

Accountability with Multidimensional Policy Experimentation

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October 7, 2022

Abstract

Politics in general and policymaking in particular are inherently multidimensional. We develop a model of accountability to study policymaking when multiple policy dimensions are available. In the model, an incumbent chooses whether and how to act on each of two correlated policy dimensions. While voters do not know what the optimal policies are, they can infer it by observing the incumbent's choices and the resulting outcomes. Thus, the incumbent influences voter learning by either focusing on a single issue or acting on multiple policy dimensions. We characterize the officeholder's decision to expand (or contract) the scope of policymaking, based on his ex-ante electoral strength and on the correlation across dimensions. Results also show how the possibility to act on multiple correlated dimensions influences policymakers' incentives to pursue moderate or extreme policies.

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1 Introduction

Contemporary politics within a large number of Western democracies, most notably the United States, are frequently described as polarized along a dominant issue or ideological dimension. Nevertheless, political pundits, practitioners, and even scholars acknowledge that politics and policy-making is inherently multidimensional, especially in polities with varied economies, racial diversity, and religious pluralism.

More than just an important descriptive feature of politics, classic accounts of power and influence frequently contend that the decision over which of these dimensions to pursue—and which to avoid (Bachrach and Baratz, 1963)—is perhaps the most consequential factor of all in determining who wins and loses in politics. Indeed, as Schattschneider famously summarizes in his classic analysis of American democracy: “Whoever decides what the game is about decides also who can get into the game” (Schattschneider, 1960). The expansion of a political appeal to include more than one issue area can dramatically influence the organization of a political party, the character of electoral campaigns, the dynamics of legislative bargaining, and the extent of policy output in a legislature. Furthermore, the choice of whether and which issues and issue positions should “go together” is a strategic consideration of tremendous political consequence, as Bawn et al. (2012) and others have shown. As a result, it stands to reason that political leaders must think carefully about which issues they will pursue. Yet, the strategic considerations behind this choice remain largely unexplored in the literature.

In this paper, we aim at addressing this question by presenting a formal model to study policy-making in a multidimensional world, within an electoral accountability framework. We investigate how the introduction of multiple, correlated issue dimensions into candidate-voter interactions influences how politicians approach policymaking. Our contribution is twofold. First, we study officeholders’ decisions to expand (or contract) the scope of policymaking. Second, we characterize how the possibility to act on multiple dimensions influences policymakers’ incentives to pursue moderate or extreme programs.

1.1 Voter Learning, Multidimensionality and Accountability

Two ingredients lie at the core of our theory. First, voters face uncertainty about their optimal policy. Second, when orienting themselves in a multidimensional world voters may obtain relevant information on one dimension by observing the content and outcome of policymaking in another.

Problems of voter information are a central challenge to popular rule. As a large body of both theoretical and empirical research has shown, voters face incentives to remain “rationally ignorant” of information useful in rendering voting decisions. As Downs (1957) puts it, gathering and processing information about candidates and issues makes little sense for any individual voter, particularly given the exceedingly low probability that she will be pivotal in an election.

Nevertheless, millions of voters cast ballots each election, and a large body of scholarship has argued that voters link their voting decisions not simply to policy information, but to their personal well-being. Although scholars disagree about the relative weight that voters place on various vote-influencing factors, they generally maintain that voters choose their optimal strategy based not only on the incumbent’s actions, but also on how such actions impact observed outcomes. Indeed, some empirical research has shown that voters can and do react to the results of policy choices—not simply the substance of the policies themselves (e.g., Fiorina 1978, Alt, Bueno de Mesquita and Rose 2011).

We build on these results, but also observe that multidimensionality in politics presents voters—and parties and candidates—with additional informational challenges. Even with available heuristics and policy feedback, voters face the problem of understanding new issue areas and how to orient themselves when navigating this multidimensional world (Izzo, Martin and Callander, Forthcoming). Here, our argument is that different issue areas are both inherently and symbolically connected to one another. What is best for a citizen in one facet of public life and policy is anything but disconnected from other facets. Thus, correlations (whether perceived or real) across issue dimensions are crucial in determining how voters address such informational challenges, and how parties approach policymaking.

That connections across issue areas are relevant for politicians and voters alike is demonstrated

by the fact that emphasizing such connections is, in many ways, precisely the function of party manifestos or platform. Each begins with a “preamble,” articulating the party’s overarching desired outcomes for society. In 2016, the Democratic Party in the U.S., for example, states a primary goal of *equality*—political, economic, and social—among Americans. This goal is then articulated in specific policy positions on a host of different dimensions, from education, to healthcare, to redistribution. Finally, the manifesto ends with the following statement: “What makes America great is our unerring belief that we can make it better. We can and we will build a more just economy, a more equal society, and a more perfect union—because we are stronger together.” If a voter agrees with those outcomes, the authors imply, she should necessarily support the party’s positions not just in one policy area, but in all those to follow.

Such claims are not confined, of course, to left-leaning causes, nor are they solely the purview of parties. Indeed, prior to the so-called Republican Revolution of the 1980s in the U.S., conservative thought leaders expended considerable energy making the case that the newly developed right-wing coalition of religious, social, and economic conservatives was *not* one of simple political convenience. Instead, right-leaning political theorists, economists, and some politicians branded the movement as the New Fusionism, wherein they argued that the societal goals of social and economic conservatives were *necessarily* intertwined.

Here, we take the connections (whether real or perceived) across issue dimensions as given, and study how they influence the inferences that guide voters’ electoral decisions and, in turn, policymakers’ strategic incentives. Our focus on voter learning and accountability contrasts with the existing literature on multidimensional policymaking, which focuses on how the presence of additional issues may facilitate logrolling or influence coalition building within legislatures (e.g., Banks and Duggan 2000; 2006). Further, this literature generally assumes that all available dimensions must be legislated on. Thus, we offer among the first formal examinations of policymakers’ decision of whether and when to expand policymaking activities to new dimensions.¹

¹The most closely related paper is Buisseret and Van Weelden (2020), which analyzes the entry and platform positioning of outsider candidates in a multidimensional world, where parties face uncertainty about the distribution of voters. Buisseret and Van Weelden (2022) also looks at a multidimensional world, and analyzes an incumbent’s decision to call a referendum on a secondary policy issue in order to reveal information about the distribution of

1.2 Our Approach

In our model, an incumbent decides whether to change policy in a primary area and, if so, where to move the policy. Beyond this baseline decision, however, the incumbent must also determine whether to legislate in an *secondary* policy area, again determining where in ideological space the new policy should be set. Importantly, players face uncertainty about which policy is optimal for the voter on each dimension. For example, we can think about a world where the mapping from policies to outcomes is unknown (e.g., Callander 2011). The key difference between the primary and secondary dimensions is that voters and politicians alike have initially more information on the former. Complicating the policymaker’s decision, the model builds on the aforementioned observation that some policy areas are “correlated” with others, meaning that voters can learn about how well a candidate’s program fits their preferences in one dimension by observing the policy outcome in another. Voters thus respond in the model by updating their priors based on the policy changes adopted by the incumbent candidate/party and the resulting outcomes (as in the tradition of career concerns models, e.g., Holmström 1999 or Ashworth, Bueno de Mesquita and Friedenberga 2017), and by their beliefs regarding the correlation of policy areas.

In this setting the inferences voters draw when observing outcomes depend on the policies implemented by the officeholder (as in Izzo Forthcoming). On each dimension, no new information is generated directly if the policy remains at the status quo. Conversely, new policies may allow voters to learn, and more extreme policies tend to generate more informative outcomes. Intuitively, if a voter obtains a good outcome from an extreme liberal policy, it must be the case that such policy is aligned with the voter’s interests. In contrast, the outcome of a moderate policy is much less informative: favorable shocks may allow the voter to enjoy a relatively high welfare even under a wrong policy, if the policy is not too radical.

Thus, the incumbent can control the amount of voter learning on each dimension, both directly (via the policy on that dimension) and indirectly (via his choices on the other correlated dimensions

voters and thus influence the equilibrium of the platform game in the following elections. In contrast, we consider multidimensional *policymaking* by officeholders, in a world where voters themselves are uncertain about the optimal policy.

that generate learning spillovers). These considerations therefore influence officeholders' choice whether to act on each available dimension and, if so, which policy to implement.

This setup enables us to examine politicians' decisionmaking over issue expansion, elucidating how the ability to expand policymaking efforts can fundamentally alter the nature of policy outcomes. Moreover, we are also able to show how these dynamics change in response to a host of relevant contextual variables—most notably in response to politicians' ideological tastes and different levels of correlation between the policy areas the politician is considering.

1.3 Preview of Main Results

We begin by analyzing a baseline case where the voter only cares about the primary dimension and learning spillovers are not possible. In this case, even if the incumbent's ideological preferences are multidimensional, his strategic problem is unidimensional: his electoral chances are only a function of his policy choice on the primary dimension. In this baseline, an incumbent who is ex-ante electorally leading always has incentives to prevent direct voter learning on the primary dimension. For this incumbent, in fact, no new information guarantees his initial advantage is preserved and the voter will choose to retain him. As a consequence, he always pursues a policy more moderate than his true ideological preferences. The opposite holds for a trailing incumbent, who has incentives to gamble for resurrection and thus implement extreme policies that facilitate voter learning.

Moving to the multidimensional setting, we find that trailing incumbents in our model have strategic incentives to pursue policymaking on the secondary issue, even when they do not have any ideological preferences over it. Electorally disadvantaged incumbents want to exploit the secondary dimension to expand opportunities for voter learning, and therefore improve their electoral chances. In particular, strong correlation across issue areas generates strategic incentives to expand the scope of policymaking: even if he only cares about a single issue, policymaking under a trailing incumbent will inevitably be multidimensional. Furthermore, this incumbent will always pursue extreme policies on the secondary dimension in order to facilitate voter learning. In our setting, extreme policies therefore need not result from extreme ideologies.

In contrast, leading incumbents face a trade-off in multiple dimensions. Here, the correlation between the policy dimension is paramount, as high correlation actually *discourages* policymaking on a secondary dimension even if the incumbent has ideological preference over it. A high correlation implies that learning spillovers from the secondary dimension have a large impact on the voter’s beliefs on the primary one. This is electorally risky for the leading incumbent, as a negative outcome on either the primary or the secondary dimension would cost him the election. Thus, high correlation pushes leading incumbent to contract the scope of policymaking so as to prevent opportunities for learning spillovers. By contrast, low correlation allows leading incumbents to legislate on the secondary dimension without risking their electoral advantage with voters.

Next, we find that the presence of a secondary dimension has an impact on the policies the incumbent pursues on the primary one. Interestingly, we uncover a strategic substitution effect between dimensions. First, consider on a leading incumbent. In the unidimensional baseline, this incumbent pursues moderate policies on the primary issue so as to suppress voter learning. Suppose instead the incumbent faces a multidimensional problem, and his ideological tastes push him to act on the secondary dimension. Recall that a leading incumbent is damaged by learning spillovers across dimensions. We find that while this induces moderation on the secondary dimension, it instead pushes the incumbent to pursue extreme policies on the primary one. He does so in hopes of generating enough positive information on the primary dimension to counteract the electoral consequences of a negative outcome on the secondary one. Policymakers who are risk-averse in a unidimensional world instead become risk-loving on the main policy issue when they choose to expand the scope of policymaking.

The opposite holds for a trailing incumbent, who always pursues extreme policies in the unidimensional baseline. In a multidimensional world, this incumbent has incentives to exploit the secondary dimension to expand opportunities for voter learning. This, in turn, induces him to pursue moderate policies on the primary dimension in order to magnify the electoral effect of learning spillovers from the secondary one.

Finally, our last result analyzes which features of a policy dimension make it more likely to be

activated by the officeholder. We show that leading incumbents will tend to expand the scope of policymaking to incorporate dimensions over which their ideological preferences are more moderate and for which voter's inference problem is more complex. In contrast, trailing incumbents will tend to open new dimensions they have extreme ideological taste on, and for which the voter's information challenges are easier to overcome.

Together, these results show how multidimensionality itself can influence the extremity of politics, and how a disconnect between policymaking areas can actually encourage more policymaking outside of the primary issue dimension.

2 The Model

Players and actions. There is an incumbent, I , a challenger, C , and a representative voter, V . In each period, the incumbent chooses whether to act on each of two policy dimensions, $D \in \{X, Z\}$. If he chooses to act on dimension D , then he selects a policy $d_t \in \mathbb{R}$ to be implemented. If he chooses not to act on dimension D , then the status quo d_{sq} remains in place in that period. For simplicity, we normalize the status quo on each dimension to 0, $x_{sq} = z_{sq} = 0$.

Information. Politicians' ideal points are common knowledge and, to streamline the analysis, symmetric around 0: $x_I = -x_C > 0$ and $z_I = -z_C > 0$.

Conversely, the policy that maximizes voter's welfare is unknown. Specifically, on each dimension d the voter's optimal policy d_v can take one of two values: $d_v \in \{-\alpha, \alpha\}$. Players share common prior beliefs that

$$\Pr(x_v = \alpha) = \pi$$

and

$$\Pr(z_v = \alpha | x_v = \alpha) = \Pr(z_v = -\alpha | x_v = -\alpha) = \rho \geq \frac{1}{2}$$

Thus, players believe the dimensions are positively correlated in a symmetric way. The ex-ante

probability that $z_v = \alpha$, which we denote as β , is then given by $\rho\pi + (1 - \rho)(1 - \pi)$.

Notice that in our setting players initially have more information about the voter's ideal policy on dimension X than on dimension Z , i.e., β is always closer to $\frac{1}{2}$ than π is. To reflect this, we will refer to X as the primary policy dimension, and Z as the secondary one.

Payoffs. Player $i \in \{I, V, C\}$'s per-period utility on dimension D is

$$u_{t,D}^i(d_t) = -\lambda_i^d(d_t - d_i)^2 + \varepsilon_{d,t},$$

where $\varepsilon_{d,t} \sim U \in \left[-\frac{1}{2\psi_d}, \frac{1}{2\psi_d}\right]$, and $\lambda_i^x = \lambda_i = 1 - \lambda_i^z$. Thus, λ_i is the weight player i puts on dimension X . The assumption that the noise $\varepsilon_{d,t}$ is uniformly distributed substantially simplifies the analysis, but is not necessary for our results. We briefly return to this point in Section 3.1 below.

Timing. The timing is as follows.

1. For each dimension $D \in \{X, Z\}$, I decides whether to act by choosing a policy $d_1 \in \mathbb{R}$, or instead keep the status quo d_{sq} .
2. V observes I 's choice and her realized utility on each dimension.
3. V chooses whether to re-elect I or replace her with C .
4. The winner of the election takes office, then chooses whether to act on each dimension or instead keep the status quo.

Before concluding this section, let us emphasize that in our setting there is no asymmetry of information between the voter and the politicians. The incumbent does not have privileged information about what policy is optimal for the voter (or his own ideological preferences). This allows us to assume away the possibility that the incumbent's policy choice directly provides information to the voter and instead, following the literature on retrospective evaluations, to focus on what

the voter learns from her lived experiences (i.e., the inferences she draws upon observing realized outcomes).

3 Equilibrium Analysis

We proceed by backward induction. In the second period, both incumbent and challenger implement their preferred policies on each dimension if elected. Thus, the voter faces a selection problem, wanting to elect the office-holder who is more aligned with her own multidimensional ideal point. The voter, however, does not know what the optimal policy is on each dimension. Further, she could be more aligned with the incumbent on one dimension and with the challenger on the other dimension. Thus, her electoral decision depends on her beliefs over the optimal policy on both dimensions X and Z .

Formally, denote by μ^d the voter's posterior that her ideal policy on dimension d is a right-wing one, $\mu^d = \Pr(d_v = \alpha)$, and recall that politicians' bliss points are symmetric around zero on each dimension. Then, the following holds:

Lemma 1. *In equilibrium, the voter reelects the right-wing incumbent if and only if*

$$\mu^x > \frac{1}{2} - \frac{(1 - \lambda_v)z_I}{\lambda_v x_I} \frac{2\mu^z - 1}{2} \equiv \hat{\mu}^x(\mu^z). \quad (1)$$

Proof. All Proofs are collected in the Appendix.² □

When the voter only cares about the primary dimension ($\lambda_v = 1$), it follows from (1) that the right-wing incumbent is reelected as long as the voter believes her optimal policy on dimension X is more likely to be a right-wing one ($\hat{\mu}^x = \frac{1}{2}$). Instead, when the voter cares about both dimensions ($\lambda_v < 1$) she becomes more lenient with the incumbent on dimension X the more she likes him on

²Whenever $\lambda_I < 1$ and $\lambda_C < 1$, so that the politicians care about both dimensions, whoever is elected in the second period would implement their bliss point on both dimension. However, in Section 4.2.1 below we will consider an extension where $\lambda_I = \lambda_C = 1$, so that both candidates only care about the primary dimension, and neither would choose to act on the secondary dimension in the second period. Thus, more generally the voter re-elects the right-wing incumbent if and only if $\mu^x > \frac{1}{2} - \mathbb{I}_z \frac{(1 - \lambda_v)z_I}{\lambda_v x_I} \frac{2\mu^z - 1}{2} \equiv \hat{\mu}^x(\mu^z)$, where $\mathbb{I}_z = 0$ if $\lambda_I = \lambda_C = 1$ and $\mathbb{I}_z = 1$ if $\lambda_I < 1$ and $\lambda_C < 1$.

dimension Z (and vice-versa). This effect is stronger the more (less) polarized candidates are on the secondary (primary) policy dimension.³

To streamline the presentation of the results, we will assume that the voter cares sufficiently about the primary dimension X . Specifically, λ_v is sufficiently large that if the voter believes her ideal point is right-wing on X (i.e., $\mu^x = 1$) but left-wing on Z ($\mu^z = 0$), she prefers to re-elect the right-wing incumbent:

Assumption 1. $\lambda_v > \frac{z_I}{x_I + z_I}$.

Before continuing with the analysis, let us introduce some useful definitions. By plugging in $\mu^x = \pi$ and $\mu^z = \beta = \pi\rho + (1 - \pi)(1 - \rho)$ into Equation 1, we can verify that at $\pi = \frac{1}{2}$ the voter is ex-ante indifferent between retaining the right-wing incumbent and replacing him with the challenger. For any $\pi > \frac{1}{2}$ the voter ex-ante prefers the incumbent, and for $\pi < \frac{1}{2}$ she instead prefers the challenger. Thus, we will say that

Definition 1. *If $\pi > \frac{1}{2}$, the incumbent is ex-ante leading. If instead $\pi < \frac{1}{2}$, the incumbent is ex-ante trailing.*

3.1 Voter Learning

Moving one step backwards, we now study how the voter forms her posterior beliefs μ^z and μ^x . Here, the voter learns about her optimal policy on each dimension from her own lived experiences. Formally, she observes her realized utility on each dimension, and updates her beliefs by applying Bayes rule (as in Izzo Forthcoming). The innovation in this model is that when policies span multiple *correlated* dimensions, the voter learning is twofold: direct, and indirect. The voter's realized utility on each dimension provides her with new information on her optimal platform on that dimension (*direct learning*), but also on the policy-relevant state of the world on the others (*indirect learning*). Thus, the voter's posterior belief on x_v is a function of her realized utility on both dimensions X and Z , and similarly for μ^z .

³Given our symmetry assumption $d_I = -d_c$, polarization on dimension D here is given simply by $2d_I$.

We begin by considering the direct channel. We characterize the voter's *interim* posterior beliefs on each dimension D , i.e., her beliefs as a function of her realized utility on that dimension only. The statements below refer to dimension X , expressions for dimension Z are analogous. The key feature of the voter learning in this setup is that new and more extreme policies generate more information:

Lemma 2 (Direct Learning). *Define $\tilde{\mu}^x$ as the voter's (interim) posterior upon observing the outcome on dimension X . We have:*

- (i) *The outcome on dimension X is either fully informative of x_v or fully uninformative, i.e., $\tilde{\mu}^x \in \{0, \pi, 1\}$;*
- (ii) *Suppose the incumbent does not act on X , i.e., $x_1 = x_{sq}$. Then, the outcome is always uninformative and $\tilde{\mu}^x = \pi$;*
- (iii) *Suppose the incumbent acts on X , i.e., $x_1 \neq x_{sq}$. Let $L_x = 1$ denote the event that the realized outcome on X is fully informative. Then $\Pr(L_x = 1) = \min \in \{1, 4\alpha|x_1|\lambda_v\psi_x\}$.*

Lemma 2 shows that, upon observing outcomes on each dimension, the voter either learns everything or nothing about her true preferences on that dimension. If the policy remains at the status quo, the voter can never observe an informative outcome on that dimension. If instead a new policy is implemented, she is more likely to discover her ideal point as the implemented platform becomes more extreme.

The logic behind this result is intuitive. Suppose the incumbent acts on the primary dimension. Then, in expectation the voter's payoff is different under the two states of the world (i.e., the two possible values of her ideal policy). However, the voter's realized utility on each dimension is also a function of a random period-specific shock $\varepsilon_{d,1}$. This, in turn, creates a partial overlap in the support of the payoff *realization*. When the policy is sufficiently moderate ($x_1 \in (-\frac{1}{4\alpha\psi_d\lambda_{vd}}, \frac{1}{4\alpha\psi_d\lambda_{vd}})$), there exists a range of payoffs that may realize (i.e., be actually observed) whether the voter's true bliss point takes a positive or a negative value. Clearly, if the payoff realization falls outside this

range, it constitutes a fully informative signal. There is only one state of the world that could have generated that specific realization: the voter simply likes the policy too much, or too little, for this to be justified as a consequence of the shock. Thus, upon observing her payoff, the voter discovers her true preferences (i.e., the value of x_v). Conversely, any payoff realization that falls inside the range of overlap is completely uninformative. Since the shock is uniformly distributed, any such realization has exactly the same probability of being observed under the two states of the world. Thus, the voter learns nothing and her interim posterior remains at her prior beliefs. As the implemented policy becomes more extreme, the range of overlap becomes smaller, and the voter is more likely to directly learn her true preferences.

Finally, suppose the policy remains at the status quo. Then, the voter can never learn anything new by observing the policy outcome. Formally, there is full overlap in the support of the payoff realizations, therefore the realized outcome is always uninformative. Notice that this result follows from our assumption that $d_{sq} = 0$. We use this normalization to simplify notation, but our qualitative results below (in particular our results on when the incumbent chooses to act on the secondary dimension) simply require that if the policy remains at the status quo, then no new (direct) information is generated on that dimension. For example, we could assume that if $d_1 = d_{sq}$, then the voter does not observe a new realization of her utility on d , so that no direct learning ever occurs. Thus, the voter may only learn indirectly via the realized outcome on other correlated dimensions.

Figure 1 provides a graphical illustration of the results in Lemma 2. The blue and orange functions represent the conditional outcome distributions (i.e., the distributions of the voter's realized utility), under a positive and a negative state of the world, respectively. In the left panel, a moderate right-wing policy $x > 0$ produces a large overlap in the conditional distributions. In the central panel, a more extreme policy $\hat{x} > x$ produces a much smaller overlap. As a consequence, the voter's inference problem is much easier in the second case. Finally, in the third panel the policy is sufficiently extreme that there is no overlap in the conditional distributions, $\tilde{x} > x'$, and the voter always learns the true value of x_v .

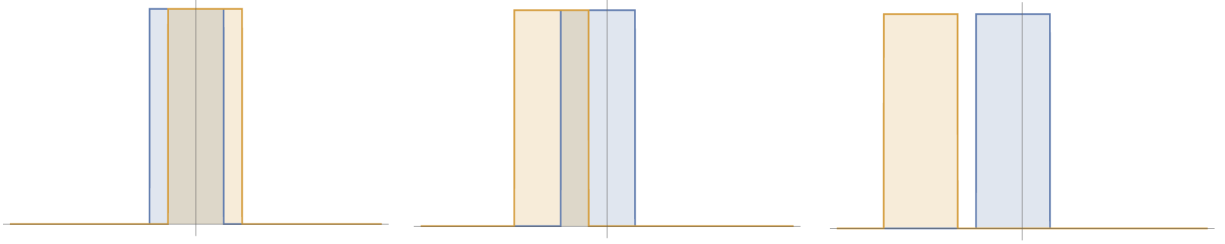


Figure 1: **Voter Learning.** The three plots display voter learning under a positive (blue function) and negative (orange function) state of the world. The policy extremism increases from the left to the right panel.

The assumption of uniformly distributed shocks simplifies the analysis by generating the stark learning environment described above. However, the crucial result that extreme (new) policies facilitate voter learning holds more generally, as it simply requires that the noise distribution satisfies the Monotone Likelihood Ratio Property. For example, Bils and Izzo (2022) shows that this result holds under normally distributed shocks. There, every outcome is somewhat informative but never fully so. Nonetheless, extreme policies continue to facilitate voter learning and thus increase the variance in the posterior distribution.

Lemma 2 indicates that the voter may learn her optimal policy on each dimension *directly* by observing how much she liked or disliked the implemented policy on that issue. Our next result indicates that such direct learning is sufficient but not necessary for the voter to obtain new information. Because dimensions are correlated, the voter will learn more about her optimal policy on the primary dimension if she observes an informative outcome on the secondary (and vice versa). Intuitively, if the voter finds that she likes liberal policies on social or economic issues, she will tend to acquire a more positive attitude towards liberal policies on healthcare as well.

Formally, the next result shows how the voter's posterior belief on X depends on the outcome of the secondary dimension. Recall that $\tilde{\mu}^x$ is the voter's *interim* posterior, as a function only of her realized utility on X . Instead, we denote μ^x the voter's posterior on x_v , as a function of her realized utility on *both* dimensions X and Z . Then, we have:

Lemma 3. *Suppose that the voter observes an uninformative outcome on Z . Then:*

$$\mu^x = \tilde{\mu}^x \quad (2)$$

Suppose that the voter observes an informative outcome on Z and learns that $z_v = \alpha$. Then:

$$\mu^x(\tilde{\mu}^x, \alpha, \rho) = \frac{\tilde{\mu}^x \rho}{\tilde{\mu}^x \rho + (1 - \tilde{\mu}^x)(1 - \rho)} \quad (3)$$

Suppose instead that the voter observes an informative outcome on Z and learns $z_v = -\alpha$. Then:

$$\mu^x(\tilde{\mu}^x, -\alpha, \rho) = \frac{\tilde{\mu}^x(1 - \rho)}{\tilde{\mu}^x(1 - \rho) + (1 - \tilde{\mu}^x)\rho} \quad (4)$$

The proof simply follows by applying Bayes rule, and is therefore omitted. This Lemma highlights that when no direct learning occurs on X (i.e., $\tilde{\mu}^x = \pi$), learning spillovers determine the voter's posterior. If the voter learns that her ideal point on Z is a right-wing (left-wing) one, she becomes more convinced that her optimal policy on X is right-wing (left-wing) as well. The higher the correlation across dimensions ρ , the stronger these learning spillovers.

4 The Incumbent's Problem

The results of the above sections highlight that, in our setting, the policy that is implemented today influences voter learning and thus her optimal retention choice. As such, the incumbent's choice on each dimension has two effects on his expected payoff. First, a *static* ideological effect: the incumbent's first-period payoff increases as the implemented policy gets closer to his ideal point. Second, a *dynamic* information effect: the incumbent's expected second-period payoff depends on the first-period policy via voter learning. In turn, this information effect emerges via two channels. The implemented policy on each dimension influences the probability of the voter *directly* learning her optimal policy on that dimension. In addition, the correlation across dimensions generates

learning spillovers, so that the implemented policy on X can also *indirectly* influence voters beliefs on Z (and vice versa).

These two effects, ideological and information, generate a potential trade-off for the incumbent: one the one hand, he wants to set a policy close to his ideal point, on the other, such policy might not generate enough information, or generate too little. This trade-off clearly appears in the incumbent maximization problem, which we can express as follows:

$$\max_{x_1, z_1} -\lambda_I(x_1 - x_I)^2 - (1 - \lambda_I)(z_1 - z_I)^2 - (1 - \mathbb{P}(x_1, z_1)) \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right), \quad (5)$$

where $\mathbb{P}(x_1, z_1)$ denotes the incumbent's retention probability which is a function of the voter posterior and the incumbent policy choices.

The first-order necessary conditions for an interior maximum are, respectively:

$$\begin{aligned} (x_1) \quad & -2\lambda_I(x_1 - x_I) + \frac{\partial \mathbb{P}(x_1, z_1)}{\partial x_1} \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right) = 0 & 2005/06/20 \\ (z_1) \quad & -2(1 - \lambda_I)(z_1 - z_I) + \frac{\partial \mathbb{P}(x_1, z_1)}{\partial z_1} \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right) = 0 & 2005/06/20 \end{aligned}$$

Recall that, from Lemma 2, more extreme policies that move farther from the status quo are more likely to generate informative outcomes. Thus, depending on whether information is electorally beneficial (i.e., $\frac{\partial \mathbb{P}(x_1, z_1)}{\partial x_1} > 0$ for right-wing X policies and $\frac{\partial \mathbb{P}(x_1, z_1)}{\partial z_1} > 0$ for right-wing Z policies) or not (i.e., $\frac{\partial \mathbb{P}(x_1, z_1)}{\partial x_1} < 0$ and $\frac{\partial \mathbb{P}(x_1, z_1)}{\partial z_1} < 0$) the incumbent will have incentives to distort his choice either to the extreme or towards the status quo $d_{sq} = 0$.

In what follows, we will see that whether one or the other distortion emerges in equilibrium depends on the incumbent's ex-ante prevailing electoral chances *and* whether he chooses to act only on a single dimension or expand the scope of policymaking.

4.1 Unidimensional Benchmark

First, suppose that $\lambda_v = 1$, so that the voter only cares about the primary dimension X . In our setting, this also implies that the voter's utility on the secondary dimension is pure noise ($u_{1,z}^v = \varepsilon_{z,1}$):

Remark 1. *Suppose $\lambda_v = 1$. Then, the voter does not learn anything upon observing her realized utility on dimension Z and $\tilde{\mu}^x = \mu^x$.*

Thus, the voter does not directly care about the secondary dimension, and there are no learning spillovers across dimensions. An immediate implication is that the incumbent's retention probability (\mathbb{P}) is only a function of his policy choice on the primary dimension. Assume without loss of generality that the voter reelects the incumbent when indifferent. Then, we have:

Remark 2. *Suppose that $\lambda_v = 1$. Then,*

- $\mathbb{P}(x_1) = \Pr(L_x(x_1) = 1)\pi = \max \in \{0, 1 - 4\alpha\psi_x\pi|x_1|\}$ when $\pi < \frac{1}{2}$;
- $\mathbb{P}(x_1) = 1 - \Pr(L_x(x_1) = 1)(1 - \pi) = \min \in \{1, 4\alpha\psi_x(1 - \pi)|x_1|\}$ when $\pi \geq \frac{1}{2}$.

Recall that π is the prior probability that the voter's optimal policy on the primary dimension is a right-wing one ($x_v = \alpha$). $L_x(x_1)$ denotes the event that the voter observes an informative outcome on this dimension, as function of the implemented policy x_1 . Notice that, given our symmetry assumption ($x_I = -x_C$), at $\pi = \frac{1}{2}$ the voter is ex-ante indifferent between reelecting the incumbent and replacing him with the challenger. Thus, when $\pi < \frac{1}{2}$ the incumbent is ex-ante *trailing*: if the voter receives no new information, she will choose to oust him. This right-wing incumbent is then reelected if and only if the voter discovers that her ideal point x_v is a right-wing policy. Recall that the probability that the voter discovers her true preferences on dimension X is (weakly) increasing as x_1 moves away from the status quo in each direction. Thus, a trailing incumbent's probability of being reelected is minimized at the status quo, and (weakly) increases as the implemented policy x_1 becomes more extreme.

The opposite holds when $\pi \geq \frac{1}{2}$, so that the incumbent is ex-ante electorally *leading*. If the voter learns nothing new, this incumbent will be reelected for sure. Thus, his probability of being retained is (weakly) decreasing as x_1 moves away from the status quo.

Thus, while the incumbent's choice on the secondary dimension is electorally inconsequential, the implemented policy on X influences the incumbent's expected payoff both statically and dynamically.

Denote d_u the incumbent's optimal policy on dimension d under $\lambda_v = 1$, where the u subscript indicates that this is the optimal policy on dimension d in a world where the incumbent's strategic problem is unidimensional. It's easy to see that in equilibrium, the incumbent always moves the secondary-dimension policy to his ideologically preferred point, $z_u = z_I$. Further, using Remark 2, we obtain:

Proposition 1.

- Suppose that $\pi \geq \frac{1}{2}$. Then, in equilibrium:

$$x_u = \max \in \left\{ x_{sq}, x_I - \frac{4\alpha\psi_x(1-\pi)}{\lambda_I} \left(\lambda_I(x_I - x_C)^2 + (1-\lambda_I)(z_I - z_C)^2 \right) \right\} < x_I.$$

- Suppose instead $\pi < \frac{1}{2}$. Then, in equilibrium:

$$x_u = \min \in \left\{ \max \in \left\{ x_I, \frac{1}{4\alpha\psi_x\lambda_v} \right\}, x_I + \frac{4\alpha\psi_x\pi}{\lambda_I} \left(\lambda_I(x_I - x_C)^2 + (1-\lambda_I)(z_I - z_C)^2 \right) \right\} \geq x_I.$$

The incumbent's policy choice on the primary dimension is always distorted away from his ideological preference. A trailing incumbent distorts policy to the extreme, away from both his static optimum and the status quo (normalized to 0), in order to facilitate voter learning.⁴ In this case, we say that the incumbent *gambles* on this policy dimension. In contrast, a leading incumbent is risk averse, and distorts policy towards 0 so as to minimize information. Notice that, since any pair of policies x and $-x$ induces the same amount of learning (Lemma 2), the right-wing incumbent never implements a policy to the left of 0.

Having characterized equilibrium policy in this unidimensional benchmark, we now move to analyzing the incumbent's policy choices in the multidimensional case (i.e., when $\lambda_v < 1$). Our

⁴If $x_I > \frac{1}{4\alpha\psi_x\lambda_v}$, the incumbent's ideal point guarantees full learning. A trailing incumbent thus faces no trade-off, and in equilibrium $x_u = x_I$.

objective is to study the conditions under which the incumbent has strategic incentives to act on the secondary policy dimension, and characterize how this influences his optimal choice on the primary one.

4.2 Multidimensional World

So far, we have assumed that $\lambda_v = 1$ so that the voter only cares about dimension X , and no indirect learning is possible via dimension Z (Lemma 2). In other words, the incumbent's retention chance is not a function of his policy choice on Z . In this section, we relax this assumption to study the incumbent's multidimensional problem.

4.2.1 A Baseline: Unidimensionally Motivated Politicians

It is useful to begin by analyzing a baseline where $\lambda_v < 1$, but $\lambda_I = \lambda_C = 1$. In other words, the voter cares about both dimensions, but politicians only care about the primary one X . Further, we will assume that when indifferent an officeholder chooses not to act on dimension d .

Thus, even though she cares about both dimensions, the voter's retention decision does not *directly* depend on her beliefs over Z (since she anticipates that neither I nor C will act on Z in the second period). More specifically, the voter's optimal retention rule is exactly the same as in the unidimensional case: she retains the right-wing incumbent if and only if $\mu^x > \frac{1}{2}$. However, by Lemma 3, the voter's posterior on X is a function of her realized utility on Z . Therefore, even though the incumbent's ideological preferences are unidimensional, his strategic problem is *multidimensional*. These assumptions allow us to isolate the strategic incentives emerging solely due to the learning spillovers across dimensions. In this section, we thus identify conditions under which officeholders have incentives to *expand* the scope of policymaking, acting on dimensions they have no ideological preferences over.

To start, we characterize the incumbent's probability of wining in this multidimensional world.

Remark 3. Suppose that $\lambda_v < 1$. Then, there exists unique $\hat{\rho}_z(\pi)$ and $\tilde{\rho}_z(\pi)$ s.t.

- $\mathbb{P}(x_1, z_1) = 1 - \Pr(L_x(x_1) = 1)(1 - \pi)$ when $\pi \geq \frac{1}{2}$ and $\rho < \tilde{\rho}_z$;
- $\mathbb{P}(x_1, z_1) = 1 - \Pr(L_x(x_1) = 1)(1 - \pi) - \left(1 - \Pr(L_x(x_1) = 1)\right) \Pr(L_z(z_1) = 1)(1 - \beta)$ when $\pi \geq \frac{1}{2}$ and $\rho > \tilde{\rho}_z$;
- $\mathbb{P}(x_1, z_1) = \Pr(L_x(x_1) = 1)\pi$ when $\pi < \frac{1}{2}$ and $\rho < \hat{\rho}_z$;
- $\mathbb{P}(x_1, z_1) = \Pr(L_x(x_1) = 1)\pi + \left(1 - \Pr(L_x(x_1) = 1)\right) \Pr(L_z(z_1) = 1)\beta$ when $\pi < \frac{1}{2}$ and $\rho > \hat{\rho}_z$.

Recall that $\Pr(L_d(d_1) = 1)$ is the probability that the voter observes an informative outcome on dimension d , which is minimized at the status quo and increases as d_1 moves away from it. First, it is easy to see that Assumption 1 implies that when ρ is too low, outcomes on Z are electorally irrelevant. The voter cares more about X than Z , and a low correlation implies that learning spillovers are too weak to dominate on the voter's prior π . In this case, the incumbent's ex-ante retention probability is as in the unidimensional baseline (Remark 2).

Suppose instead the correlation is sufficiently strong. Then, the secondary dimension is strategically relevant. In contrast to the unidimensional case, a leading incumbent will lose the election even when no direct learning occurs on X , if the outcome on Z is informative and unfavorable. If the voter learns that her optimal z -policy is a left-wing one, she becomes more skeptical of the merits of a conservative policy on the primary dimension as well. Absent direct learning on X , this then induces the voter to oust the initially advantaged incumbent.

For an analogous but symmetric logic, a trailing incumbent will be able to be reelected even when the outcome on the primary dimension is uninformative, if he generates favorable information on the secondary one. As an aside, we note that this Remark applies even in the case in which $\lambda_I < 1$, analyzed in the next section.

From this, we can easily identify conditions under which learning spillovers create strategic incentives for the incumbent to act on the secondary policy dimension:

Proposition 2. *Suppose $\lambda_v < 1$ and $\lambda_I = \lambda_C = 1$. Then, the incumbent chooses to act on Z (i.e., $z_1 \neq z_{sq}$) if and only if $\pi < \frac{1}{2}$ and $\rho > \hat{\rho}_z$. Further, we have that $\hat{\rho}_z = 1 - \pi > \frac{1}{2}$.*

Recall that, under our assumption that $\lambda_I = \lambda_C = 1$, the voter's retention choice is only a function of her posterior on the primary dimension μ^x . Thus, the incumbent acts on Z if and only if he has strategic incentives to facilitate *indirect* learning on X . It follows from Remark 3 that a leading incumbent (i.e., $\pi > \frac{1}{2}$) never wants to act on Z , since he wants to prevent the voter from obtaining any new information. Suppose instead that $\pi < \frac{1}{2}$, so that the incumbent is ex-ante trailing. Then, he wants to facilitate learning spillovers, in hopes of overcoming his initial disadvantage and jumping above the retention threshold. As highlighted above, however, outcomes on the secondary dimension remain electorally irrelevant if the correlation ρ is too small. Thus, a trailing incumbent is indifferent between acting on the secondary dimension and keeping the status quo and (by assumption) chooses not to act. If instead ρ is sufficiently large, the trailing incumbent can exploit learning spillovers to increase his probability of resurrecting himself. In equilibrium he will therefore always choose to expand the scope of policymaking to the secondary dimension, even if he has no ideological taste for it.

The next Corollary follows straightforwardly from the above discussion and Lemma 2:

Corollary 1. *Suppose that in equilibrium the incumbent chooses to act on the secondary dimension Z . Then, he always implements a fully informative policy on this dimension, i.e., $z_1^* \geq z'$.*

Even though the incumbent does not have ideological preferences over dimension Z , his strategic incentives to facilitate voter learning induce policy extremism on this secondary dimension. Thus, in equilibrium we either observe inaction on the secondary dimension (when ρ is low), or we observe the incumbent pursuing extreme policies on this dimension (when ρ is high). In our setting, extreme policymaking need not follow from extreme ideological preferences.

Next, we characterize how the possibility to exploit learning spillovers from the secondary dimension influences the incumbent's policy choice on the primary one. Recall that x_u is the incumbent's optimal policy choice in the unidimensional benchmark. Then, we have:

Proposition 3. *Suppose that in equilibrium the incumbent chooses to act on the secondary dimension Z . Then, his policy choice on the primary one x_1^* satisfies $x_1^* < x_I \leq x_u$.*

This Proposition highlights that the correlation ρ generates a strategic substitution effect between policy dimensions. When a trailing incumbent cannot exploit the secondary dimension (i.e., $\lambda_v = 1$ or $\rho < 1 - \pi$), then he always has strategic incentives to gamble on the primary one. Recall that policies farther from the status quo generate more information, therefore this incumbent always implements a policy more extreme than his ideological preference, $x_u > x_I$. When instead this trailing incumbent has the possibility to exploit learning spillovers (i.e., $\lambda_v < 1$ and $\rho > 1 - \pi$), this induces moderation on the primary dimension: $x_1^* < x_I$.

To understand this, recall that the outcome on Z may influence the voter's retention decision only if she does not learn about x_v directly (as otherwise she reaches a degenerate interim posterior $\tilde{\mu}^x$). Therefore, in order to benefit from the learning spillovers, the incumbent must avoid generating an informative outcome on X . In sharp contrast with the results of the unidimensional baseline, this generates incentives for the trailing incumbent to pursue moderate policies on the primary dimension.

The above discussion implies that if the incumbent chooses to gamble on Z , he becomes risk-averse on X to magnify the impact of the learning spillovers. However, in principle the trailing incumbent may find it optimal to forego the learning spillovers and gamble on X instead. Proposition 3 indicates that this never occurs in equilibrium. If ρ is sufficiently large that outcomes on the secondary dimension are electorally relevant, the trailing incumbent always chooses to gamble on Z and pursue moderate policies on X . The reason lies in the fact that, ex-ante, the players have less information on the secondary dimension than on the primary one. Recall that the incumbent is trailing if and only if $\pi < \frac{1}{2}$. Thus, even though the trailing incumbent needs to generate information in order to be reelected, an informative outcome is more likely to reveal to the voter that she is aligned with the challenger's preferences. This is true on both dimensions, but since $\rho < 1$, we have that $\pi < \frac{1}{2}$ implies $\beta < \pi$, where β is the prior probability that the voter's optimum on Z is a right-wing one. In other words, since the players have less accurate information on the secondary dimension, the state $z_v = \alpha$ is ex-ante more likely than $x_v = \alpha$, therefore the incumbent is more likely to generate a favorable outcome on Z than on X . Thus, the trailing right-wing incumbent

prefers to gamble on Z , hoping to exploit a false positive: generate a favorable outcome on Z and thus induce the voter to positively update on x_v , even if the state of the world on the primary dimension is actually a left-wing one.

These results therefore highlight that, in equilibrium, the incumbent will never gamble on both dimensions. Rather, if the correlation is too low to exploit the learning spillovers, he will have no strategic incentives to act on Z and will continue gambling on X . If instead the correlation is high, he will gamble on Z but become risk averse on X .

Finally, we characterize how the magnitude of the correlation ρ influences the incumbent's policy choice on the primary dimension:

Corollary 2. *Suppose that in equilibrium the incumbent chooses to act on the secondary dimension Z . Then, we have that $\frac{\partial x_1^*}{\partial \rho} > 0$.*

The stronger the correlation across dimensions, the weaker the substitution effect described above. Recall that in equilibrium the incumbent acts on Z only when $\pi < \frac{1}{2}$, i.e., the true state on X is more likely to be unfavourable for the right-wing incumbent. Thus, as the correlation increases it becomes more and more likely that $z_v = -\alpha$, so that if the voter observes an informative outcome on Z she updates against the incumbent. Thus, as ρ increases the likelihood that the incumbent is able to resurrect his reelection chances by gambling on Z decreases, and the incentives to prevent direct learning on X become weaker. As a consequence, whenever the incumbent chooses to act on Z in equilibrium, the policy on X becomes more extreme as ρ increases.

4.2.2 General Model

The results of the previous section are useful to isolate the strategic incentives generated by the learning spillovers. Here, we complete the analysis by studying the incumbent's policy choice on each dimension in the general model, where both the voter and the politicians care about both dimensions (i.e., $\lambda_i < 1$ for $i \in \{I, V, C\}$) In contrast with the analysis presented above, the incumbent now has both strategic *and ideological* preferences over the secondary dimension Z . As

above, our goal is to characterize the conditions under which the incumbent chooses to open the secondary dimension, and study how this influences his policy on the primary one.

Proposition 4. *Suppose $\pi < \frac{1}{2}$. Then the incumbent always acts on the secondary dimension Z . Suppose instead $\pi > \frac{1}{2}$. Then the incumbent acts on Z if and only if*

- *The correlation ρ is sufficiently low, or*
- *The correlation ρ is high and λ_I is sufficiently small.*

A trailing incumbent has both ideological and strategic reasons to act on the secondary dimension. In equilibrium, he will therefore always choose to do so. In contrast, a leading incumbent faces a trade-off. On one hand, he has ideological preferences over the secondary dimension and would therefore statically find it optimal to act on it. On the other hand, as the results of the previous section demonstrate, acting on the secondary dimension hurts his retention chances and thus his expected future payoff. If the correlation between dimensions is sufficiently low, the leading incumbent can survive reelection even if the outcome on the secondary dimension reveals damaging information. He can therefore implement his preferred policy on the secondary dimension while avoiding the negative electoral consequences. Suppose instead the correlation ρ is high, then the trade-off discussed above is binding, and the incumbent only acts on Z if his ideological preferences are sufficiently strong (i.e., λ_I is low).

Our second result describes how strategic incentives to act on the secondary policy dimension influence the incumbent's policy choice on the primary one. As above, a trailing incumbent becomes more moderate on the primary dimension when he can act on a secondary one. However, the result is reversed for a leading incumbent: here, the presence of dimension Z generates more extremism on X .

Proposition 5. *Suppose that the incumbent chooses to act on the secondary dimension. Then*

- *When $\pi < \frac{1}{2}$, we have $z_1^* \geq z_u = z_I$ and $x_1^* \leq x_u$;*
- *when $\pi > \frac{1}{2}$, we have $z_1^* \leq z_u = z_I$ and $x_1^* \geq x_u$.*

Recall that d_u is the equilibrium policy on dimension d in the unidimensional baseline, i.e., the world in which the voter only cares about the primary dimension ($\lambda_v = 1$).

The intuition for the case in which $\pi < \frac{1}{2}$ is exactly as described in the previous section. The trailing incumbent has incentives to gamble on Z , where a false positive is more likely. Furthermore, the correlation across dimensions generates a substitution effect, whereby the incumbent has incentives to moderate on the primary dimension in order to exploit learning spillovers from the secondary one.

Suppose instead $\pi > \frac{1}{2}$, i.e., the incumbent is leading. As discussed above, the leading incumbent does not have strategic incentives to act on Z , since his retention chances are maximized when the voter learns nothing new. However, because $\lambda_I < 1$, the incumbent has ideological preferences over Z and therefore sometimes chooses to act on this secondary dimension. When he does so, he is undertaking more electoral risk, since he may generate an informative and unfavorable outcome on Z and hurt his retention chances. Further, since $\pi > \frac{1}{2}$ implies $\beta < \pi$, an unfavourable outcome is ex-ante more likely on dimension Z than on X . Also recall that, given our Assumption 1, direct learning on X renders outcomes on Z electorally irrelevant. Taken together, these two observations imply that in order to counteract the detrimental effects of learning on the secondary dimension, the leading incumbent has incentives to facilitate direct learning on the primary one. In other words, a leading incumbent - who is always risk-averse in a unidimensional world - becomes risk-loving on the primary dimension when he chooses to expand the scope of policymaking. As a consequence, the leading incumbent will distort policy towards 0 on the secondary dimension, while on the primary one will implement policies more extreme than in the unidimensional world.

Before concluding, we characterize the features that render a policy dimension more likely to be addressed by the incumbent. Suppose that multiple secondary dimensions are available, each characterized by a different correlation ρ_d with the primary dimension X . Suppose the incumbent is resource constrained, so that he cannot act on all available dimensions. To restrict our attention to the most interesting cases, we also assume that for any available dimension ρ_d is sufficiently large that learning spillovers are always electorally relevant. Then, we have:

Proposition 6. *All else equal, a leading incumbent ($\pi > \frac{1}{2}$) will (weakly) prefer to open the dimensions:*

- *He is less extreme on, or*
- *That have higher correlation with the primary dimension, or*
- *For which outcomes are more noisy.*

Suppose instead the incumbent is trailing ($\pi < \frac{1}{2}$). Then, all else equal, the incumbent (weakly) prefers to open the dimensions:

- *He is more extreme on, or*
- *That have lower correlation with the primary dimension, or*
- *For which outcomes are less noisy.*

The intuition is as follows. Consider first a leading incumbent. If he chooses to act on a secondary policy dimension, he will find it optimal to implement a moderate policy on this dimension so as to mitigate the electoral downsides. This is less costly if his ideologically preferred policy is a moderate one. At the same time, a dimension for which outcomes are more noisy carries less electoral risk and will therefore be more appealing. Finally, recall that an incumbent is leading when $\pi > \frac{1}{2}$, i.e., the state of the world on the primary dimension is likely to be in his favor. The higher the correlation with this primary dimension, the higher the likelihood that the policy outcome on the additional dimension would also be favorable to the incumbent. Again, this makes the dimension more appealing.⁵ The opposite intuition underlies the results for a trailing incumbent.

5 Conclusion

As our analysis underscores, the introduction of multidimensionality within an accountability setting dramatically influences the incentives that incumbents face, as they make decisions about whether

⁵Let us emphasize again that this is due to the assumption that there is no available dimension for which the correlation is so low that outcomes become electorally irrelevant, i.e., $\rho_d > \max\{\tilde{\rho}_z, \hat{\rho}_z\}$ for all available dimensions.

and how to change policy. Indeed, whereas incumbents in the unidimensional setting frequently pursue extreme policies in order to encourage voter learning, the presence of a secondary dimension complicates this decision. For the trailing incumbent, the possibility of policymaking in multiple dimensions presents greater opportunities for voter learning. As a result, such incumbents always expand their policymaking—and, at the same time, *moderate* on the primary dimension. For a leading incumbent, instead, the problem is reversed. This incumbent tends to pursue moderate policy in a unidimensional world, and multidimensionality generates incentives for extremism on the primary policy dimension.

Beyond multidimensionality’s overall effect on policymaking and accountability, however, our incorporation of a correlation term between policymaking dimensions generates interesting nuances in policymaking and voter learning. Indeed, in order for trailing incumbents to leverage spillover learning—or for leading incumbents to avoid it—voter learning *between* issue areas is highly consequential. That is, if correlation between the issues is high, voter learning will spill over between the dimensions, altering the politician’s willingness to pursue policy change.

Together, we believe these findings underscore the importance of incorporating multiple dimensions into models of policymaking, especially those involving voters. Given the rise of populism and the consequent interest in issue areas such as immigration and trade policy, both politicians and voters appear to clearly care about and pursue more issues than typical left-right economic ones. We believe models of policymaking should reflect these changes, even in cases when logrolling or bargaining is not involved. Voters can learn about policymaking in one area by observing activity in another, and they are often *encouraged* to do so by activists, partisan media, and other outlets in modern political life.

Moreover, in order to understand not only whether to expand policymaking but to *where*, we believe it is imperative for theoretical and empirical models to think carefully about this correlation between issue areas. Indeed, particularly in a polarized era in which parties and candidates are thought to be more consistent across issue areas than in eras past, the possibility for spillover learning is important both substantively and strategically. Political scientists have long pointed to

the importance of understanding “what goes with what” and the extent to which voters relate issue areas in their mind (e.g., Converse 1964). We show that this association is consequential for policymaking, and we hope that similar parameters are incorporated into other models policymaking, accountability, and delegation.

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Appendix

Main Results - Proofs

Proof of Lemma 1. The voter re-elects the right-wing incumbent if and only if the expected payoff from voting for I given the information received in $t = 1$ is greater than that of voting for C . Formally:

$$\begin{aligned} & -\lambda_v[\mu^x(x_I - \alpha)^2 + (1 - \mu^x)(x_I + \alpha)^2] - (1 - \lambda_v)[\mu^z(z_I - \alpha)^2 + (1 - \mu^z)(z_I + \alpha)^2] > \\ & -\lambda_v[\mu^x(x_C - \alpha)^2 + (1 - \mu^x)(x_C + \alpha)^2] - (1 - \lambda_v)[\mu^z(z_C - \alpha)^2 + (1 - \mu^z)(z_C + \alpha)^2]. \end{aligned} \quad (8)$$

Plugging in the assumption that $d_I = -d_C$, the above reduces to

$$2\lambda_v\mu^x x_I \alpha - \lambda_v x_I \alpha + 2(1 - \lambda_v)\mu^z z_I \alpha - (1 - \lambda_v)z_I \alpha > 0$$

which rearranged yields:

$$\mu_v^x > \frac{1}{2} + \frac{(1 - \lambda_v)z_I}{\lambda_v x_I} \frac{(1 - 2\mu_v^z)}{2} \equiv \widehat{\mu_v^x}(\mu^z). \quad (9)$$

□

Proof of Lemma 2. We prove the statements for dimension X . Let $\mu^x \in [0, 1]$ denote V 's posterior that the state of the world on dimension X is positive.

(i) A possible payoff realization for V given the incumbent's choice (x_t) has to fall within:

$$\left[-\lambda_v(x_t - \alpha)^2 - \frac{1}{2\psi_x}, -\lambda_v(x_t + \alpha)^2 + \frac{1}{2\psi_x} \right]. \quad (10)$$

Thus, if V observes $u_v^t > -\lambda_v(x_t + \alpha)^2 + \frac{1}{2\psi_x}$, she knows for sure that she likes the right policy, i.e., $\mu^x = 1$. Similarly, if V observes $u_v^t < -\lambda_v(x_t - \alpha)^2 - \frac{1}{2\psi_x}$, then $\mu^x = 0$.

The last case to consider is when u_v^t falls within the interval in Equation 10. Denote by $f(\cdot)$ the

PDF of the error term $\varepsilon_{x,t}$. We have:

$$\Pr(x_v = \alpha | u_v^t) = \frac{f(u_v^t + \lambda_v(x_t - \alpha)^2) \pi}{f(u_v^t + \lambda_v(x_t - \alpha)^2) \pi + f(u_v^t + \lambda_v(x_t + \alpha)^2) (1 - \pi)}.$$

Since $\varepsilon_{x,t}$ is uniformly distributed, we have $f(u_v^t + \lambda_v(x_t + \alpha)^2) = f(u_v^t + \lambda_v(x_t - \alpha)^2)$, hence

$$\Pr(x_v = \alpha) = \pi.$$

(ii)-(iii) Now, denote by $L \in \{0, 1\}$ players' learning of x_v . There exists a value of policy x'_t such that, for any $x_t > x'_t$, the realization of u_v^t is fully informative, i.e., the interval (10) is empty. This requires:

$$-\lambda_v(x_t + \alpha)^2 + \frac{1}{2\psi_x} + \lambda_v(x_t - \alpha)^2 + \frac{1}{2\psi_x} \leq 0 \quad (11)$$

which rearranged yields:

$$x_t \geq \frac{1}{4\alpha\lambda_v\psi_x}. \quad (12)$$

Define $x' \equiv \frac{1}{4\alpha\lambda_v\psi_x}$, and assume $x_t \in [0, x']$. We have:

$$\begin{aligned} \Pr(L = 1 | \pi, 0 < x_t < x') &= \pi \Pr\left(-\lambda_v(x_t - \alpha)^2 + \varepsilon_{x,t} > -\lambda_v(x_t + \alpha)^2 + \frac{1}{2\psi_x}\right) \\ &\quad + (1 - \pi) \Pr\left(-\lambda_v(x_t + \alpha)^2 + \varepsilon_{x,t} > -\lambda_v(x_t - \alpha)^2 - \frac{1}{2\psi_x}\right). \end{aligned}$$

Since the two probabilities are symmetric, we have

$$\begin{aligned} \Pr(L = 1 | \pi, 0 < x_t < x') &= \Pr\left(-\lambda_v(x_t - \alpha)^2 + \varepsilon_{x,t} > -\lambda_v(x_t + \alpha)^2 + \frac{1}{2\psi_x}\right) \\ &= \Pr\left(\varepsilon_{x,t} < 4\lambda_v\alpha x_t - \frac{1}{2\psi_x}\right) \\ &= 4\alpha x_t \lambda_v \psi_x, \end{aligned} \quad (13)$$

where notice that the probability that V learns her true preference is increasing in x_t .

The proof for dimension Z is analogous therefore omitted. \square

Proof of Remark 2. From Lemma 2 we know that $\Pr(L_x = 1 | \pi, 0 < x_t < x') = 4\alpha\psi_x|x_1|$. It follows that, if $\pi \geq \frac{1}{2}$ (if $\pi < \frac{1}{2}$), $\mathbb{P}(x_1)$ is weakly decreasing (increasing) in x_1 . \square

Proof of Proposition 1. When $\pi \geq \frac{1}{2}$ we can express I 's problem as

$$-\lambda_I(x_1 - x_I)^2 - 4\alpha\psi_x x_1(1 - \pi) \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right), \quad (14)$$

which yields the following FONC (which is also sufficient since the problem is concave):

$$-2\lambda_I(x_1 - x_I) - 4\alpha\psi_x(1 - \pi) \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right) = 0, \quad (15)$$

Rearranging (15) yields:

$$x_1 = x_I - \frac{4\alpha\psi_x(1 - \pi)}{\lambda_I} \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right).$$

It follows that

$$x_1 = \max \left\{ 0, x_I - \frac{4\alpha\psi_x(1 - \pi)}{\lambda_I} \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right) \right\}. \quad (16)$$

When instead I is trailing, we can express I 's problem as

$$-\lambda_I(x_1 - x_I)^2 - 4\alpha\psi_x x_1 \pi \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right), \quad (17)$$

which yields the following FONC (which is also sufficient):

$$-2\lambda_I(x_1 - x_I) - 4\alpha\psi_x \pi \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right) = 0,$$

which rearranged yields:

$$x_1 = x_I + \frac{4\alpha\psi_x \pi}{\lambda_I} \left(\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2 \right).$$

It follows that

$$x_1 = \min \left\{ x_I + \frac{4\alpha\psi_x\pi}{\lambda_I} (\lambda_I(x_I - x_C)^2 + (1 - \lambda_I)(z_I - z_C)^2), \frac{1}{4\alpha\lambda_v\psi_x} \right\}. \quad (18)$$

□

Proof of Proposition 2. First, suppose $\pi > \frac{1}{2}$. Recall that this implies that the incumbent is always reelected if the voter receives no new information. Further, Assumption 1 implies that if she observes an informative outcome on X , then the outcome on Z is electorally irrelevant. Suppose that the voter observes an uninformative outcome on X , and an informative outcome on Z . If $z_v = \alpha$, I is always re-elected with $\pi > \frac{1}{2}$. Consider now the case in which the voter observes an uninformative outcome on X , but learns that $z_v = -\alpha$. Denote $\mu^x(\emptyset, -\alpha, \rho)$ the voter's posterior that the state of the world on dimension X is positive in this case, i.e, if she observes an uninformative outcome on dimension X but learns that the state on dimension Z is $-\alpha$. Then, we must consider two cases. If the prior π is sufficiently high relative to the correlation ρ so that $\mu^x(\emptyset, -\alpha, \rho) > \widehat{\mu}_v^x(0)$, then the incumbent is reelected. In this case, the incumbent's retention chances are not a function of the policy on dimension Z , therefore he is indifferent between acting and not acting and by assumption chooses not to. If instead the prior π is sufficiently low relative to the correlation ρ so that $\mu^x(\emptyset, -\alpha, \rho) < \widehat{\mu}_v^x(0)$, the incumbent's retention chances are hurt by information on Z , and he chooses not to act on this secondary dimension.

Next, suppose $\pi < \frac{1}{2}$. Then, the incumbent is always ousted if the voter receives no new information. As above, if the voter observes an informative outcome on X , the outcome on Z is electorally irrelevant. Similarly, if the voter observes an uninformative outcome on X , and an informative outcome on Z such that $z_v = -\alpha$, I is always ousted with $\pi < \frac{1}{2}$. Suppose instead that the voter observes an uninformative outcome on X , but learns that $z_v = \alpha$. Again, we must consider two cases. If the correlation ρ is low, so that that $\mu^x(\emptyset, \alpha, \rho) < \widehat{\mu}_v^x(1)$, then the incumbent is ousted. Under this condition, the incumbent's ex-ante retention chances are not a function of the policy on dimension Z , therefore he is indifferent between acting and not acting and by assumption

chooses not to. If instead the correlation ρ is sufficiently high that $\mu^x(\emptyset, \alpha, \rho) > \widehat{\mu}_v^x(1)$, generating an informative outcome on Z can only help the incumbent's retention chances. In other words, the incumbent's ex-ante retention chances increase as z_1 moves away from 0. Thus, he always chooses to act on Z .

Therefore, $\widehat{\rho}_T$ satisfies:

$$\mu^x(\emptyset, \alpha, \rho) = \widehat{\mu}_v^x(1), \quad (19)$$

where

$$\mu^x(\emptyset, \alpha, \rho) = \frac{\pi\rho}{\pi\rho + (1-\pi)(1-\rho)}. \quad (20)$$

Combining the above, we have

$$\widehat{\rho}_T = \frac{(1-\pi)\widehat{\mu}_v^x(1)}{\pi(1-2\widehat{\mu}_v^x(1)) + \widehat{\mu}_v^x(1)} \quad (21)$$

□

Proof of Corollary 1. Recall that under $\lambda_I = 1$ the incumbent's utility depends on z_1 only via the voter learning. Further, if the incumbent chooses to act on Z in equilibrium it must be the case that his probability of winning is increasing in the probability of generating an informative outcome on Z . This yields that in equilibrium the incumbent will always choose to implement a fully informative policy $z_1^* > z'$. □

Proof of Proposition 3. Consider the incumbent's choice on X . When I is trailing and $\rho > \widehat{\rho}_T$, we have $\mathbb{P} = 4\alpha\psi_x x_1 \pi + (1 - 4\alpha\psi_x x_1)4\alpha\psi_z z_1 \beta$. Plugging in $z_1^* = \frac{1}{4\alpha\psi_z(1-\lambda_v)}$, the trailing incumbent's retention probability reduces to

$$4\alpha\psi_x \pi x_1 + (1 - 4\alpha\psi_x x_1)\beta. \quad (22)$$

Note that, given $\pi < \frac{1}{2}$, $\beta = \pi\rho + (1 - \pi)(1 - \rho) > \pi$, therefore the incumbent's probability of winning is decreasing in x_1 . It follows from Equation 6 that $x_1^* < x_I$. \square

Proof of Corollary 2. Applying the implicit function theorem, we have that

$$\frac{\partial x_1^*}{\partial \rho} = -\frac{\frac{\partial FOC}{\partial \rho}}{\frac{\partial FOC}{\partial x_1}}. \quad (23)$$

In equilibrium, we have that $\frac{\partial FOC}{\partial x_1} < 0$, therefore $\frac{\partial x_1^*}{\partial \rho} > 0$ if and only if $\frac{\partial FOC}{\partial \rho} > 0$:

$$\frac{\partial^2 \mathbb{P}(x_1, z_1)}{\partial x_1 \partial \rho} (\lambda_I 4x_I^2 + (1 - \lambda_I) 4z_I^2) > 0. \quad (24)$$

Recall that $\mathbb{P}(x_1, z_1) = 4\alpha\psi_x x_1 \lambda_v \pi + (1 - 4\alpha\psi_x x_1 \lambda_v) 4\alpha\psi_z z_1 (1 - \lambda_v)\beta$, therefore the above reduces to

$$-16\alpha^2 \psi_x \psi_z \lambda_v (1 - \lambda_v) \frac{\partial \beta}{\partial \rho} > 0, \quad (25)$$

which is always true when $\pi < \frac{1}{2}$. \square

Proof of Proposition 4. Recall that $\widehat{\mu}_v^x(\mu^z)$ defines the value of μ_v^x such that the voter is indifferent between replacing and keeping the incumbent, for a given μ^z . We must consider four cases, which differ in whether the voter posterior upon observing an uninformative outcome on X and for a given value of ρ is above or below the retention threshold $\widehat{\mu}_v^x(\mu^z)$, given the outcome on the secondary dimension Z . The first two cases correspond to an ex-ante leading incumbent, the last two ones to a trailing one:

1. $\pi = \mu_v^x(\emptyset, \emptyset, \rho) > \mu_v^x(\emptyset, -\alpha, \rho) > \widehat{\mu}_v^x(0) > \frac{1}{2}$
2. $\pi = \mu_v^x(\emptyset, \emptyset, \rho) > \widehat{\mu}_v^x(0) > \mu_v^x(\emptyset, -\alpha, \rho)$ (which implies $\pi > \frac{1}{2}$)
3. $\pi = \mu_v^x(\emptyset, \emptyset, \rho) < \widehat{\mu}_v^x(1) < \mu_v^x(\emptyset, \alpha, \rho)$ (which implies $\pi < \frac{1}{2}$)
4. $\pi = \mu_v^x(\emptyset, \emptyset, \rho) < \mu_v^x(\emptyset, \alpha, \rho) < \widehat{\mu}_v^x(1) < \frac{1}{2}$.

Case 1: $\pi > \mu_v^x(\emptyset, -\alpha, \rho) > \widehat{\mu}_v^x(0) > \frac{1}{2}$.

First, suppose that the incumbent is leading and that the voter re-elects the incumbent even if she knew with certainty to be aligned with the challenger on dimension Z , i.e.:

$$\mu_v^x(\emptyset, -\alpha, \rho) > \widehat{\mu}_v^x(0). \quad (26)$$

Substituting $\mu_v^x(\emptyset, -\alpha, \rho) = \frac{\pi(1-\rho)}{\pi(1-\rho)+(1-\pi)\rho}$ yields the following condition on the correlation coefficient ρ :

$$\rho < \frac{\pi(1 - \widehat{\mu}_v^x(0))}{\pi + \widehat{\mu}_v^x(0)(1 - 2\pi)}, \quad (27)$$

where notice that the denominator $\pi + \widehat{\mu}_v^x(0)(1 - 2\pi) > 0$ (the RHS is linear in π , and the condition is always satisfied at $\pi = 0$ and $\pi = 1$). The condition implies that the incumbent is always reelected unless the voter observes an informative outcome on X and learns that $x_v = -\alpha$. Then, it is easy to see that the incumbent's probability of winning is not a function of the outcome on Z , therefore not a function of his policy choice z_1 . This also implies that the incumbent's maximization problem reduces to the unidimensional one, and the incumbent sets his ideologically preferred policy on dimension Z , $z_1^* = z_I$.

Case 2: $\pi = \mu_v^x(\emptyset, \emptyset, \rho) > \widehat{\mu}_v^x(0) > \mu_v^x(\emptyset, -\alpha, \rho) > \frac{1}{2}$.

Suppose (as in the first case) that I is leading, and that—differently from the first case—the voter replaces the incumbent when she learns to be aligned with the challenger on dimension Z :

$$\widehat{\mu}_v^x(0) > \mu_v^x(\emptyset, -\alpha, \rho).$$

Substituting $\mu_v^x(\emptyset, -\alpha, \rho) = \frac{\pi(1-\rho)}{\pi(1-\rho)+(1-\pi)\rho}$ produces:

$$\rho > \frac{\pi(1 - \widehat{\mu}_v^x(0))}{\pi + \widehat{\mu}_v^x(0)(1 - 2\pi)}. \quad (28)$$

Straightforwardly, the incumbent chooses to open the secondary dimension if and only if his utility is increasing in z_1 at $z_1 = 0$.

Under the assumption on ρ , I 's retention probability is given by:

$$\begin{aligned}\mathbb{P}(x_1, z_1) &= 1 - (1 - \pi) \Pr(L = 1 | \pi, x_1) - (1 - \Pr(L = 1 | \pi, x_1))(1 - \beta) \Pr(L = 1 | \beta, z_1) \\ &= 1 - (1 - \pi)4\alpha x_1 \lambda_v \psi_x - (1 - 4\alpha x_1 \lambda_v \psi_x)(1 - \beta)4\alpha z_1 (1 - \lambda_v) \psi_z\end{aligned}$$

Denote $K = 4\lambda_I x_I^2 + 4(1 - \lambda_I)z_I^2$. Plugging the value of $\mathbb{P}(x_1, z_1)$ into I 's objective and differentiating with respect to z_1 , we get that I opens Z if and only if:

$$2(1 - \lambda_I)z_I - (1 - \beta)(1 - 4\alpha \hat{x} \lambda_v \psi_x)4\alpha \psi_z(1 - \lambda_v) [\lambda_I 4x_I^2 + (1 - \lambda_I)4z_I^2] > 0, \quad (29)$$

where \hat{x} solves

$$-2\lambda_I(x_1 - x_I) - 4\alpha \psi_x \lambda_v \left[1 - \pi - 4\alpha \psi_z(1 - \lambda_v)z_1(1 - \beta) \right] K = 0. \quad (30)$$

and is equal to:

$$\hat{x} = x_I - \frac{4\alpha \psi_x \lambda_v (1 - \pi) [\lambda_I 4x_I^2 + (1 - \lambda_I)4z_I^2]}{\lambda_I}. \quad (31)$$

Condition 29 is satisfied for $\lambda_I < \hat{\lambda}_I$. The expression for $\hat{\lambda}_I$ is lengthy therefore omitted. Intuitively, the incumbent opens the secondary dimension when he sufficiently cares about it.

Case 3: $\pi = \mu_v^x(\emptyset, \emptyset, \rho) < \widehat{\mu}_v^x(1) < \mu_v^x(\emptyset, \alpha, \rho) < \frac{1}{2}$.

Suppose now that the incumbent is trailing and

$$\widehat{\mu}_v^x(1) < \mu_v^x(\emptyset, \alpha, \rho), \quad (32)$$

which, plugging in $\mu_v^x(\emptyset, \alpha, \rho) = \frac{\pi\rho}{\pi\rho + (1-\pi)(1-\rho)}$, reduces to

$$\rho > \frac{\widehat{\mu}^x(1)(1-\pi)}{\widehat{\mu}^x(1)(1-2\pi) + \pi}. \quad (33)$$

Recall that $\pi < \frac{1}{2}$ implies the incumbent is always ousted if the voter learns nothing new. This also implies that the incumbent is ousted if the voter observes an uninformative outcome on X and learns $z_v = -\alpha$. Finally, $\pi = \mu_v^x(\emptyset, \emptyset, \rho) < \widehat{\mu}_v^x(1) < \mu_v^x(\emptyset, \alpha, \rho)$ implies that the incumbent is re-elected if the voter observes an uninformative outcome on X but learns $z_v = \alpha$.

Thus we have that in this case:

$$\mathbb{P} = 4\alpha\psi_x\lambda_v x_1\pi + (1 - 4\alpha\psi_x\lambda_v x_1)4\alpha\psi_z(1 - \lambda_v)z_1\beta. \quad (34)$$

Thus, the FOCs are

$$(x_1) - 2\lambda_I(x_1 - x_I) + 4\alpha\psi_x\lambda_V \left[\pi - 4\alpha\psi_z(1 - \lambda_V)z_1\beta \right] \left[4\lambda_I x_I^2 + 4(1 - \lambda_I)z_I^2 \right] = 0 \quad (35)$$

$$(z_1) - 2(1 - \lambda_I)(z_1 - z_I) + (1 - 4\alpha\psi_x\lambda_V x_1)4\alpha\psi_z(1 - \lambda_V)\beta \left[4\lambda_I x_I^2 + 4(1 - \lambda_I)z_I^2 \right] = 0 \quad (36)$$

Recalling that in equilibrium $1 - 4\alpha\psi_x\lambda_I x_1 \geq 0$, we notice that the incumbent's utility is always increasing in z_1 at $z_1 = 0$. Thus, the incumbent z_1 is always strictly larger than 0 in equilibrium.

Case 4: $\mu_v^x(\emptyset, \emptyset, \rho) = \pi < \mu_v^x(\emptyset, \alpha, \rho) < \widehat{\mu}_v^x(1) < \frac{1}{2}$.

Lastly, suppose that the incumbent is trailing and that the voter ousts the incumbent even if she knew with certainty to be aligned with him on dimension Z , i.e.:

$$\mu_v^x(\emptyset, \alpha, \rho) < \widehat{\mu}_v^x(1). \quad (37)$$

The condition implies that the incumbent is always ousted unless the voter observes an informative outcome on X and learns $x_v = \alpha$. Then, analogously to case 1, the incumbent's probability of

winning is not a function of the outcome on Z , therefore not a function of his policy choice z_1 . Thus, in equilibrium the incumbent always sets $z_1^* = z_I > 0$. \square

Proof of Proposition 5. From the proof of Proposition 4, we know that in Cases 1 and 4 $x_1^* = x_u$ and $z_1^* = z_I = z_u$.

Consider instead Case 2, i.e., a leading incumbent ($\pi > \frac{1}{2}$) under a high ρ . First, consider the equilibrium choice on Z . The incumbent's utility is always decreasing in z_1 at $z_1 \geq z_I$:

$$-2(1 - \lambda_I)(z_1 - z_I) - (1 - \beta)(1 - 4\alpha x_1 \lambda_v \psi_x)4\alpha \psi_z(1 - \lambda_v)K \quad (38)$$

Therefore, in equilibrium it must be the case that $z_1 < z_I$. Next, consider the incumbent's choice on X . First, suppose that the problem is concave and the FOC is sufficient to identify the equilibrium policy. The result follows from inspection of 30. Suppose instead that the problem is not concave (or it is, but the equilibrium policy is at a corner). Then, depending on parameters, the equilibrium policy can take one of three values: $\{0, \hat{x}, x'\}$, where \hat{x} is the interior solution. Then, to conclude the proof for Case 2 is sufficient to show that $x_1^* = 0 \implies x_u = 0$. This follows from the fact that if the incumbent's utility is decreasing in x_1 at $x_1 = 0$ under $\lambda_v < 1$, then it must also be decreasing under $\lambda_v = 1$:

$$2\lambda_I x_I - 4\alpha \psi_x(1 - \pi)K \leq 2\lambda_I x_I - 4\alpha \psi_x \lambda_v \left[1 - \pi - 4\alpha \psi_z(1 - \lambda_v)z_1(1 - \beta)\right]K, \quad (39)$$

which reduces to

$$1 - \pi \geq \lambda_v \left[1 - \pi - 4\alpha \psi_z(1 - \lambda_v)z_1(1 - \beta)\right], \quad (40)$$

which is always satisfied.

Finally, consider Case 3, i.e., a trailing incumbent ($\pi < \frac{1}{2}$) under a high ρ . We proceed as above. Focus first on the equilibrium choice on Z . The incumbent's utility is always increasing in z_1 at $z_1 \leq z_I$ (follows from inspection of 36) therefore in equilibrium it must be the case that

$z_1 > z_I$. Next, consider the incumbent's choice on X . First, suppose that the problem is concave and the FOC is sufficient to identify the equilibrium policy. The result follows from inspection of 35. Suppose instead that the problem is not concave (or it is, but the equilibrium policy is at a corner). Then, depending on parameters, the equilibrium policy can take one of three values: $\{0, \hat{x}, x'\}$, where \hat{x} is the interior solution. Then, to conclude the proof for Case 3 is sufficient to show that $x_1^* = x' \implies x_u = x'$. This follows from the fact that if the incumbent's utility is increasing in x_1 at $x_1 = x'$ under $\lambda_v < 1$, then it must also be increasing under $\lambda_v = 1$:

$$-2\lambda_I(x' - x_I) + 4\alpha\psi_x\pi K \geq -2\lambda_I(x' - x_I) + 4\alpha\psi_x\lambda_v\left[\pi - 4\alpha\psi_z(1 - \lambda_v)z_1\beta\right]K \quad (41)$$

which reduces to

$$\pi \geq \lambda_v\left[\pi - 4\alpha\psi_z(1 - \lambda_v)z_1\beta\right], \quad (42)$$

which is always satisfied. □

Proof of Proposition 6. Suppose that the incumbent has multiple secondary dimensions \tilde{D} available to open, but can only choose one. Applying the envelope theorem, we can characterize how the incumbent's equilibrium utility changes if he chooses to open dimensions with different features in the first period. For simplicity, we will assume that in the second period the officeholder implements his ideologically preferred policy on all dimensions, and denote \tilde{K} the cost of losing the election in this augmented multidimensional world. Further, we denote \tilde{d}_I the incumbent's ideal point on dimension \tilde{d} , $\rho_{\tilde{d}}$ the correlation between X and \tilde{D} , and $\psi_{\tilde{d}}$ the precision of the shock term on dimension \tilde{D} . Then, we have

$$\frac{\partial U_I^*}{\partial \tilde{d}_I} = 2(d_1 - \tilde{d}_I). \quad (43)$$

From Proposition 5 we know that $d_1 \geq \tilde{d}_I$ iff $\pi < \frac{1}{2}$. Therefore $\frac{\partial U_I^*}{\partial \tilde{d}_I} \geq 0$ iff $\pi > \frac{1}{2}$. As an aside,

note that here we are not treating \tilde{K} as a function of \tilde{d}_I , since we are comparing utility across dimensions and the cost of losing does not depend on which dimension the incumbent chooses to open in the first period.

Looking at the correlation $\rho_{\tilde{d}}$, we have

$$\frac{\partial U_I^*}{\partial \rho_{\tilde{d}}} = \tilde{K}(1 - 4\alpha\psi_x x_1 \lambda_v) 4\alpha\psi_{\tilde{d}} \tilde{d}_1 (1 - \lambda_v) \frac{\partial \beta}{\partial \rho_{\tilde{d}}} > 0 \quad (44)$$

for a leading incumbent (β is increasing in $\rho_{\tilde{d}}$ under $\pi > \frac{1}{2}$), and

$$\frac{\partial U_I^*}{\partial \rho_{\tilde{d}}} = \tilde{K}(1 - 4\alpha\psi_x x_1 \lambda_v) 4\alpha\psi_{\tilde{d}} \tilde{d}_1 (1 - \lambda_v) \frac{\partial \beta}{\partial \rho_{\tilde{d}}} < 0 \quad (45)$$

for a trailing one (β is decreasing in $\rho_{\tilde{d}}$ under $\pi < \frac{1}{2}$).

Finally, consider the precision of the outcomes:

$$\frac{\partial U_I^*}{\partial \psi_{\tilde{d}}} = -(1 - 4\alpha\psi_x \lambda_v x_1) (1 - \beta) 4\alpha \tilde{d}_1 (1 - \lambda_v) < 0 \quad (46)$$

for a leading incumbent, and

$$\frac{\partial U_I^*}{\partial \psi_{\tilde{d}}} = (1 - 4\alpha\psi_x \lambda_v x_1) \beta 4\alpha \tilde{d}_1 (1 - \lambda_v) > 0 \quad (47)$$

for a trailing one. □