tag{45}$$

Substituting $x^D = \beta x_M + (1-\beta)x_I$ and solving for γ gives us condition:

$$\gamma > 1 + \frac{x_M - 4\alpha\psi[\beta x_M + (1-\beta)x_I][2x_M - \beta x_M - (1-\beta)x_I]}{(4\alpha\psi)^2 x_I[\beta x_M + (1-\beta)x_I][2x_M - \beta x_M - (1-\beta)x_I]}. \tag{46}$$

Thus, the conjectured equilibrium exist if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \in \{\frac{1}{2}, 1 + \frac{x_M - 4\alpha\psi[\beta x_M+(1-\beta)x_I][2x_M-\beta x_M-(1-\beta)x_I]}{(4\alpha\psi)^2 x_I[\beta x_M+(1-\beta)x_I][2x_M-\beta x_M-(1-\beta)x_I]}\} < \gamma < \frac{1+4\alpha\psi((\beta x_M+(1-\beta)x_I)(4\alpha\psi x_I-1))}{(4\alpha\psi)^2 x_I(\beta x_M+(1-\beta)x_I)} = \overline{\gamma}$

2. $\underline{\beta} = \frac{1-4\alpha\psi x_I}{4\alpha\psi(x_M-x_I)} \leq \beta < min \in \{1, \frac{1+4\alpha\psi x_I(2\alpha\psi x_I-1)}{4\alpha\psi(x_M-x_I)(1-2\alpha\psi x_I)}\} = \overline{\beta}$

3. $x_M > \frac{1}{4\alpha\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x_M ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 2: $x_d^* = \frac{1}{4\alpha\psi}$, $x_{nd}^* = (\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma)$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2}, \tag{47}$$

$$\beta < \frac{1-4\alpha\psi x_I}{4\alpha\psi(x_M - x_I)}, \tag{48}$$

$$\gamma > \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi(\beta x_M + (1-\beta)x_I)} - 1). \tag{49}$$

Additionally, the equilibrium condition for the dissenter is

$$\begin{aligned} -(\frac{1}{4\alpha\psi} - x_M)^2 - \gamma(x_I - x_M)^2 - (1-\gamma)(x_I + x_M)^2 > \\ -[(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma)) - x_M]^2 \\ -[1 - 4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I - x_M)^2 \\ -[4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I + x_M)^2. \end{aligned} \tag{50}$$

Let $I = 4\alpha\psi(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))$. The above can be rewritten as:

$$-(\frac{1}{4\alpha\psi} - x_M)^2 - \gamma(x_I - x_M)^2 - (1-\gamma)(x_I + x_M)^2 > \tag{51}$$
$$-(\frac{I}{4\alpha\psi} - x_M)^2 - (1 - I(1-\gamma))(x_I - x_M)^2 - I(1-\gamma)(x_I + x_M)^2,$$

which reduces to

$$(1 - I)(\frac{x_M}{2\alpha\psi} - 4x_I x_M(1-\gamma) - \frac{1+I}{(4\alpha\psi)^2}) > 0. \tag{52}$$

By substituting $I = 4\alpha\psi(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))$ and solving for γ we get condition:

$$1 + \frac{-1 + 4\alpha\psi(2x_M - x_I - \beta(x_M - x_I)}{-2(4\alpha\psi)^2 x_I(2x_M - x_I - \beta(x_M - x_I)} < \gamma < 1. \tag{53}$$

Thus, the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \in \{\frac{1}{2}, 1 + \frac{-1+4\alpha\psi(2x_M - x_I - \beta(x_M - x_I)}{-2(4\alpha\psi)^2 x_I(2x_M - x_I - \beta(x_M - x_I)}, \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi(\beta x_M + (1-\beta)x_I)} - 1)\} < \gamma < 1 = \overline{\gamma}$

2. $\underline{\beta} = max \in \{0, \frac{1 - 4\alpha\psi x_I - 2(4\alpha\psi x_I)^2}{4\alpha\psi(x_M - x_I)(8\alpha\psi x_I + 1)}\} < \beta < \overline{\beta} = min \in \{\frac{1 - 4\alpha\psi x_I}{4\alpha\psi(x_M - x_I)}, \frac{4\alpha\psi(2x_M - x_I) - 1}{4\alpha\psi(x_M - x_I)}\}$

3. $x_M > \underline{x_M} = max \in \{\frac{1+4\alpha\psi x_I}{8\alpha\psi}, \frac{1+4\alpha\psi x_I}{4\alpha\psi(1+8\alpha\psi x_I)}\}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The condition on x_M ensures that the range $[\underline{\beta}, \overline{\beta}]$ exists.

Case 3: $x_d^* = (\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi x_I \gamma)$**,** $x_{nd}^* = (\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma)$

The equilibrium conditions for the incumbent are:

$$\gamma > \frac{1}{2}, \tag{54}$$
$$\beta < \frac{1 - 4\alpha\psi x_I}{4\alpha\psi(x_M - x_I)}, \tag{55}$$
$$and \quad \gamma < \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi(\beta x_M + (1-\beta)x_I)} - 1). \tag{56}$$

Additionally, the equilibrium condition for the dissenter is

$$\begin{aligned}
&-[(\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi x_I\gamma)) \\
&-x_M]^2 - [1 - 4\alpha\psi\gamma(\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi x_I\gamma)](x_I + x_M)^2 \\
&-[4\alpha\psi\gamma(\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi x_I\gamma)](x_I - x_M)^2 > \\
&-[(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma)) - x_M]^2 \\
&-[1 - 4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I - x_M)^2 \\
&-[4\alpha\psi(1-\gamma)(\beta x_M + (1-\beta)x_I)(1 - 8\alpha\psi x_I(1-\gamma))](x_I + x_M)^2.
\end{aligned} \tag{57}$$

Let $x^D = (\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi x_I\gamma)$. We can rewrite the above as:

$$\begin{aligned}
&-(x^D - x_M)^2 - (1 - 4\alpha\psi x^D\gamma)(x_I + x_M)^2 - 4\alpha\psi x^D\gamma)(x_I - x_M)^2 > \\
&-(x^D - 8\alpha\psi x_I(\beta x_M + (1-\beta)x_I) - x_M)^2 \\
&-(1 - 4\alpha\psi(1-\gamma)(x^D - 8\alpha\psi x_I(\beta x_M + (1-\beta)x_I))(x_I - x_M)^2 \\
&-4\alpha\psi(1-\gamma)(x^D - 8\alpha\psi x_I(\beta x_M + (1-\beta)x_I)(x_I + x_M)^2,
\end{aligned} \tag{58}$$

which reduces to

$$\begin{aligned}
&-4x_I x_M + 16\alpha\psi x^D x_I x_M\gamma > \\
&-(8\alpha\psi x_I(\beta x_M + (1-\beta)x_I))^2 + 16\alpha\psi x_I(\beta x_M + (1-\beta)x_I)(x^D - x_M) \\
&-16\alpha\psi x_I x_M(1-\gamma)(x^D - 8\alpha\psi x_I(\beta x_M + (1-\beta)x_I)).
\end{aligned} \tag{59}$$

By substituting $x^D = (\beta x_M + (1-\beta)x_I)(1 + 8\alpha\psi(x_I)^2\gamma)$ and solving for γ we obtain condition:

$$\gamma > \frac{x_M + 4\alpha\psi(\beta x_M + (1-\beta)x_I)(1 - 4\alpha\psi x_I)(\beta x_M + (1-\beta)x_I - 2x_M)}{2x_I(4\alpha\psi)^2(\beta x_M + (1-\beta)x_I)(-\beta x_M - (1-\beta)x_I + 2x_M)}. \tag{60}$$

Thus the conjectured equilibrium exists if and only if the following conditions are satisfied:

1. $\underline{\gamma} = max \in \{\frac{1}{2}, \frac{x_M+4\alpha\psi(\beta x_M+(1-\beta)x_I)(1-4\alpha\psi x_I)(\beta x_M+(1-\beta)x_I-2x_M)}{2x_I(4\alpha\psi)^2(\beta x_M+(1-\beta)x_I)(-\beta x_M-(1-\beta)x_I+2x_M)}\} < \gamma <$
   $min \in \{1, \frac{1}{8\alpha\psi x_I}(\frac{1}{4\alpha\psi(\beta x_M+(1-\beta)x_I)} - 1)\} = \overline{\gamma}$

2. $\underline{\beta} = max \in \{0, 1-\frac{1}{2}\sqrt{\frac{x_I(4\alpha\psi x_M)^2+4\alpha\psi(x_M)^2-x_M}{\alpha\psi(x_M-x_I)^2(1+4\alpha\psi x_I)}}\} < \beta < min \in \{\frac{1+2x_I(4\alpha\psi)^2(x_M-x_I)-\sqrt{1+4(4x_M x_I\alpha\psi)^2(4\alpha\psi)^2}}{32\alpha^2\psi^2 x_I(x_M-x_I)},$
   $\frac{1-4\alpha\psi x_I(1+4\alpha\psi x_I)}{4\alpha\psi(x_M-x_I)(1+4\alpha\psi x_I)}\} = \overline{\beta}$

3. $x_M > max \in \{\frac{1}{4\alpha\psi(1+4\alpha\psi x_I)}, \frac{x_I(1-(4\alpha\psi x_I)^2)}{1-2(4\alpha\psi x_I)^2}, \frac{1+4\alpha\psi x_I}{4\alpha\psi(1+8\alpha\psi x_I)}\}$

4. $x_I < \frac{\sqrt{5}-1}{8\alpha\psi}$

The conditions on β ensure that the range $[\underline{\gamma}, \overline{\gamma}]$ exists. The conditions on x_M and x_I ensure that the range $[\underline{\beta}, \overline{\beta}]$ exists. □

Corollary 2. *Suppose that* $\frac{1}{8\alpha\psi} < x_I$ *and* $\frac{1}{4\alpha\psi} < x_M < \frac{1}{4\alpha\psi(1-2\alpha\psi x_I)}$*. Then, for all* $\beta \in [0,1)$*, there exists a non-measure zero set* $\Gamma(\beta)$ *such that if* $\gamma \in \Gamma(\beta)$ *public dissent occurs in equilibrium.*

Proof. From an analysis of the cases above we can verify that sufficient conditions for the claim (for all $\beta \in [0,1)$, there exists a non-measure zero set $\Gamma(\beta)$) to hold are:

- The binding upper bound $\overline{\beta}$ in case 1 is $= 1$
- The binding lower bound $\underline{\beta}$ in case 2 is $= 0$
- The binding upper bound $\overline{\beta}$ in case 2 is $= \frac{1-4\alpha\psi x_I}{4\alpha\psi(x_M-x_I)}$ (which is also the lower bound from case 1)

For the three conditions to be satisfied we need $\frac{1}{4\alpha\psi} < x_M < \frac{1}{4\alpha\psi(1-2\alpha\psi x_I)}$ and $x_I > \frac{1}{8\alpha\psi}$ □

Proposition 3. *There exists a unique* $\widehat{x_M}$ *s.t. if* $x_M > \widehat{x_M}$*, then the misaligned copartisan's incentives to dissent are stronger under higher office rents. Otherwise, if* $x_M < \widehat{x_M}$*, then the incentives to dissent decrease as office rents increase.*

Proof. Proceeding as in the baseline, we obtain that

$$x_d^* = \min \in \{x_I + 2\bar{\alpha}\psi\gamma(\rho\xi + 4x_I^2), \frac{1}{4\bar{\alpha}\psi}\}, \tag{61}$$

and

$$x_{nd}^* = \max \in \{0, x_I - 2\bar{\alpha}\psi(1-\gamma)(\rho\xi + 4x_I^2)\}. \tag{62}$$

For simplicity, we will assume that x_I is sufficiently small that $x_d^* = x_I + 2\bar{\alpha}\psi\gamma(\rho\xi + 4x_I^2)$ and $x_{nd}^* = x_I - 2\bar{\alpha}\psi(1-\gamma)(\rho\xi + 4x_I^2)$. Notice that we can write $x_{nd}^* = x_d^* - \lambda$, where $\lambda = 2\bar{\alpha}\psi(\rho\xi + 4x_I^2)$. Then, we have that dissent emerges in equilibrium if and only if

$$\begin{aligned} -(x_d^* - x_M)^2 - (1 - 4\bar{\alpha}\psi x_d^*\gamma)(x_I + x_M)^2 - 4\bar{\alpha}\psi x_d^*\gamma[(x_I - x_M)^2 - (1-\rho)\xi] > \\ -(x_d^* - \lambda - x_M)^2 - 4\bar{\alpha}\psi(x_d^* - \lambda)(1-\gamma)(x_I + x_M)^2 - [1 - 4\bar{\alpha}\psi(x_d^* - \lambda)(1-\gamma)][(x_I - x_M)^2 - (1-\rho)\xi]. \end{aligned} \tag{63}$$

The above reduces to

$$-4x_I x_M + \lambda^2 - 2\lambda(x_d^* - x_M) + 16\bar{\alpha}\psi x_I x_M[x_d^* - \lambda(1-\gamma)] - \xi(1-\rho)[1 + 4\bar{\alpha}\psi(\lambda(1-\gamma) - x_d^*)] > 0. \tag{64}$$

Differentiating the LHS with respect to ξ we obtain

$$\begin{aligned} 2\frac{\partial\lambda}{\partial\xi}\lambda - 2\frac{\partial\lambda}{\partial\xi}(x_d^* - x_M) - 2\lambda\frac{\partial x_d^*}{\partial\xi} + 16\bar{\alpha}\psi x_I x_M[\frac{\partial x_d^*}{\partial\xi} - \frac{\partial\lambda}{\partial\xi}(1-\gamma)] \\ -(1-\rho)[1 + 4\bar{\alpha}\psi(\lambda(1-\gamma) - x_d^*)] + \xi(1-\rho)4\bar{\alpha}\psi[\frac{\partial x_d^*}{\partial\xi} - \frac{\partial\lambda}{\partial\xi}(1-\gamma)] \end{aligned} \tag{65}$$

Where $\frac{\partial \lambda}{\partial \xi} = 2\bar{\alpha}\psi\rho > 0$, $\frac{\partial x_d^*}{\partial \xi} = 2\bar{\alpha}\psi\rho\gamma > 0$ and $\frac{\partial x_d^*}{\partial \xi} - \frac{\partial \lambda}{\partial \xi}(1-\gamma) = 2\bar{\alpha}\psi\rho(2\gamma - 1) > 0$. Therefore, 65 is increasing in x_M and always positive for a sufficiently large x_M. □

Appendix B: Welfare Analysis

Proposition 4. *Suppose that* $x_I < \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$*, and the conditions in Proposition 1 are satisfied. Then, in equilibrium the voter benefits from the presence of an extreme faction in the incumbent party if:*

- *The voter's dislike of dissent is sufficiently small (i.e.,* δ *is sufficiently small)*
- *The value of information is sufficiently high*
  - *The prior* γ *is sufficiently close to* $\frac{1}{2}$
  - *Incumbent and challenger are sufficiently polarized (i.e.,* x_I *is sufficiently large)*
  - *Learning the true state has a sufficiently large impact on the voter's preferences (i.e.,* $\bar{\alpha}$ *is sufficiently large)*
- *The faction is sufficiently extreme (i.e.,* x_M *is sufficiently large)*

Proof. In equilibrium, the voter benefits from dissent if and only if

$$-\gamma(x_d^* - \bar{\alpha})^2 - (1-\gamma)(x_d^* + \bar{\alpha})^2 - (1 - 4\bar{\alpha}\psi x_d^*)[\gamma(-x_I - \bar{\alpha})^2 + (1-\gamma)(-x_I + \bar{\alpha})^2] - 4\bar{\alpha}\psi x_d^*(\bar{\alpha} - x_I)^2 - \delta >$$
$$-\gamma(x_{nd}^* - \bar{\alpha})^2 - (1-\gamma)(x_{nd}^* + \bar{\alpha})^2 - (1 - 4\bar{\alpha}\psi x_{nd}^*)[\gamma(x_I - \bar{\alpha})^2 + (1-\gamma)(x_I + \bar{\alpha})^2] - 4\bar{\alpha}\psi x_{nd}^*(\bar{\alpha} - x_I)^2. \tag{66}$$

Suppose $x_I < \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$, and plug in the values of x_d^* and x_{nd}^*. The above reduces to

$$\delta < \frac{(1-2\gamma)(1 - 8\alpha\psi x_I + 2(4\alpha\psi x_I)^2 + 16\alpha\psi^2(x_I)^3) - 4\psi(x_I)^2 + 4(4\alpha\psi x_I\gamma)^2}{\psi\gamma(1 + 8\alpha\psi x_I\gamma)} = \delta_w. \tag{67}$$

Recall that dissent emerges only if $\delta > 4\bar{\alpha}x_I(2\gamma - 1)$. Therefore, necessary condition for dissent to emerge in equilibrium under $\delta < \delta_w$ is that $\delta_w > 4\bar{\alpha}x_I(2\gamma - 1)$, which reduces to:

$$(1 - 2\gamma)(1 + 4\alpha\psi x_I(8\alpha\psi x_I - 2 + 4\psi(x_I)^2 + 8\alpha\psi x_I\gamma^2 + \gamma)) - 4\psi(x_I)^2 + 4(4\alpha\psi x_I\gamma)^2 > 0. \quad (68)$$

The LHS is decreasing in γ, therefore the condition establishes an upper bound γ_w. Further, notice that the condition is satisfied at $\gamma = \frac{1}{2}$ iff $\bar{\alpha} > \frac{1}{2\sqrt{\psi}} = \alpha_w$. Finally, from the proof of Case 2 we can verify that if $x_I > \frac{1}{8\alpha\psi} = x_{Iw}$ and $x_M > \frac{4\alpha\psi x_I^2}{8\alpha\psi x_I - 1} = x_{Mw}$, then $\underline{\gamma} = \frac{1}{2}$.

Thus, assuming that $x_I < \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$ and that the conditions in Proposition 1 are satisfied, the voter benefits from dissent in equilibrium if $\delta < \delta_w$, $\bar{\alpha} > \bar{\alpha}_w$, $\gamma < \gamma_w$, $x_I > x_{Iw}$ and $x_M > x_{Mw}$. This concludes the proof.

□

Appendix C: Public Dissent by a Moderate Faction

In this section I consider a dissenter whose bliss point is to the left of the incumbent: $0 < x_M < x_I$. In line with the rest of the paper, I maintain the assumption that $x_I < \frac{1}{4\alpha\psi}$.

Lemma .6. *Public dissent by a moderate member emerges in equilibrium only if, absent dissent, the incumbent is trailing, i.e.,* $\gamma < \frac{1}{2}$.

Proof. Suppose that $\gamma > \frac{1}{2}$, i.e., the incumbent is leading. From the proof of Lemma 4, we know that dissent can never emerge (given $x_M > 0$) if it turns the incumbent into certain loser (i.e., $\delta > 4\alpha\psi x_I$). Suppose instead dissent would turn the leading incumbent into a trailing one. Here, the proof of Proposition 1 shows that dissent can never emerge when $x_M < x_I$. □

Lemma .7. *The incumbent's moderate copartisan chooses to publicly attack him only if its electoral cost is sufficiently high to turn the trailing incumbent into a certain loser, i.e.,* $\delta > 4\bar{\alpha}\psi x_I$.

Proof. Suppose that $\delta < 4\bar{\alpha}\psi x_I$. Then, whether or not he experiences dissent, the incumbent remains electorally trailing: he wins reelection if and only if the voter observes an informative outcome and learns that her ideal policy is a right-wing one. Thus, $x_d^* = x_{nd}^*$, and dissent never emerges in equilibrium. □

Proposition 5. *Suppose that* $\gamma < \frac{1}{2}$ *and* $\delta > 4\bar{\alpha}\psi x_I$. *Then, there exist unique* $\bar{x_I}$, $\bar{x_M}(x_I)$ *and* $\bar{\gamma}(x_I)$ *such that public dissent emerges in equilibrium if and only if*

- *The incumbent's disadvantage is sufficiently large,* $\gamma < \bar{\gamma}$, *and*
- *Both the incumbent and the dissenter are sufficiently moderate,* $x_I < \bar{x_I}$ *and* $x_M < \bar{x_M}$.

Proof. From Lemma .7 we know that if the incumbent experiences dissent in equilibrium, it must be the case that $x_d^* = x_I$. Suppose instead the incumbent does not experience dissent. As for Proposition 1, we must consider three cases:

- If $x_I > \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$, then $x_{nd}^* = \frac{1}{4\bar{\alpha}\psi}$

- If $x_I < \frac{\sqrt{8\gamma+1}-1}{16\bar{\alpha}\psi}$, then $x^*_{nd} = x_I + 8\alpha\psi x_I^2\gamma$

I will analyse each case separately.

Case 1: $x_I > \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$

Given the incumbent's best response, the dissenter attacks if and only if

$$-(x_I - x_M)^2 - (x_I + x_M)^2 > -(\frac{1}{4\alpha\psi} - x_M)^2 - \gamma(x_I - x_M)^2 - (1-\gamma)(x_I + x_M)^2, \tag{69}$$

which reduces to

$$\gamma < \frac{-8\alpha\psi x_M(1 - 4\alpha\psi x_I) - (4\alpha\psi x_I)^2 + 1}{(8\alpha\psi)^2 x_I x_M}. \tag{70}$$

Since $\gamma > 0$, the above requires

$$x_M < \frac{1 - (4\bar{\alpha}\psi x_I)^2}{(1 - 4\bar{\alpha}\psi x_I)8\bar{\alpha}\psi}, \tag{71}$$

which in turns requires $x_I < \frac{1}{4\bar{\alpha}\psi}$.

case 2: $x_I < \frac{\sqrt{8\gamma+1}-1}{16\gamma\bar{\alpha}\psi}$

In this case, the dissenter attacks if and only if

$$\begin{aligned} -(x_I - x_M)^2 - (x_I + x_M)^2 > \\ -(x_I + 8\alpha\psi(x_I)^2\gamma - x_M)^2 - (1 - 4\alpha\psi\gamma(x_I + 8\alpha\psi(x_I)^2\gamma))(x_I + x_M)^2 \\ -4\alpha\psi\gamma(x_I + 8\alpha\psi(x_I)^2\gamma)(x_I - x_M)^2, \end{aligned} \tag{72}$$

which reduces to

$$x_M < \frac{x_I}{2}. \tag{73}$$

Combining all of the above, we obtain that there exist unique $\bar{x_I}$, $\bar{x_M}(x_I)$ and $\bar{\gamma}(x_I)$ such that dissent emerges in equilibrium if and only if $x_I < \bar{x_I}$, $x_M < \bar{x_M}(x_I)$ and $\gamma < \bar{\gamma}(x_I)$. □