Ideology For The Future

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Abstract

Political parties sometimes adopt unpopular positions that condemn them to electoral defeat. This phenomenon is usually ascribed to *expressive* motives, namely parties' desire to maintain their ideological purity. Could ideological parties instead have *strategic* incentives to lose? To answer this question, I present a model of repeated spatial elections in which voters face uncertainty about their preferred policy and learn via experience. The amount of voter learning, I show, depends on the location of the implemented policy: a more radical policy generates more information. This creates a trade-off for a party whose ideological stance is unpopular with the electorate, between winning the upcoming election so as to secure policy influence, and changing voters' preferences so as to win with a better platform in the future. Under some conditions the party gambles on the future. It chooses to lose today to possibly change voters' views and win big tomorrow.

Introduction

Whether political parties want power for power's sake, or as a mean to implement their preferred policy, an instrumental desire to win elections is typically expected to drive their strategic behavior. Yet, parties sometimes appear to deviate from this law of electoral politics. The most prominent example is the case of Barry Goldwater, 1964 Republican presidential candidate. Goldwater ran on an extreme right-wing platform, despite the widespread belief that it would be too unpopular with the American public to be electorally viable. Goldwater himself admitted he never expected to win (Goldwater 1988: 154). Indeed, he lost in a landslide against Lyndon Johnson.

This and other examples suggest that political parties sometimes *choose* to settle for electoral defeat: they adopt unpopular positions, even if this means losing the upcoming election for sure. From a rational choice perspective, this is quite puzzling. Existing models predict that instrumentally rational parties will not sow the seeds of their own demise. Even if a party is motivated solely by ideology, it should never accept guaranteed electoral defeat. Indeed, extant explanations for these and other cases rely on the assumption that parties (i.e., their members, activists, or candidates themselves) have *expressive* rather than strategic motivations, and value ideological purity. Thus, a party may be willing to lose if winning comes at purity's expense (Harmel and Janda 1994, Roemer 2001, Strom 1990, Budge et. al 2010).

This paper's main contribution is to show that ideologically motivated parties may instead choose to lose for *entirely strategic* reasons, even without expressive concerns for purity. A party whose ideology is unpopular with the electorate faces a trade-off, between compromising to win the upcoming election, and changing the voters' preferences to be able to win with a better platform in the future. Under some conditions, the party gambles on the future by choosing to lose today to change voters' views and win big tomorrow. Crucially, one such condition is that parties are ideological not only in their preferences, but also in their beliefs about which policy is best for voters. Thus, this paper shows that phenomena typically ascribed to expressive motivations can instead arise from strategic considerations coupled with behavioral tendencies such as parties agreeing to disagree.

To micro-found this intuition, I analyze a model of repeated spatial elections with two time periods. Two policy-motivated parties compete for the support of a representative voter by proposing a platform along the left-right spectrum.[1] The voter elects the party whose platform provides her with the highest expected payoff. The model introduces two novel features. First, the voter (*and* both parties) are uncertain about which policy is best for her. Here, the players are faced with what Tavits (2007) defines as pragmatic policy issues, and are unsure of 'what types of policies are related to what sorts of outcomes' (*ibid*: 155). In short, the uncertainty refers to the expected consequences of the various policies for the voter's welfare. For example, high taxation may be good for the representative voter, as it improves the provision of public goods, or bad for her, if it hampers economic growth. Second, the players hold different prior beliefs about which policy is best for the voter. In my setup, these priors represent a second dimension of ideology: the players' 'political beliefs systems' (Sartori 1989: 400). Going back to our redistribution example, a left-wing party believes the net effect of government intervention to be positive for the average voter, whereas a right-wing one is convinced of the virtues of trickle-down economics. Importantly, my players recognize that they hold different worldviews, but do not infer anything from the existence of this disagreement: they agree to disagree.

In this setting, the voter's preferences may change as she experiences the consequences of the first-period implemented policy:[2] she observes how much she liked (or disliked) this policy's outcome, and accordingly revises her expectation over the location of the optimal platform. Policy outcomes, however, are noisy (i.e., their realization is subject to an idiosyncratic shock), and this complicates the voter's inference problem. A consequence of this technology, I show, is that the voter learns more about her ideal policy when more radical platforms (i.e., platforms farther away from the center of the ideological policy space, normalized to zero in the model) are enacted. Formally, radical platforms make it easier for the voter to separate information from noise. Substantively,

[1] Here, parties are unitary actors. See p. 11 for a discussion of this assumption.

[2] This is analogous to the notion that party identification evolves as a running tally of political experiences (e.g., Fiorina 1981).

suppose that following the implementation of a radical progressive platform, involving very high taxation and public spending, the voter sees her condition improve. Then, she infers that this platform is likely close to the optimal policy and revises her preferences accordingly. Conversely, because the voter's learning is imperfect, her payoff from a more moderate policy is much less informative. No learning occurs, and the voter's policy preferences remain unchanged.

Let's now consider the incentives facing the parties. In each period, the party proposing the platform closer to the voter's preferred policy (as a function of the voter's own beliefs, which are common knowledge) wins the election with certainty. Thus, in the second (and last) period parties behave as in standard one-shot spatial elections: both platforms converge on the voter's preferred point. Not so much in the first period.

Suppose that the voter is initially right-leaning (i.e., under her prior beliefs she ex-ante prefers a right-leaning policy) and consider the left-wing party's problem in the first-period election. The party always has incentives to cater to the voter's preferences, in order to win the upcoming election and move the implemented platform closer to its own ideal policy. This is the usual centripetal tendency arising in spatial elections models. However, this initially unpopular party also has an incentive to facilitate voter learning, in hopes of changing the voter's future policy preferences and being able to win with a better platform tomorrow. The unpopular party's dilemma is that it cannot achieve both goals simultaneously.

This is a direct consequence of the voter's bias against the party. Precisely because the voter's initial preferences are right-leaning, in the first period the popular right-wing party can win with relatively more radical platforms,[3] which would generate more information. This creates the unpopular party's trade-off. The party could move closer to the voter and win, thus minimizing the immediate policy losses. But then, a less informative policy is implemented, the voter's preferences are unlikely to change, and the party will probably have to compromise on a right-wing platform again tomorrow. Conversely, if the unpopular party allows its opponent to win with a more extreme right-wing platform today, the voter learns more. If the voter dislikes such platform's realized out-

[3]For any pair of platforms equidistant from the voter, the right-wing one is farther from zero.

come (thus learning that the platform is not aligned with her optimal policy), then the unpopular party can win with a left-wing policy in the future.

In other words, the unpopular party must choose between compromising to minimize immediate losses, but at the cost of compromising again tomorrow, versus standing firm to facilitate voter learning and potentially win with a better platform in the future. If the incentives to change the voter's preferences are sufficiently strong, the unpopular party gambles on the future: loses today to win big tomorrow. This paper characterizes the conditions under which this occurs in equilibrium.

I show that extreme policy preferences are not enough for an instrumentally rational party to choose to lose. Gambling equilibria require that both parties are also sufficiently ideological in their beliefs, i.e., sufficiently confident that the true optimal policy for the voter aligns with their own preferences. Intuitively, the unpopular party is willing to throw today's election only when it believes this will move the voter's future preferences to the left. However, this is not enough. In a spatial setting, it takes two to gamble: the popular party must also be willing to increase voter learning. This party has a lot to lose from generating additional information. If it is not sufficiently confident that doing so would move the voter even further to the right, the popular party does not take up the gamble and platform convergence always emerges in equilibrium. Thus, open conflict of ideological beliefs is an essential component of the story.

The nature of electoral competition in this model is distinct from dynamics typically emerging in spatial elections. In a gambling equilibrium, the unpopular party's behavior is driven by incentives to change the voter's future preferences. As the voter's (ex-ante) preferences shift further rightward such incentives increase. The party is therefore willing to move further to the left and allow its opponent to win with an even more extreme (and radical) right-wing platform, thus ensuring more information is generated. My model's comparative statics therefore show that parties may respond to shifts in public opinion by moving *away* from the electorate, providing a result that goes in sharp contrast with the standard spatial logic. Thus we can - and do, as I discuss below - observe empirical patterns potentially consistent with my model, but difficult to reconcile with alternative theories (such as the findings in Schumacher et al. 2013, Adams et al. 2009).

While this project focuses on political parties' strategic platform choice, the insights may apply beyond this specific context. My contribution is to demonstrate that behavior consistent with (and typically ascribed to) expressive motives – namely, a desire to express one's own true ideological stance – can instead arise from dynamic strategic considerations, when these are coupled with ideological beliefs. This potentially extends beyond the specific platform game considered here. In concluding the paper I briefly discuss how the theory may be relevant for our understanding of candidates' entry decisions, as well as legislative bargaining. Incorporating the non-common priors assumption within standard political economy models is a (relatively) small and well-defined deviation from Bayesian rationality, but it potentially allows us a richer understanding of several real-world phenomena.

Contribution to Existing Literature

The study of parties' strategic positioning is the subject matter of a large number of both theoretical and empirical works in the spatial theory tradition. This literature originated with the work of Anthony Downs (1957), which posits that office-motivated parties always propose convergent platforms, catering to the preferences of the median voter. Successive work has noted that parties may not be merely office-seeking. Instead, parties are often motivated by ideology, and only see power as a means to policy influence (Chappell and Keech 1986, Calvert 1985, Wittman 1983, Muller and Strom 1999). While such ideological motivations may prevent full platform convergence,[4] 'even ideologues have to give some weight to electoral success' (Budge et al. 2010: 972), as it is necessary to achieve their policy goals.

I contribute to this literature by showing that, when we take into account *dynamic* considerations, ideological parties may sacrifice their short-term policy goals in order to pursue the objective of changing voters' future policy preferences. In 1990, Strøm described formal theorists' focus on static models of electoral competition as one of the main shortcomings in this literature. Three decades later, dynamic spatial elections models remain an exception. This paper emphasizes the

[4]See discussion in Stokes 1999, pp. 251-253.

importance of considering dynamic incentives, demonstrating how (and under which conditions) doing so may substantially alter our understanding and predictions about parties' strategic positioning.

My theory produces two novel results. First, I propose a rationale for why *instrumentally rational* ideological parties may adopt unpopular positions that condemn them to certain electoral defeat in the short-run, even absent frictions (Walgrave et al. 2009), constraints (Dalton et al. 2015) or concerns for ideological purity (Schumacher et al. 2013). Second, I show that, in contrast with the standard spatial logic, ideological parties may respond to shifts in the electorate by moving their platform in the opposite direction, *away* from the median voter. I further elaborate on the model's empirical implications in a separate section, where I show that my theory may provide a rationale for observed empirical patterns hard to reconcile with the the purely spatial theory of elections.

A related paper is Eguia and Giovannoni (2019), which also analyzes parties' platform choice within a dynamic game. They show that an *office-motivated* party with a valence disadvantage[5] may adopt an extreme (and unpopular) policy today, in order to acquire ownership of that platform. An *exogenous* shock to voters' preferences that makes such platform more appealing may then allow the party to win with higher probability in the future. I analyze an analogous dynamic tradeoff. However, my parties are *policy-motivated*, and voter learning is *endogenous* to their platform choice. Furthermore, Eguia and Giovannoni (2019) assume politicians choose between one of two platforms (a mainstream one and an extreme one), that do not have any ideological connotation. Instead, I consider (a continuum of) policy choices along the ideological spectrum. Thus, my model complements Eguia and Giovannoni (2019) by allowing us to analyze how voters' (ex-ante) ideological leaning, as well as parties' own ideological preferences and beliefs, influence parties' incentives to gamble with extreme platforms (p. 7).

A separate contribution of my paper is to propose a theory of policy-induced voter learning and preference formation. The theory builds on the assumption that voters lack information about which policy is optimal for them, and therefore form preferences on the basis of their beliefs over

[5]E.g., lower policy competence.

'what is a good way to get to' their favorite outcomes (Stimson 1999: 28). In the formal literature, several works analyze elections under such policy-relevant uncertainty. However, these models typically assume that politicians have privileged information about the possible consequences of the various policies, and engage in a signaling game with the electorate (e.g., Maskin and Tirole 2004, Canes-Wrone et al. 2001, Kartik et al. 2015).

I adopt a different perspective, analyzing a setting in which voters learn via experience: observe the consequences of the implemented platform, and revise their policy preferences accordingly. This assumption builds on the literature on partisan identification which argues that voters form (and change) their preferences on the basis of their objective experiences (e.g., Fiorina 1981, Achen 1992). I innovate on this literature by modeling voter learning as a function of the ideological location of the implemented policy along the left-right spectrum.[6] In turns, this allows me to study how the desire to influence voters' future preferences impacts political parties' incentives in the platform positioning game. Notice that, in my setting, voters base their electoral choice on two elements: the past policy outcome generated by the party in power (which determines their updated beliefs over their optimal platform), *and* parties' campaign promises (which they expect the election winner to fulfill). This brings together two perspectives that are often seen as antithetical.

Callander (2011) also analyses a spatial election model where voters learn about the optimal policy by observing realized outcomes. However, the assumptions about the nature of uncertainty are fundamentally different from my paper. In my model, players learn about the *expected* consequences of the various policies.[7] Callander (2011) assumes voters face no uncertainty about expected outcomes, but try to learn about the *exact* effects of *each* specific policy.[8] As a consequence, the learning process is very different in the two settings. Here, voter learning increases when radical

[6] A related argument sees public mood as a thermostatic response (Wlezien 1995): when the government moves too much to the left (right), moderate liberals (conservatives) acquire a preference for less (more) government intervention. While this theory refers to the public's *relative* tastes (i.e., relative to the status quo), my model speaks to voters' *absolute* ideological preferences.

[7] E.g., the average impact of increasing taxation on the representative voter's welfare.

[8] E.g., voters know the expected impact of increasing taxation, but must discover the exact consequences of each specific redistributive policy program.

platforms (i.e., platforms far from the center of the ideological policy space) are enacted. Instead, in Callander's (2011) setting, small moves away from the status quo generate the most information. Furthermore, focusing on the statically optimal choice for a policy maker, Callander (2011) assumes myopic parties. Therefore, he does not analyze parties' dynamic incentives to control voter learning, nor how these incentives impact their platform choice.

The Model

The model consists of two periods, with an election in each. The players are two policy-motivated parties, L and R, and a representative voter V. Before each election, each party commits to a policy along the real line, $x_t^i \in \mathbb{R}$. The voter decides whom to elect. The winner implements the announced platform.

The voter faces uncertainty about the exact location of her ideal policy (hereafter, the state of the world). This policy can take one of two values that, for simplicity, are symmetric around zero: $x_V \in \{\underline{\alpha}, \overline{\alpha}\}$ where $\overline{\alpha} = -\underline{\alpha} \geq 0$. We can think about this uncertainty as referring to the expected consequences of the various policy choices. In other words, the voter does not know which policy is most likely to produce her preferred outcome.

The realization of the state of the world is unknown to all players, but they hold heterogeneous prior beliefs: they assign different probabilities, γ_i for each $i \in \{L, V, R\}$, to the voter's bliss point taking a positive value. Such heterogeneous priors are common knowledge but players agree to disagree, i.e. they do not update on each other's beliefs. Because this assumption is an important point of departure from the standard tenets of Bayesian rationality, I discuss it further below.

Given common knowledge of heterogeneous priors, the voter only learns via experience. Formally, the voter's payoff realization is a noisy signal of the state of the world:

$$U_t^V = -(x_V - x_t)^2 + \varepsilon_t \tag{1}$$

where

$$\varepsilon_t \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$$

The assumption that the noise is distributed uniformly simplifies the analysis but is not necessary for the results.

Finally, parties are policy motivated with quadratic loss utility:

$$U_t^i = -(x_i - x_t)^2 \tag{2}$$

where $x_L \leq 0 \leq x_R$. Here, parties are fully patient, i.e., do not discount their future payoffs. In Appendix B, I show that the model's conclusions hold substantively when this assumption is relaxed.

In turn, the game proceeds as follows:

1. Nature draws $x_V \in \{\underline{\alpha}, \overline{\alpha}\}$ (that remains unknown to all players)
2. The parties simultaneously commit to a policy platform $x_1^i \in \mathbb{R}$, $\forall i \in \{L, R\}$
3. The voter decides whom to elect
4. The winner implements the announced platform
5. The voter's first-period payoffs realize
6. The second period begins, and proceeds as above

Notice that my parties are unitary actors, strategically selecting a platform along the left-right spectrum. While this is a standard assumption in spatial elections models, political parties are complex organization (Aldrich 2011), and their strategic positioning is often governed by rich internal dynamics. Fully incorporating such dynamics is beyond the scope of this paper. However, it is worth noting that this setting can be interpreted as a reduced-form version of a citizen-candidate model with a primary stage. Here, choosing the party's electoral platform is equivalent to selecting a primary candidate who then runs on his true ideological bliss point. The unitary party thus stands in lieu of strategic primary voters and candidates. In this perspective, the paper speaks to a recurrent argument in the literature, according to which primaries represent a polarizing force because ideological activists are unwilling to compromise (Aldrich 1983, Coleman 1971, Brady 2007, Hall 2015).

Finally, let me emphasize that the voter has no private information: given any pair of platforms, the parties face no uncertainty over the current period's electoral outcome. However, there is uncertainty – and, due to heterogeneous priors, disagreement – over what the voter will learn upon observing the first period policy outcome.

Heterogeneous Priors and Beliefs as Ideology

Before delving into equilibrium analysis, it is important to discuss in more depth the key assumption underpinning the results: players hold heterogeneous priors on the state of the world, and 'agree to disagree' (Aumann 1976). This represents a departure from canonical models based on the common priors assumption, i.e., the assumption that heterogeneous beliefs can only be due to information asymmetries. If a conflict of beliefs becomes common knowledge in a common priors setting, it is immediately resolved: individuals revise their own priors according to those held by others, and eventually reach full mutual agreement.

I adopt a different perspective, conceptualizing prior beliefs as a person's innate convictions. In this perspective, 'individuals may simply be endowed with different prior beliefs (just as they may be endowed with different preferences)' (Che and Kartik 2009). Here, such beliefs represent players' deep-rooted mental models of the world. For example, political actors may have different views about the functioning of society or the economy. Indeed, Callander argues that 'much political disagreements is over beliefs (...), that we may think of as ideology' (2011: 657).[9] Hafer and Landa (2005, 2007) also see ideology and beliefs as closely connected, thinking of a player's ideology as the

[9] Benabou and Tirole (2006) and McMurray (2016) present analogous intuitions.

likelihood of being persuaded by a left-wing argument versus a right-wing one. Beyond the formal theory literature, Converse (1954) and Sartori (1969) also discuss the notion of ideology as political beliefs, and Gerring argues that several scholars see ideology as 'virtually undistinguishable from worldview' (1997: 96). This conceptualization is also consistent with empirical results highlighting that different political groups hold polarized beliefs and disagree about important factual questions (see discussion in Levy and Razin 2017).

In line with these arguments, I model parties' beliefs as a second dimension of their ideology. The left-wing party always prefers left-wing policies being implemented (this is the standard notion of ideology in electoral models). However, the party also believes that such policies are in line with the voter's optimum. The converse holds for the right-wing party. In short, ideological parties are convinced that the true state of the world is aligned with their own policy preferences. Formally, $\gamma_L = 1 - \gamma_R = \epsilon$, where $\epsilon > 0$ is arbitrarily small.

Conceptualizing priors as ideology, open conflicts of beliefs can now be sustained in equilibrium. Players have different worldviews, which translate into different beliefs about the true state. Simply becoming aware of this conflict is not enough to solve it. Indeed, quite the opposite. 'Individuals with belief conflicts think that they can persuade each other by taking actions that will produce more information, each expecting it to prove that they were right' (Hirsch, 2016: 70).[10]

Analysis: Learning

Before analyzing the parties' equilibrium behavior, let us focus on the voter's learning process. Here, the voter learns by experience: she considers how much she liked or disliked the first-period policy, and updates her beliefs using Bayes' rule. Formally, her payoff realization is a noisy signal of the state of the world (i.e., the location of her ideal policy). In this setting, I show, the amount of voter learning depends on the policy implemented in the first period. The voter learns more

[10] In addition to the scholars mentioned above, several others have allowed players to 'agree to disagree' (see Yildiz 2004, Smith and Stam 2004, Minozzi 2013, Ashworth and Sasso 2017). Thus, while somewhat unorthodox, this approach is not unprecedented in the literature.

about the state of the world when more radical platforms – that is, platforms farther away from the center of the ideological policy space (normalized to zero) – are enacted. As the implemented policy moves away from zero, the distance in the expected outcome as a function of the true state increases. Thus, each signal is more informative. Substantively, if the voter likes (dislikes) the outcome of a radical policy, it is likely that such policy is (is not) in line with her true preferences. Instead, the outcome of a moderate policy is much less informative. It is harder for the voter to distinguish whether a good outcome stems from a policy closely matching the state, or instead from a temporary idiosyncratic shock salvaging a bad policy.

This feature emerges starkly when the noise ε_t is uniformly distributed. Denote as μ_V the voter's posterior that $x_V = \overline{\alpha}$, given her own payoff realization U_1^V, the first-period policy x_1 and her prior γ_V.

Lemma 1. *The voter learning satisfies the following properties:*

(i) her posterior μ_V takes one of three values: $\mu_V \in \{0, \gamma_V, 1\}$;

(ii) the more radical (i.e., the farther away from zero) the policy implemented in the first period x_1, the higher the probability that $\mu_V \neq \gamma_V$ and

(iii) there exists a policy x' such that $|x_1| \geq |x'|$ implies that $\mu_V \neq \gamma_V$ with probability 1.

After observing her first-period payoff realization, the voter learns either everything or nothing about the true state. Further, a more radical implemented policy is more likely to generate an informative signal. Appendix A contains a formal proof, but the logic for Lemma 1 is easily illustrated graphically.

In Figure 1, the solid lines represent the voter's expected payoff as a function of the implemented policy x_1, for the two possible values of x_V. The thick increasing solid curve is $-(x_1 - \overline{\alpha})^2$ and the thin decreasing solid curve is $-(x_1 - \underline{\alpha})^2$. For any policy different from zero, the voter's expected payoff is always different in the two states of the world. The *realized* payoff, however, also depends on the realization of the shock ε_1. The dashed curves represent the maximum and minimum possible values of the payoff realization, accounting for the shock. Suppose that the true state is positive ($x_V = \overline{\alpha}$). Then, for any policy x_1 the actual payoff realization can fall anywhere on the vertical

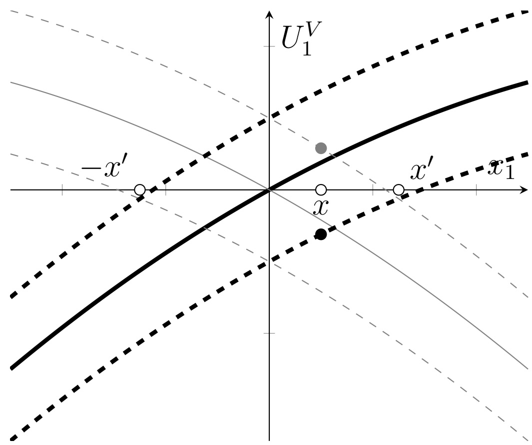

Figure 1: Voter's payoff realization as a function of first-period policy. The thick (thin) curves represent the case in which $x_V = \overline{\alpha}$ ($x_V = \underline{\alpha}$). Solid curves are the voter's expected payoff $E[U_1^V]$, dashed ones represent $E[U_1^V] - \frac{1}{2\psi}$ and $E[U_1^V] + \frac{1}{2\psi}$

line between the two thick increasing dashed curves (representing, respectively, $-(x_1 - \bar{\alpha})^2 + \frac{1}{2\psi}$ and $-(x_1 - \bar{\alpha})^2 - \frac{1}{2\psi}$). Analogously, if $x_V = \underline{\alpha}$, then the payoff realization can be anywhere on the line between the thin dashed curves.

The shock creates a partial overlap in the support of the payoff realization for the two states of the world. Formally, for each policy $x_1 \in (-x', x')$, there exist values of the voter's payoff that may be observed whatever the true state. Consider for example policy x, as represented in the graph. Any payoff realization falling between the gray and black bullets may be observed with positive probability under both states of the world. Suppose that the voter observes a payoff realization outside this range of overlap. There is only one state of the world that could have generated that specific realization: the voter likes the policy too much, or too little, for this to be justified as a consequence of the shock. Thus, the signal is fully informative, and the voter learns the true value of x_V. Conversely, any payoff realization inside the range of overlap is uninformative. Because the shock is uniformly distributed, any such realization has exactly the same probability of being observed under either state of the world. Thus, the voter learns nothing and her beliefs remain at her prior. The more radical (i.e., the further away from 0) the implemented policy, the smaller the range of overlap (i.e., the distance between the black and gray dots in Figure 1) and the more likely the voter is to discover the true state.

I emphasize that my results only require that policies more distant from the center of the policy space are more informative. They do not require that noise is uniformly distributed. The critical assumption is that distribution of noise satisfies the monotonic likelihood ratio property (normally distributed errors, e.g., would satisfy this condition).

The Voter

In what follows, I assume without loss of generality that the voter's prior is biased in favor of the right-wing party, so that her ex-ante preferred policy is positive: $\gamma_V > \frac{1}{2}$. Thus, I refer to the left-wing (right-wing) party as the unpopular one (popular one). To avoid trivialities, the voter's preferred policy is always between the two parties' per-period bliss points, irrespective of her beliefs: $x_L \leq \underline{\alpha} \leq 0 \leq \overline{\alpha} \leq x_R$. For ease of presentation, in the main text I consider a myopic voter. In Appendix B, I show that the (qualitative) results are robust to assuming a forward looking, and fully patient, voter. Finally, to restrict the number of cases under consideration, I assume that $\overline{\alpha} < x'$.

The voter's equilibrium behavior is straightforward:

Lemma 2. *In each period, the voter elects the party whose platform is closer to her preferred policy (given her own beliefs).*

The voter's preferred first-period policy is a function of her prior: $\overline{\alpha}(2\gamma_V - 1)$. In the second period, it instead depends on her updated beliefs: $\overline{\alpha}(2\mu_V - 1)$.

The Parties

Consider now the parties' platform choice. Absent any future concerns, the second-period subgame is equivalent to a one-shot Downsian game:

Lemma 3. *The second-period subgame has a unique equilibrium, in which both parties commit to the voter's preferred policy:* $x_2^{L^*} = x_2^{R^*} = \overline{\alpha}(2\mu_V - 1)$.

The proof follows the usual argument, and is therefore omitted.

It is easy to see that Downsian convergence can be extended to the first period. Thus, the game always has an equilibrium in which the parties propose the voter's preferred policy in both periods. However, the key argument of this paper is that this classic equilibrium is not always unique and does not always capture the nature of electoral competition. In what follows, I show that the unpopular party's strategic behavior is sometimes driven by the incentives to change the voter's future preferences, even at the cost of losing for sure.

The Parties' Utility

Lemma 1 shows that the location of the first-period implemented policy has a crucial impact on the voter learning. As the policy moves away from zero, the variance in the distribution of her posterior beliefs increases (i.e., the likelihood that $\mu_V \neq \gamma_V$ increases). The voter's posterior in turns determines the second-period equilibrium platforms (Lemma 3). Thus, the first-period implemented policy has a twofold effect on the parties' expected utility. A direct effect on their first-period payoff, and an indirect one on their expected future utility (via voter learning). The direct effect is clear. Each party's utility decreases as the platform moves away from its per-period bliss point. The indirect effect is more subtle. Each party believes the true state of the world to be in line with its own policy preferences (i.e., $\gamma_L = 1 - \gamma_R = \epsilon$, where ϵ takes an arbitrarily small value). Thus, each anticipates that information will move the voter's future preferences closer to its own. Each party's expected future utility therefore increases as the policy implemented in the first period becomes more radical, *both to the left and to the right of 0.* Recall that this expectation is the subjective one, as a function of the party's own prior.

The combination of direct and indirect effects determines the overall impact of the first-period policy on the parties' expected utility. Focus again on the unpopular left-wing party (symmetric results hold for the right-wing one). If we consider a left-wing policy ($x_1 < 0$) moving to the right away from x_L, direct and indirect effects go in the same direction. The party's immediate payoff decreases, and as the policy moves closer to zero it also (weakly) reduces the amount of voter

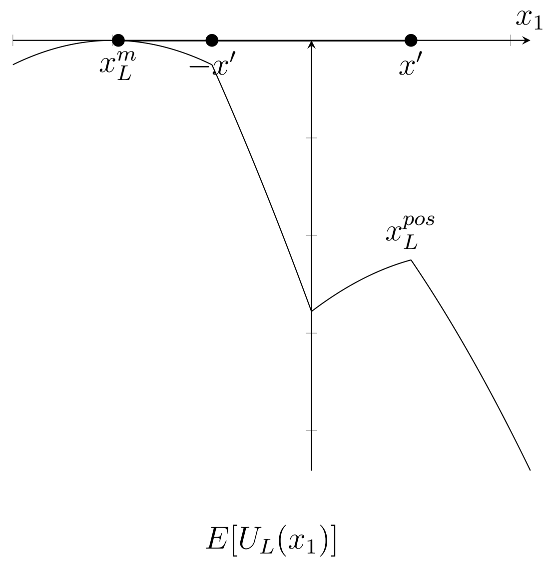

Figure 2: L's expected utility as a function of first-period policy

learning. This also implies that the policy maximizing the party's expected utility – which I denote as x_L^m – is (weakly) to the left of x_L. Conversely, shifting a right-leaning policy farther rightward has competing direct and indirect effects: the party's first-period payoff decreases, but a more radical policy being implemented implies that the voter is more likely to learn the true state of the world, which increases the party's expected future utility. If the indirect effect is sufficiently strong, the party's expected utility has a second (local) maximum above zero, which I denote as x_L^{Pos}.

Lemma 4. *There exist unique* $\overline{\alpha}^{NMon}$ *and* ${x_L}^{NMon}$ *such that if* $\overline{\alpha} > \overline{\alpha}^{NMon}$ *and* $x_L < {x_L}^{NMon}$*, then L's expected utility on* $[0, \infty]$ *is non monotonic with a maximum at* $x_L^{Pos} > 0$*. Otherwise, L's expected utility is monotonically decreasing on* $[0, \infty]$*.*

The indirect effect is stronger if information has a large impact on the voter's policy preferences (i.e., as $\overline{\alpha}$ increases). Additionally, a more extreme party expects to benefit more from from shifting the voter's future preferences to the left (given concave utility). Thus, if the conditions in Lemma 4 are satisfied, the indirect effect dominates, and the left-wing party's overall utility increases as the implemented policy shifts rightward over $[0, x_L^{Pos}]$, as depicted in Figure 2.[11] In what follows, I

[11]Because the probability of learning is not smooth in x_1, neither is the utility function: it kinks at $-x'$, 0 and x' (see Lemma 1).

show that this non-monotonicity can generate gambling behavior in equilibrium.

Gambling on the Future

I now study the incentives facing the parties in the first-period platform game. Consider the popular party R. Recall that (by assumption) $x_R > \overline{\alpha}$, where x_R is the party's static bliss point (i.e., the policy maximizing its utility in the current period). Additionally, since the party's expected future utility increases with the amount of voter learning, its welfare maximizing policy x_R^m is (weakly) to the right of x_R. This implies that, in equilibrium, the winning platform must always be (weakly) larger than the voter's ex-ante preferred policy, $\overline{\alpha}(2\gamma_V - 1)$. Given any policy to the left of this point, the right-wing party can always find a different platform that increases both its own and the voter's payoff. In particular, for any policy $x < 0$, the party can move to $-x > 0$. This guarantees the same amount of learning, but increases both the voter's and the party's immediate payoff. The popular right-wing party would therefore never allow its opponent to win with a policy left of the voter.

Should the same reasoning apply to the left-wing party, the usual Downsian dynamics would emerge, thereby leading to a unique equilibrium in full convergence. Instead, the unpopular party faces a trade-off between securing policy influence (i.e., winning the upcoming election) and increasing the amount of voter learning. This is a direct consequence of the voter's 'bias' against the party. Given $\gamma_V > \frac{1}{2}$, for any pair of platforms making the voter indifferent, the right-wing one is always farther from zero. Thus, the popular party can win with relatively more radical platforms (i.e., platforms farther from the center of the policy space), that would therefore generate more information. This generates the unpopular party's dylemma.

The unpopular party could compromise, and converge towards the voter's preferred platform, so as to win the upcoming election and move the implemented policy to the left. Yet, this would imply that little information is generated, the voter is unlikely to change her beliefs, and the party will have to compromise on a right-wing platform again tomorrow. Conversely, if the party allows its opponent to win with a more extreme right-wing policy, the voter is more likely to learn the true state and the party is more likely to be able to win with a left-wing platform in the future.

If the incentives to change the voter's preferences are sufficiently strong, the unpopular party gambles on the future. It allows the right-wing opponent to win, hoping that the voter will learn that its policies are not aligned with the true state. The unpopular party chooses to lose today to change voters' views and win big tomorrow. In what follows, I establish the conditions under which this behavior can be sustained in equilibrium.

A *gambling equilibrium* is such that, in the first period:

(i) the parties adopt platforms on opposite sides of the voter's preferred policy: $x_1^{L^*} < \overline{\alpha}(2\gamma_V - 1) < x_1^{R^*}$;

(ii) the unpopular party L loses with probability 1.

Notice that any equilibrium satisfying (i) must also meet condition (ii). As mentioned above, the popular party would never allow its opponent to win with a policy to the left of the voter. Thus, any divergence equilibrium must be a gambling equilibrium.

Proposition 1 identifies necessary and sufficient conditions for gambling equilibria to exist:

Proposition 1. *There exist unique $x_L^g \leq {x_L}^{NMon}$ and $\overline{\alpha}^{NMon}$ such that gambling equilibria exist if and only if:*

1. *the unpopular party is sufficiently extreme: $x_L < x_L^g$, and*

2. *learning the true state has a sufficiently large impact on the voter's preferences: $\overline{\alpha} > \overline{\alpha}^{NMon}$*

Recall that ${x_L}^{NMon}$ and $\overline{\alpha}^{NMon}$ are the thresholds defined in Lemma 4. The conditions in Proposition 1 ensure that L's expected utility is increasing in x_1 at $x_1 = \overline{\alpha}(2\gamma_V - 1)$.[12] The intuition is straightforward. If the voter receives no additional information, the parties converge

[12]When these conditions are not satisfied, the game has a unique equilibrium, in which the parties converge on the voter's bliss point in both periods. If the conditions are satisfied, then there exist other convergence equilibria, in which both parties adopt the same platform in the range $[\overline{\alpha}(2\gamma_V - 1), 2\overline{\alpha}(2\gamma_V - a1) - x_L^{Min}]$, where x_L^{Min} is as defined in Proposition 2.

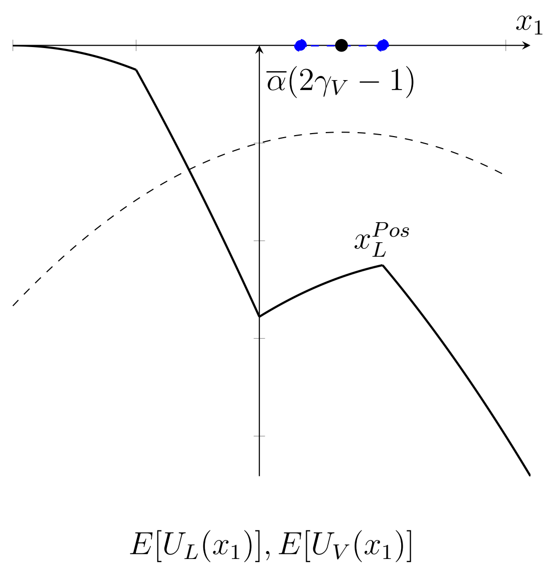

Figure 3: Players' utility as a function of first-period policy. The solid line represents the left-wing party's expected utility in the whole game, while the dashed line represents the voter's first-period expected utility.

on $\overline{\alpha}(2\gamma_V - 1)$ in the second period. Suppose instead that the voter learns that the true state of the world aligns with the left-wing party's ideology. Then, the second-period equilibrium policy moves to $\underline{\alpha}$. The gain from a successful gamble thus increases in $\overline{\alpha} = -\underline{\alpha}$. Additionally, the value of moving tomorrow's equilibrium policy increases as the party's bliss point x_L shifts leftward.

Having established conditions under which gambling can emerge, Propositions 2 and 3 identify the range of platforms that can be sustained in a gambling equilibrium.

Proposition 2. *There exists a unique* $x_L^{Min}(\bar{\alpha}, \gamma_V, x_L) \geq 2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos}$ *such that any pair of platforms which satisfy the following two properties:*

1. *Platforms are symmetric around the voter* ($x_1^{R^*} - \overline{\alpha}(2\gamma_V - 1) = \overline{\alpha}(2\gamma_V - 1) - x_1^{L^*}$)
2. *The left-wing platform is (weakly) to the right of* x_L^{Min} ($x_1^{L^*} \geq x_L^{Min}$)

can be sustained in a gambling equilibrium.

Notice that in these *symmetric* gambling equilibria the voter must be breaking indifference in favour of the popular party R. With any other indifference breaking rule, R has a profitable

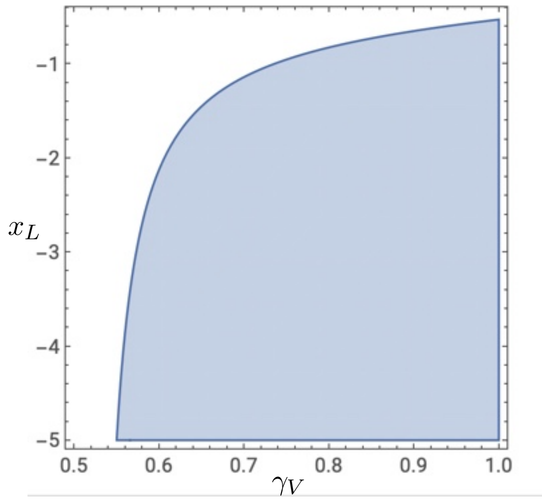

Figure 4: The shaded region identifies the parameter region in which gambling equilibria exist.

deviation to move slightly closer to the voter and win for sure. Thus, the unpopular party chooses to lose the election with probability one, even if an arbitrarily small deviation would guarantee victory.

Next, Proposition 3 shows that (under some conditions) there also exist *asymmetric* gambling equilibria, in which the unpopular party's platform is more extreme than his opponent's (i.e., farther from the voter).

Proposition 3. *There exists a unique x_L^{Asym} such that if and only if $x_L < x_L^{Asym}$, then any pair of platforms satisfying the following two properties:*

1. *the right-wing party commits to its global optimum ($x_1^{R^*} = x_R^m$)*
2. *the left-wing party is strictly farther from the voter ($x_1^{L^*} < 2\bar{\alpha}(2\gamma_V - 1) - x_R^m$)*

can also be sustained in a gambling equilibrium.

Two things are worth noticing. First, asymmetric equilibria emerge only when the unpopular party is sufficiently extreme. Second, in any asymmetric equilibrium, the popular party wins by proposing exactly the policy that maximizes its global utility (x_R^m). This highlights that ideological extremism does not necessarily induce fierce opposition or divergence of interests between the parties. Quite the opposite:

Corollary 1. *Both parties' expected utility in any asymmetric equilibrium is (weakly) higher than in all symmetric equilibria.*

Notice that in one such asymmetric equilibrium (which always exists under $x_L < x_L^{Asym}$) *both* parties propose their global optimum. (i.e., $x_1^{R^*} = x_R^m$ and $x_1^{L^*} = x_L^m$).[13] Intuitively, this equilibrium represents (when it exists) a natural focal point of the game, on which we may expect parties to coordinate.

Robustness and Alternative assumptions.

Degenerate priors. Here, I have assumed that both parties assign probability (arbitrarily close to) one to the state of the world being in line with their own policy preferences (i.e., $\gamma_R = 1-\gamma_L = 1-\epsilon$). In Appendix B, I show that while this is not necessary to sustain the results, heterogeneous priors are a crucial part of the story: gambling equilibria require *both* parties to be sufficiently ideological in their beliefs. Intuitively, the unpopular party is willing to lose the first-period election only when it is sufficiently confident that the gamble will succeed (i.e., that the true state aligns with its own preferences). It is less straightforward to understand why the popular right-wing party may have a profitable deviation. After all, in a gambling equilibrium the party wins for sure, running on a right-wing platform. However, the popular party has a lot to lose from facilitating voter learning. If γ_R is too low the popular party is afraid that learning will move the voter preferences to the left. The party then has an incentive to prevent information generation, and the conjectured equilibria collapse. Interestingly, this implies that gambling equilibria can be sustained when the voter and the popular party share the same beliefs. However, a disagreement between the voter and the unpopular party is always necessary. Finally, I show that ideological beliefs and extreme policy preferences are, to a certain extent, substitutes. As the parties become more ideological in their beliefs, gambling equilibria can be sustained under more moderate policy preferences (Figure 5).

Purely policy motivated parties. To simplify the presentation, I maintain several of the key features of the standard spatial model. In particular, both parties must move simultaneously,

[13]Notice that $x_L^{Pos} \geq x_R^m$ implies $|x_L^m| \geq x_R^m$, therefore $x_L^m \leq 2\overline{\alpha}(2\gamma_V - 1) - x_R^m$.

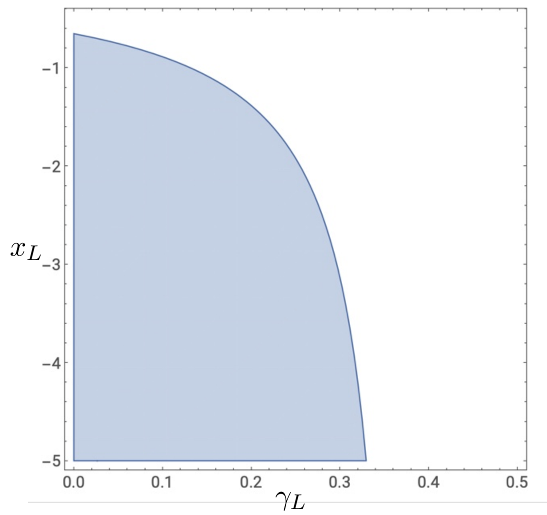

Figure 5: The shaded region identifies the parameter region in which gambling equilibria exist.

and the left-wing (right-wing) party can credibly commit even to radical right-wing (left-wing) platforms. These assumptions are restrictive, but they usually bear no impact on equilibrium results. Not so much in this model. Indeed, in the current set-up gambling equilibria exist only if parties are purely policy motivated. However, if we relax either of these assumptions (simultaneous moves or full commitment ability), gambling equilibria survive even if parties care about office as well as policy. Suppose for example that parties have full commitment ability, but can choose the timing of their platform announcement. Then, gambling equilibria survive as long as office rents are not too large. This is because each party's (policy) utility in a gambling equilibrium exceeds that under full convergence.Alternatively, we could assume that the parties move simultaneously but have limited commitment ability. Budge's 'New Spatial Theory' (1994) highlights the role of ideological consistency as a constraint, with parties only able to move within a subset of the policy space. Similarly, Levy (2004) argues that parties can only commit to policies in the Pareto set of their members (see also Krasa and Polborn 2018). Under such limited commitment assumptions, gambling equilibria survive for sufficiently low office rents as long as the right-most (left-most) platform that the left-wing (right-wing) party can promise is not too radical. Importantly, this is true even if both parties can commit to the voter's (expected) ideal policy.

Electoral volatility. In the baseline model, learning about the state of the world is the only source of electoral volatility across periods. Suppose instead that, from one period to the next, voters' preferences may also be subject to an ideological shock. Would this make gambling equilibria easier or harder to sustain? Interestingly, the answer depends on the shock's expected direction (see Appendix B). Suppose that, in expectation, the shock will move the voter's future preferences to the right. Then, the unpopular left-wing party's gain from changing the voter's beliefs increases in the expected magnitude of the shock (due to concave utility). Thus, gambling equilibria are easier to sustain (in the sense of set inclusion) the larger the average shock. The opposite holds if the shock is expected to move the voter's future preferences to the left. Notice that these findings align with Corollary 1 (despite the underlying mechanism being very different). Taken together, these results imply that an increase in the voter's initial bias against the unpopular party (whether via beliefs about policy consequences or an ideological shock) increases the likelihood of gambling emerging in equilibrium.

Two periods. The baseline model describes a two-period game. In Appendix B I analyze an extension of the model where the game is repeated for infinitely many periods. I show that the strategic incentives arising here mirror the two-period game, and gambling equilibria survive if (and only if) the unpopular party is sufficiently patient and extreme. In such equilibria, the unpopular party continues to gamble until the voter learns the true state of the world. Once an informative outcome is observed, the parties converge on the voter's preferred policy in every period. Interestingly, if the unpopular party is arbitrarily patient (as it is the case in the baseline model), gambling equilibria are easier to sustain than in the two-period baseline.

Parties' response to losses. How should we think about parties' post-election behavior, within the framework of this model? If a party *chooses* to lose an election, then why would it oust the leader, reorganize, or change its platform position following such a loss? In principle, both changing course and sticking to the status quo can be consistent with the party rationally expecting to lose. To see this, consider the infinite-horizon version of the model. At $t = 1$, the unpopular party gambles on the future, rationally and willingly losing the election. Depending on the voter's payoff realization, one of two outcomes may occur. First, the voter may observe an uninformative payoff realization: no learning occurs, and the voter maintains her prior beliefs and preferences. In this case, the game remains in the gambling phase: the parties adopt the same set of platforms again at $t = 2$, with the unpopular one again choosing to lose for sure. In this scenario, no realignment or reorganization has to occur, and we may expect the losing party to confirm the former leadership. Second, the voter may observe an informative payoff realization and thus discover the location of her ideal policy. The game moves to a convergence phase in period 2, and we observe platform convergence on the voter's true optimum in every period thereafter. Suppose that the voter learns that her optimal policy is misaligned with the unpopular party: the gamble has failed, moving the voter away from the party's own bliss point. In this case, the losing party needs to change course. We may therefore expect it to replace the former leader with a new one, willing and able to adopt positions appealing to the voter's newly discovered preferences. If instead the gamble succeeds, the party may choose to confirm the old leadership or opt to replace it with an ideologically aligned but even more extreme one.Thus, the party's response to electoral loss depends on the magnitude and sign of the shift in voter's preferences across periods.

Empirical Implications

Having established the existence of gambling equilibria, I now delve into the theory's empirical implications. So far, I focused on the case in which $\gamma_V > \frac{1}{2}$ (i.e., the left-wing party is the unpopular one). This is without loss of generality: all the results hold symmetrically when $\gamma_V < \frac{1}{2}$. Nonetheless, for clarity of exposition it is useful to explicitly consider both $\gamma_V > \frac{1}{2}$ and $\gamma_V < \frac{1}{2}$ in this section. For simplicity, I will focus on symmetric gambling equilibria (Proposition 2), but all the empirical implications hold under asymmetric equilibria as well.[14]

In a gambling equilibrium, electoral competition is driven by the unpopular party's desire to change the electorate's future preferences, even at the cost of losing today. This has important implications for our understanding of how parties may respond to shifts in voters' preferences:

[14]See Corollaries 2A and 3A in Online Appendix.

Corollary 2.

- *Suppose* $\gamma_V > \frac{1}{2}$ *(i.e., the left-wing party is the unpopular one). Then, the left-most platform that can be sustained in a symmetric gambling equilibrium is decreasing in* γ_V*, and the right-most platform is increasing in* γ_V*;*
- *Suppose instead that* $\gamma_V < \frac{1}{2}$ *(i.e., the right-wing party is the unpopular one). Then, the left-most platform that can be sustained in a symmetric gambling equilibrium is increasing in* γ_V*, and the right-most platform is decreasing in* γ_V *.*

To understand these results (summarized in Table 1 below), suppose that $\gamma_V > \frac{1}{2}$. As the voter's initial preferences move rightward (i.e., γ_V *increases*), the unpopular left-wing party has more to gain and less to lose from taking a gamble. Thus, the party is willing to allow its opponent to win with an even more extreme (and radical) right-wing platform, that further increases the amount of voter learning. In order to do so, the unpopular party must be willing to move further to the left, away from the voter. The opposite holds if γ_V *decreases*: the voter moves to the left, reducing the left-wing party's disadvantage. This unpopular party thus has lower incentives to gamble, and its platform shifts to the right. Thus, under $\gamma_V > \frac{1}{2}$, the left-most platform emerging in a gambling equilibrium is always decreasing in γ_V. A symmetric reasoning applies to the right-wing party when $\gamma_V < \frac{1}{2}$. As the voter moves left (right), the party has stronger (weaker) incentives to gamble and is therefore willing (unwilling) to move further to the right. Here, the right-most platform emerging in a gambling equilibrium is decreasing in γ_V. Thus Corollary 2 shows that, in a gambling equilibrium, the unpopular party may respond to shifts in the voter's preferences by moving *in the opposite direction*.[15]

This emphasizes that the nature of electoral competition in this model is distinct from the dynamics typically emerging in spatial elections. Probabilistic voting models[16] analyze an analogous tradeoff, whereby policy-motivated parties may adopt a platform that decreases their probability of winning (although they would never accept to lose for sure) (Wittman 1983, Calvert 1985).

[15] Corollary 2A estabslishes this result for asymmetric gambling equilibria.

[16] Where voter's behavior, and thus the outcome of the *upcoming* election, is probabilistic.

	Rigtward shift in V's preferences (increase in γ_V)		Leftward shift in V's preferences (decrease in γ_V)	
	$\gamma_V > \frac{1}{2}$	$\gamma_V < \frac{1}{2}$	$\gamma_V > \frac{1}{2}$	$\gamma_V < \frac{1}{2}$
R party	$\rightarrow$	$\leftarrow$	$\leftarrow$	$\rightarrow$
L party	$\leftarrow$	$\rightarrow$	$\rightarrow$	$\leftarrow$

Table 1: Responses to shift in voter's preferences (change in γ_V), gambling equilibrium.

Yet, electoral competition is still driven by the parties' (instrumental) desire to win. Thus, both equilibrium platforms always move in the same direction as the (expected) median voter. Other theories hypothesize that parties are constrained in this adaptation process, but nonetheless predict that, if parties move at all, they follow the electorate (e.g., Dalton 2015).

Thus, Corollary 2 may provide a rationale for patterns hard to reconcile with the purely spatial theory of elections. Indeed, Schumacher et al. (2013) show that, contrary to the classic spatial logic, 'activist-dominated parties' respond to shifts in the electorate by moving in the opposite direction (p. 474). A recurrent argument paints political activists as true ideologues (Enos 2015). This resonates with my model's predictions that (unpopular) parties may be willing to gamble only when sufficiently ideological in both beliefs and preferences. Similarly, Adams et al. (2009) find that left-wing parties do not respond to shifts in public opinion as spatial theories predict. Interestingly, the authors' discussion of what may explain this mismatch aligns with the mechanism uncovered in my model: left-wing parties are more ideological, and might forego platform shifts 'that could confer short-term electoral advantages, because they instead aim to influence voter preferences in the long run' (*ibid.* p. 615).

Let me emphasize that, while it is reassuring to observe patterns potentially consistent with Corollary 2 (but less so with competing theories), qualifying these as evidence of my mechanism requires further analysis. Future research should delve deeper into the specific cases and verify whether the divergent platform shifts occurred in unpopular and extreme parties, as predicated in the model.

Furthermore, one caveat must be kept in mind: observing evidence of parties shifting away from the electorate is *not* a necessary implication of the model. Corollary 2 gives us comparative statics on the unpopular party's platform *in a gambling equilibrium.* However, when the conditions in Proposition 1 fail, the equilibrium takes the familiar form of Downsian convergence. Furthermore, gambling equilibria are not unique: an additional equilibrium in convergence always exists. Finally, platform convergence always occurs in the second period of the two-period model (and in any period following an informative outcome realization in the infinite-horizon model). Therefore, even if my model correctly describes the data generating process, we may observe a mix of equilibria (i.e., gambling and convergence) when we consider data aggregated across different contexts (or periods). Then, any statement about the model's observable implications in the aggregate must be a statement about average effects (where the average is across different equilibria).

In this perspective, we obtain clear predictions if we compare how popular and unpopular parties respond to shifts in the electorate:

Implication 1. *Consider the following regression:*

$$Plat_{it} = \alpha + \beta_1 V_t + \beta_2 Unpop_{it} + \beta_3 V_t \times Unpop_{it} + \epsilon_{it} \tag{3}$$

Where $Plat_{it}$ is the left-right position of party i's platform at time t, V_t is the position of the (median) voter at time t, and $Unpop_{it}$ is a binary indicator taking value one if party i at time t is unpopular, and zero otherwise. Then, β_3 should have a negative sign.

A discussed above, when considering aggregate data, researchers will obtain a mix of different equilibria, i.e., gambling and convergence. To illustrate how this influences our expectations over the sign of the coefficients in 3, Table 2 considers a thought experiment (each cell describes the expected effect of a one-unit *increase* - i.e., rightward shift - in the voter's ex-ante preferred policy). First, suppose all the observations in the dataset feature parties playing a convergence equilibrium. Then, because in such equilibrium both parties always move in the same direction as the electorate (and by exactly the same amount), we should obtain $\beta_1 > 0$ and $\beta_3 = 0$. Suppose instead all observations

	Gambling		Convergence	
	$\gamma_V > \frac{1}{2}$	$\gamma_V < \frac{1}{2}$	$\gamma_V > \frac{1}{2}$	$\gamma_V < \frac{1}{2}$
R party	$\beta_1 > 0$	$\beta_1 + \beta_3 < 0$	$\beta_1 > 0$	$\beta_1 > 0, \beta_3 = 0$
L party	$\beta_1 + \beta_3 < 0$	$\beta_1 > 0$	$\beta_1 > 0, \beta_3 = 0$	$\beta_1 > 0$

Table 2: Parties' response to one-unit *rightward* shift in voter's preferences.

are drawn from gambling equilibria. Corollary 3 indicates that, in this case, the popular party will move in the same direction as the voter, therefore $\beta_1 > 0$. In contrast, the unpopular one will move in the opposite direction. Thus, we should obtain $\beta_1 + \beta_3 < 0$ (which implies $\beta_3 < 0$). β_3 will therefore be equal to 0 in a convergence equilibrium, and take negative value in a gambling one. Notice that this holds regardless of whether $\gamma_V > \frac{1}{2}$ or $\gamma_V < \frac{1}{2}$ (i.e., of the identity of the unpopular party).

Substantively, this has two implications for what researchers should observe when considering aggregate data. First, the model does not discipline our (absolute) directional expectations on how unpopular parties respond to shifts in voters' preferences. In a convergence equilibrium, the unpopular party moves with the voter. In a gambling one, it moves in the opposite direction. Thus, $\beta_1 + \beta_3$ may have, on average, a positive or a negative sign (or even be a zero). Second, and most importantly, the prediction on how unpopular parties respond *relative* to popular ones is instead well defined. Because in a convergence equilibrium both parties move in the same direction, and by the same amount, the sign of β_3 will only capture the platform shifts occurring in gambling equilibria. Thus, when aggregating across equilibria, the model's prediction is unambiguous: on average, we should obtain $\beta_3 < 0$.

While this party-level implication is theoretically well-defined, it may be challenging to evaluate empirically (as it requires a measure of party popularity). A more promising avenue may be to look at election-level implications:[17]

Implication 2. *As the voter's ex-ante preferences become more radical, platform polarization should*

[17] Corollary 3A presents a formal statement of this result.

increase on average.

This follows straightforward from Corollary 2. Notice that, when $\gamma_V > \frac{1}{2}$, the voter's preferences become more radical (i.e., move away from zero) as γ_V *increases*. In contrast, if $\gamma_V < \frac{1}{2}$, voter radicalism increases as γ_V *decreases*. Then, Table 1 clearly shows that, in a gambling equilibrium, the parties' platforms move away from each other as γ_V moves away from $\frac{1}{2}$. Recall that, in a convergence equilibrium, platform polarization is instead constant. Therefore, the sign of the average effect (across equilibria) is again unambiguous: the maximum amount of platform polarization sustainable in equilibrium increases as the voter becomes more radical. To the best of my knowledge, no empirical work has investigated the link between voter radicalismand platform polarization. Indeed, Curini et al. (2012) lament the literature's almost exclusive focus on the institutional determinants of platform polarization. As such, this is a promising avenue for future research.

Finally, consider the theory's implications for electoral consequences of parties' strategic positioning. Suppose that scholars compare two sets of elections: one where the parties are playing a gambling equilibrium and therefore selecting more extreme platforms, and another where the parties are converging on the voter's preferences. Recall that in a convergence equilibrium both parties win with positive probability. Conversely, in any gambling equilibrium the popular party wins with probability one. Implication 3 follows straightforwardly:

Implication 3. *On average, popular parties should perform better in the elections in which they select a more extreme platform. In contrast, the opposite holds for unpopular parties.*

Empirical scholars have often emphasized a surprising lack of consistent findings on the electoral consequences of parties' platform positioning. Adams et al. (2006), for example, conclude that (mainstream) parties don't perform better (or worse) on average when they moderate their platforms in the direction of the electorate. Implication 3 indicates that this null result may emerge from averaging across coefficients with different signs: positive for unpopular parties and negative for popular ones. Indeed, we have some evidence suggesting that the effect may be heterogeneous along this dimension. Bawn et al. (2012) find a negative correlation between platform extremism and electoral performance for opposition parties, but a positive one for governing parties. While incumbency status is not an ideal proxy for popularity (as defined in my model), it is reasonable to speculate that the two variables are positively correlated.[18]

Conclusion

In concluding, I revisit the main motivating example through the lenses of the model. Republican candidate Barry Goldwater espoused an extreme right-wing platform during the 1964 Presidential campaign, despite a consensus that this would condemn the party to electoral failure. Indeed, he suffered a burning defeat. According to my theory, Goldwater gambled on the future: faced with a left-leaning electorate, he knowingly adopted an electorally unviable position in hopes of changing the voters' future preferences. Indeed, historians and political commentators alike maintain that Goldwater's 1964 strategy aimed at 'a higher goal than president of the United States' (Volle 2010: 45). His 'was a radical plan, not calculated to win (...) but to challenge the minds and hearts of voters and produce a Conservative wave in America' (Edwards 2014: 8).

Looking at public opinion and parties' platforms in the lead-up and aftermath of the 1964 election, we also see evidence aligning with the theory. The election took place within the context of the so-called Liberal Consensus in American politics (see, e.g., Perlstein 2001: xi). Indeed, we see both parties' platforms shifting in the same direction between the 1956 and 1960 elections, precisely as we would expect under the classic spatial logic (see Figure 6).[19] Instead, if we look at the 1964 race, things appear to be very different.

Following the logic of my model, Goldwater was presented with a trade-off: continue compromising and adopt an electorally viable position close to the center of the policy space (with little hopes of generating an informative outcome and changing the voter's future preferences), or allow

[18] E.g., if the voter's prior over her optimal policy is the correct one (i.e., reflect the true state distribution), this prior is more likely to be confirmed than not from one period to the next, resulting in a positive correlation between popularity and incumbency.

[19] The electorate also continued to move marginally to the left in this period (see for example Stimson's (1999) Policy Mood Index).

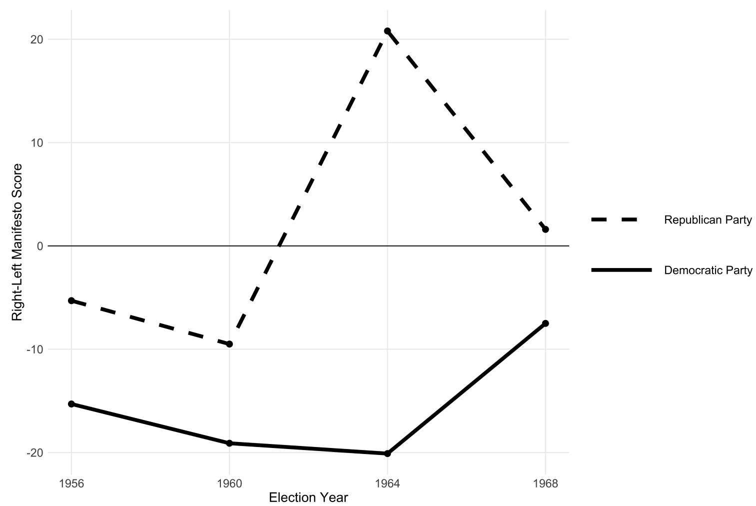

Figure 6: right-left positioning (RILE score, Comparative Manifesto Project) of Republican and Democratic platforms.

its opponent to win and implement a more radical platform, in hopes of facilitating voter learning and obtaining a better policy in the future. Notice that, while Goldwater's platform was electorally untenable (in the model's language, too extreme), it was no more radical than its opponent's (i.e, the two platforms are equally distant from the center of the policy space). This illustrates the source of the unpopular party's tradeoff: its popular opponent can win with relative more radical, and thus more informative, platforms. Moreover, in line with the theory, scholars argue that Goldwater's gamble was induced by extreme preferences coupled with ideological beliefs: he was willing to lose because had faith that 'history would prove him right' (Volle 2010: 50). Evidence suggests that this was one instance in which this strategy paid off. The model predicts a return to platform convergence in the second period, tilted in the direction of the unpopular party in case of a successful gamble (i.e., if the voter learns that her optimal policy is aligned with the party's). This is precisely what we observe in the 1968 elections. The Republican and Democratic platforms converged towards each other, both moving significantly to the right compared to the 1960 campaign.[20] This suggests that Goldwater's strategy successfully moved the center of the political space to the right. Indeed, Stimson's (1999) Policy Mood Index shows a rightward shift in the electorate between 1964

[20] And, precisely as we would expect, by exactly the same amount.

and 1968.[21] This shift, scholars argue, would prove to be long-lasting: Goldwater's gamble is often credited for paving the way for Reagan's election (Will 1998) making this 'one time, at least, in which history was written by the losers' (Perlstein 2001: x).

While this project has focused on political parties' strategic platform positioning, its key insights may extend beyond this specific context. For example, Pons and Tricaud (2018) look at data from run-off elections in France to analyze how the presence of third candidates impacts electoral outcomes. They show that third entrants often end up hurting their own ideological camp, since 'in 19.2 percent of the elections, the presence of the third candidate causes the loss of the candidate among the top two that is ideologically closest to her' (p. 1623). As the authors themselves argue, these results are 'difficult to rationalize (...) in particular when the third candidate appears to have slim chances of being a front-runner in the second round' (p. 1623). This paper suggests a mechanism under which concerns for future policy, combined with ideological beliefs over possible policy consequences, may generate this type of behavior. Moreover, the model provides a framework to understand under which conditions such behavior is more or less likely to emerge.

Finally, similar dynamics may be at play in the context of legislative bargaining. Bargaining players sometimes appear unwilling to compromise, and this rigidity is typically interpreted as a desire to maintain their ideological purity (Mann and Ornstein 2012). My theory suggests an alternative rationale. Forward-looking actors may accept a worse policy today if they are convinced that the resulting outcome will alter the electorate's beliefs in a way that provides them a stronger bargaining position in the future. Consider for example the bargaining between the US Congress and President Donald Trump over the repeal of Obamacare. Soon after the Republican bill to repeal the Affordable Care Act was pulled from the House for insufficient support, Trump adopted the strategy of keeping the status quo rather than looking for a compromise. 'The best thing politically is let Obamacare explode', Trump argued. The logic behind this strategy seems to align with the mechanism advocated in this paper: Trump believed that keeping the policy in place would show American voters the flaws of the current system, and thus generate stronger support for a reform

[21] See Figure 1OB in Online Appendix.

(Bryan 2017).

In short, the contribution of this paper is to show that political actors' behavior typically considered as expressive, and thus explained 'in its own terms' rather than 'in terms of preferences over outcomes' (Brennan and Buchanan 1984: 187), may instead arise from strategic considerations coupled with fundamental behavioral tendencies, such as heterogenous (and ideological) beliefs.

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Appendix A: Proofs

Lemma 1: *voter learning satisfies the following properties:*

(i) Her posterior μ_V *takes one of three values:* $\mu_V \in \{0, \gamma_V, 1\}$;

(ii) The more radical (i.e., the farther away from zero) the policy implemented in the first period x_1*, the higher the probability that* $\mu \neq \gamma_V$;

(iii) There exists a policy x' *such that if* $|x_1| \geq |x'|$*, then* $\mu_V \neq \gamma_V$ *with probability 1.*

Proof. The proof of Claims 1 and 2 below is necessary and sufficient to prove Lemma 1.

Claim 1: *Let* $x_t \geq 0$.

(i) A payoff realization $U_t^v \notin [-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$ *is fully informative. Upon observing* $U_t^v > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}$*, the players form posterior beliefs that* $x_V = \bar{\alpha}$ *with probability 1. Similarly, upon observing* $U_t^v < -(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}$ *the players form beliefs that* $x_V = \underline{\alpha}$ *with probability 1.*

(ii) A payoff realization $U_t^v \in [-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$*, is uninformative. Upon observing* U_t^v*, players confirm their prior belief that* $x_V = \bar{\alpha}$ *with probability* γ_i, $\forall i \in \{R, V, L\}$.

Symmetric results apply when $x_t < 0$.

Proof. The proof of part (i) is trivial given the boundedness of the distribution of e, and is therefore omitted. Part (ii) follows straightforwardly from applying Bayes rule. Recall that the voter's payoff realization U_t^v is a function of the implemented policy (x_t) the voter's true bliss point (x_V) and the noise term (e): $U_t^v = -(x_V - x_t)^2 + e$. Denote as $f(\cdot)$ the PDF of e. Then,

$$prob(x_V = \bar{\alpha}|U_t^v) = \frac{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma_i}{f(U_t^v + (x_t - \bar{\alpha})^2)\gamma_i + f(U_t^v + (x_t - \underline{\alpha})^2)(1 - \gamma_i)} \tag{1}$$

Given the assumption that ϵ is uniformly distributed

$$f(U_t^v + (x_t - \bar{\alpha})^2) = f(U_t^v + (x_t - \underline{\alpha})^2) \tag{2}$$

Therefore the above simplifies to

$$prob(x_V = \bar{\alpha}|U_t^v) = \gamma_i \tag{3}$$

This concludes the proof of Claim 1. □

Claim 1 proves that players either observe an uninformative or a fully informative signal. Claim 2 shows that the policy choice determines the expected probability that the signal will be informative. The farther from zero the implemented policy, the higher such probability.

Claim 2: *Let L be a binary indicator, taking value 1 if the players learn the true value of x_V at the end of period 1, and 0 otherwise. There exists $x' = \frac{1}{4\bar{\alpha}\psi}$ such that*

- *For all $|x_1| \geq |x'|$*

$$Prob(L = 1|x_1) = 1 \tag{4}$$

- *For all $x_1 \in [0, x')$*

$$Prob(L = 1|x' \geq x_1 \geq 0) = 4\bar{\alpha}\psi x_1 \tag{5}$$

- *For all $x_1 \in (-x', 0]$*

$$Prob(L = 1| - x' \leq x_1 \leq 0) = -4\bar{\alpha}\psi x_1 \tag{6}$$

Proof. Let me first prove the existence of point x'. From Claim 1, x' is the point such that for any policy $|x| \geq |x'|$, the interval $[-(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}, -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}]$ is empty. This requires

$$-(x_t - \underline{\alpha})^2 + \frac{1}{2\psi} + (x_t - \bar{\alpha})^2 + \frac{1}{2\psi} \leq 0 \quad (7)$$

Recall that $\bar{\alpha} = -\underline{\alpha}$, thus the above reduces to

$$x \geq \frac{1}{4\bar{\alpha}\psi} = x' \quad (8)$$

To complete the proof, assume $x_1 \in (0, x')$. The expected probability of the realized outcome being informative is:

$$Prob(L = 1|\gamma_i, 0 < x_1 < x') =$$

$$\gamma_i[Prob(-(x_t - \bar{\alpha})^2 + \varepsilon_1 > -(x_t - \underline{\alpha})^2 + \tfrac{1}{2\psi})] + (1 - \gamma_i)[Prob(-(x_t - \underline{\alpha})^2 + \varepsilon_1 < -(x_t - \bar{\alpha})^2 - \tfrac{1}{2\psi})] \quad (9)$$

Given the symmetry

$$Prob(-(x_t - \bar{\alpha})^2 + \varepsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi}) = Prob(-(x_t - \underline{\alpha})^2 + \varepsilon_1 < -(x_t - \bar{\alpha})^2 - \frac{1}{2\psi}) \quad (10)$$

(15) simplifies to

$$Prob(L = 1|x_1 > 0) = Prob(-(x_t - \bar{\alpha})^2 + \varepsilon_1 > -(x_t - \underline{\alpha})^2 + \frac{1}{2\psi})) = 4\bar{\alpha}\psi x_1 \quad (11)$$

Similar calculations produce the result for $x_1 \in (-x', 0]$.

This concludes the proof of Claim 2 □

and thus of Lemma 1. □

The Parties' Utility

In this section I characterize the policies x_L^m and x_L^{pos} (symmetric results apply for the right-wing party), and present the proof of Lemma 4.

Denote as $\beta(x_1)$ the probability of the voter learning the true state of the world (as a function of the policy implemented in the first period). Given $\gamma_L = \epsilon \approx 0$, the left-wing party's (subjective) expected utility can be written as:

$$-(x_1 - x_L)^2 - (1 - \beta(x_1))(\bar{\alpha}(2\gamma_V - 1) - x_L)^2 - \beta(x_1)(\underline{\alpha} - x_L)^2 \tag{12}$$

Notice that the party's utility is increasing in $\beta(x_1)$, given the assumption on γ_L. From Lemma 1 we know that $\beta(x_1)$ is not a smooth function of x_1: it kinks at $-x'$, 0 and x'. Thus, we must analyze the utility function piecewise.

Consider first the case in which $x_L \leq -x'$. Then, L's expected utility as a function of x_1 has the following properties:

- In the range $[-\infty, -x']$ it is concave and non monotonic with global maximum at $x_L^m = x_L$. Every policy in this range guarantees learning with probability 1. Thus, as x_1 moves away from x_L it only has a negative direct effect on the party's payoff.

- In the range $[-x', 0]$ it is strictly decreasing. As the policy moves to the right the party's immediate utility decreases. The probability of the voter learning the true state is also reduced, which implies lower expected future utility

- In the range $[0, x']$ the party faces a trade-off, that is analyzed in more details below.

- In the range $[x', \infty]$ it is strictly decreasing. Every policy in this range guarantees learning with probability 1. Thus, as x_1 moves to the right it only has a negative direct effect on the party's payoff.

Consider now the case in which $x_L > -x'$. Then, L's expected utility as a function of x_1 has the following properties:

- In the range $[-\infty, -x']$ it is strictly increasing. Every policy in this range guarantees learning with probability 1. Thus, as x_1 moves closer to x_L it only has a positive direct effect on the party's payoff.
- In the range $[-x', 0]$ it is concave and non-monotonic with global maximum at $x_L^m \in [-x', x_L]$. This is the policy that solves the following maximization problem:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -(\mathrm{x_1} - \mathrm{x_L})^2 - (1 + 4\overline{\alpha}\psi \mathrm{x_1})(\overline{\alpha}(2\gamma_\mathrm{V} - 1) - \mathrm{x_L})^2 + 4\overline{\alpha}\psi \mathrm{x_1}(\underline{\alpha} - \mathrm{x_L})^2 \\ \text{subject to} \quad & x_1 \in [-\frac{1}{4\overline{\alpha}\psi}, 0] \end{aligned} \tag{13}$$

- In the range $[0, x']$ the party faces a trade-off, that is analyzed in more details below.
- In the range $[x', \infty]$ it is strictly decreasing. Every policy in this range guarantees learning with probability 1. Thus, as x_1 moves away from x_L it only has a negative direct effect on the party's payoff.

Lemma 4: *There exist unique $\overline{\alpha}^{NMon}$ and ${x_L}^{NMon}$ such that if $\overline{\alpha} > \overline{\alpha}^{NMon}$ and $x_L < {x_L}^{NMon}$ then L's expected utility on $[0, \infty]$ is non monotonic with a maximum at $x_L^{Pos} > 0$. Otherwise, L's expected utility is monotonically decreasing on $[0, \infty]$.*

Proof. From the discussion above we know that L's utility is always monotonically decreasing in the range $[x', \infty]$. Conversely, in the range $[0, x']$ the party faces a trade off. As the policy moves to the right the party's immediate payoff decreases, while its future expected payoff increases. The maximization problem is:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -(\mathrm{x_1} - \mathrm{x_L})^2 - (1 - 4\overline{\alpha}\psi \mathrm{x_1})(\overline{\alpha}(2\gamma_\mathrm{V} - 1) - \mathrm{x_L})^2 - 4\overline{\alpha}\psi \mathrm{x_1}(\underline{\alpha} - \mathrm{x_L})^2 \\ \text{subject to} \quad & x_1 \in [0, \frac{1}{4\overline{\alpha}\psi}] \end{aligned} \tag{14}$$

The solution to this maximisation problem is $x^* = \min \in \{\max \in \{0, x_L - 8\bar{\alpha}^2\psi(x_L\gamma_V + \overline{\alpha}\gamma_V(1 - \gamma_V))\}, \frac{1}{4\overline{\alpha}\psi}\}$. Thus, if $x_L - 8\bar{\alpha}^2\psi(x_L\gamma_V + \overline{\alpha}\gamma_V(1 - \gamma_V)) \leq 0$, the function is monotonically decreasing on

$[0, \infty]$. Otherwise, it is non monotonic with maximum at $x_L^{pos} = \min \in \{x_L - 8\bar{\alpha}^2\psi(x_L\gamma_V + \bar{\alpha}\gamma_V(1 - \gamma_V)), \frac{1}{4\bar{\alpha}\psi}\}$. Therefore, the condition for non-monotonicity is $x_L - 8\bar{\alpha}^2\psi(x_L\gamma_V + \overline{\alpha}\gamma_V(1 - \gamma_V)) > 0$. This yelds:

$$x_L < \frac{-8\bar{\alpha}^3\psi\gamma_V(1 - \gamma_V)}{8\bar{\alpha}^2\psi\gamma_V - 1} \tag{15}$$

and

$$\bar{\alpha}^2 > \frac{1}{8\psi\gamma_V} \tag{16}$$

□

Proposition 1: *There exist unique $x_L^g \leq {x_L}^{NMon}$ and $\overline{\alpha}^{NMon}$ such that Gambling equilibria exist if and only if:*

- *The unpopular party is sufficiently extreme: $x_L < x_L^g$*
- *Learning the true state has a sufficiently large impact on the voter's preferences: $\overline{\alpha} > \overline{\alpha}^{NMon}$*

Proof. Necessary and sufficient condition for gambling equilibria to exist is that L's expected utility is increasing at $x_1 = \bar{\alpha}(2\gamma_V - 1)$, i.e. $x_L^{pos} > \bar{\alpha}(2\gamma_V - 1)$.[1] Notice that $\bar{\alpha}(2\gamma_V - 1) < \frac{1}{4\bar{\alpha}\psi}$ (given the assumption that $\bar{\alpha} < \frac{1}{4\bar{\alpha}\psi}$). Thus, we do not have to worry about the case in which (13) has a corner solution at $\frac{1}{4\bar{\alpha}\psi}$, and the condition is:

$$x_L - 8\bar{\alpha}^2\psi(x_L\gamma_V + \bar{\alpha}\gamma_V(1 - \gamma_V)) > \bar{\alpha}(2\gamma_V - 1) \tag{17}$$

[1]Proposition 2 will establish that any gambling equilibrium must involve symmetric platforms (given the assumption that $x_R > x'$). Since the right-wing party's global maximum is to the right of the voter, this implies that we can always find a pair of platforms such that the right-wing party has no profitable deviation (with the voter breaking indifference in the party's favor).

The above can be satisfied if and only if the LHS id decreasing in x_L. Thus, we obtain:

$$x_L < \frac{-\bar{\alpha}(2\gamma_V - 1) - 8\bar{\alpha}^3\psi\gamma_V(1-\gamma_V)}{8\bar{\alpha}^2\psi\gamma_V - 1} \tag{18}$$

And

$$\bar{\alpha}^2 > \frac{1}{8\psi\gamma_V} \tag{19}$$

Recall that throughout the paper I maintain that $x_L < \overline{\alpha}$. However, it is easy to see that (18) is always binding, i.e., $\frac{-\bar{\alpha}(2\gamma_V-1)-8\bar{\alpha}^3\psi\gamma_V(1-\gamma_V)}{8\bar{\alpha}^2\psi\gamma_V-1} < \overline{\alpha}$.[2]

□

Proposition 2: *There exists a unique* $x_L^{Min}(\bar{\alpha}, \gamma_V, x_L) \geq 2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos}$ *such that any pair of platforms*

1. *symmetric around the voter* ($x_1^{R^*} - \overline{\alpha}(2\gamma_V - 1) = \overline{\alpha}(2\gamma_V - 1) - x_1^{L^*}$)*, and*

2. *such that the left-wing party is (weakly) to the right of* x_L^{Min} ($x_1^{L^*} \geq x_L^{Min}$)

can be sustained in a gambling equilibrium.

Proof. Denote as $\tilde{x}$ the policy that maximises R's expected utility in the range $[0, x']$.[3]

Notice that if the conditions in Proposition 1 are satisfied, x_L^{pos} is decreasing in x_L. As such, there exists a unique threshold $\widetilde{\widetilde{x_L}} < 0$ s.t. $\tilde{x} < x_L^{pos} \iff x^L < \widetilde{\widetilde{x_L}}$.

Case 1: $x^L > \widetilde{\widetilde{x_L}}$**.** Consider first the case in which $x^L > \widetilde{\widetilde{x_L}}$, and thus $\tilde{x} > x_L^{pos}$ and the right-wing party's expected utility is increasing on $[0, x_L^{pos}]$. In this case, any gambling equilibrium must involve platforms symmetric around the voter. The proof is straightforward: for any pair of asymmetric policies at least one of the parties can deviate to a winning platform that strictly increases its own expected utility. If $x_1^{R^*} \leq x_L^{pos}$, R can make an arbitrarily small move to the right and continue to win with probability 1, while strictly increasing expected its utility. If $x_1^{R^*} > x_L^{pos}$, the left-wing

[2]Recall that, by assumption, $\overline{\alpha} < \frac{1}{4\overline{\alpha}\psi}$.

[3]If $x_R < x'$, then $\tilde{x} \in (0, x')$ and it represents the right-wing party's global optimum. If instead $x_R \geq x'$, then $\tilde{x} = x'$ and is only a local maximum.

party can move to x_L^{pos} and win, while strictly increasing its expected utility. Indeed, notice that the unpopular party would never allow its opponent to win with a policy to the right of x_L^{pos}. The lower bound of the range of (left-wing) policies that can be sustained in equilibrium is therefore always (weakly) larger than the symmetric $2\overline{\alpha}(2\gamma_V - 1) - x_L^{pos}$. In particular, $x_L^{Min} = 2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos}$ when $2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos} \geq 0$. Recall, in fact, that the left-wing party's utility is monotonically increasing on $[0, x_L^{pos}]$. Therefore, for any $x_1^{R^*} \in (\overline{\alpha}(2\gamma_V - 1), x_L^{pos}]$, the left-wing party cannot find a winning platform that strictly increases its expected utility.

Suppose instead that $2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos} < 0$. Then, the following Corollary holds:

Corollary 1A: *Suppose that* $2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos} < 0$. *Then,* $x_L^{Min} = \max \in \{2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos}, \hat{x}\}$, *where* $\hat{x} \leq 0$ *is such that* $E[U_L(\hat{x})] = E[U_L(2\overline{\alpha}(2\gamma_V - 1) - \hat{x})]$

Proof. First of all let me prove the existence of a (unique) policy $\hat{x}$.

Claim 1. *There exists a unique policy* $\hat{x} \leq 0$ *such that: (i)* $E[U_L(\hat{x})] = E[U_L(2\overline{\alpha}(2\gamma_V - 1) - \hat{x})]$, *(ii) for any* $x < \hat{x}$, $E[U_L(x)] > E[U_L(2\overline{\alpha}(2\gamma_V - 1) - x)]$ *and (iii) for any* $\hat{x} < x < 0$, $E[U_L(x)] < E[U_L(2\overline{\alpha}(2\gamma_V - 1) - x)]$.

Proof. Given $x_L^{pos} > 2\overline{\alpha}(2\gamma_V - 1)$, L's expected utility is monotonically increasing on $[0, \overline{\alpha}(2\gamma_V - 1)]$. It follows straightforwardly that:

$$E[U_L(x)] < E[U_L(2\overline{\alpha}(2\gamma_V - 1) - x)] \tag{20}$$

When $x = 0$. Additionally, it is easy to see that the following holds:

$$E[U_L(x)] > E[U_L(2\overline{\alpha}(2\gamma_V - 1) - x)] \tag{21}$$

When $x \leq -x'$ (since both x and $2\overline{\alpha}(2\gamma_V - 1) - x$ guarantee learning with probability 1, but x is always closer to x_L).

Thus, there must exist (at least) one policy $\hat{x} \in (-x', 0)$ such that

$$E[U_L(\hat{x})] = E[U_L(2\overline{\alpha}(2\gamma_V - 1) - \hat{x})] \tag{22}$$

The uniqueness of $\hat{x}$ follows straightforwardly from the fact that $E[U_L(2\overline{\alpha}(2\gamma_V - 1) - x]$ is monotonically decreasing on $[-x', 0]$, while $E[U_L(x)]$ is either monotonically decreasing or concave with maximum at x_L^m.

$\square$

Claim 1 (along with Point 1 in Proposition 2) implies that $x_L^{Min} \geq \hat{x}$: for any pair of platforms symmetric around the voter and such that $x_1^{L^*} < \hat{x}$, L has a profitable deviation to make an arbitrarily small move to the right and win for sure. Further, recall that L's expected utility is monotonically decreasing on $[2\overline{\alpha}(2\gamma_V - 1) - x_L^{pos}, 0]$[4] and monotonically increasing on $[0, x_L^{pos}]$. Additionally (as discussed in the main body), notice that x_L^{Min} must always be to the right of $2\overline{\alpha}(2\gamma_V - 1) - x_L^{pos}$. Thus, it follows straightforwardly from Claim 1 that $x_L^{Min} = \max \in \{2\bar{\alpha}(2\gamma_V - 1) - x_L^{pos}, \hat{x}\}$.

This concludes the proof of Corollary 1A. $\square$

Case 2: $x^L < \widetilde{\widetilde{x_L}}$**.** Suppose instead that $x^L < \widetilde{\widetilde{x_L}}$, and therefore $\tilde{x} < x_L^{pos}$. In this case, the right-wing party is never wiling to commit to x_L^{pos}. It could always deviate to $\tilde{x}$ and strictly increase both its own and the voter's payoff. Indeed (given the definition of $\tilde{x}$) the same reasoning applies to any platform in $[\tilde{x}, x']$. Further, recall that $x_L^{pos} \leq x'$ therefore no platform to the right of x' can ever be sustained in equilibrium. As such, in any gambling equilibrium $x_1^{R^*} \leq \tilde{x}$. Straightforwardly, in any equilibrium in which $x_1^{R^*} < \tilde{x}$, the two parties must be adopting symmetric platforms. The right-wing party can otherwise always find a winning policy that strictly increases its expected utility. Thus, the left-most platform that can be sustained in a symmetric equilibrium is always larger than $2\bar{\alpha}(2\gamma_V - 1) - \tilde{x}$. The proof that such policy x_L^{Min} exists and is always unique proceeds

[4]It is straightforward to verify that $2\overline{\alpha}(2\gamma_V - 1) - x_L^{pos}$ is always to the right of the function's maximum on $[-x', 0]$.

as for Case 1, and is therefore omitted. $\square$

Proposition 3: *There exists a unique x_L^{Asym} such that if and only if $x_L < x_L^{Asym}$, then any pair of platforms such that*

1. *the right-wing party commits to its global optimum ($x_1^{R^*} = x_R^m$), and*

2. *the left-wing party is strictly farther from the voter ($x_1^{L^*} < 2\bar{\alpha}(2\gamma_V - 1) - x_R^m$)*

can also be sustained in a gambling equilibrium.

Proof. The proof of Proposition 2 shows that asymmetric platforms can never be sustained in equilibrium if $\tilde{x} > x_L^{pos}$. Suppose instead that $x^L < \widetilde{\overline{x_L}}$, and therefore $\tilde{x} < x_L^{pos}$, and conjecture an asymmetric gambling equilibrium. First, notice that $\tilde{x} < x_L^{pos}$ implies that $\tilde{x} < x'$ (since $x_L^{pos} \leq x'$). Thus, $\tilde{x}$ is the policy that maximizes the right-wing party's expected utility from the whole game (i.e., $\tilde{x} = x_R^m$). As such, there can be no asymmetric equilibrium in which $x_1^{R^*} \neq \tilde{x}$. The right-wing party could in fact always find a winning policy closer to $\tilde{x}$ and thus strictly increase its expected payoff. Thus, conjecture an asymmetric gambling equilibrium in which the right-wing party proposes $\tilde{x} = x_R^m$, and the left-wing party commits to a policy x_1^L further from the voter's bliss point. Trivially, the popular party never has any profitable deviation. Consider now the unpopular left-wing party. The conjectured equilibrium exists if and only if

$$E[U_L(2\overline{\alpha}(2\gamma_V - 1) - \tilde{x})] \leq E[U_L(\tilde{x})], \tag{23}$$

where $E[U_L(x)] = -(x - x_L)^2 - (1 - 4\overline{\alpha}\psi x)(\overline{\alpha}(2\gamma_V - 1) - x_L)^2 - 4\overline{\alpha}\psi x(\overline{\alpha} + x_L)^2$.

Recall in fact that L's expected utility is monotonically decreasing on $[2\overline{\alpha}(2\gamma_V - 1) - \tilde{x}, 0]$[5] and monotonically increasing on $[0, \tilde{x}]$. As such, if (23) is satisfied, the unpopular party can do nothing better than allow its opponent to win and implement $\tilde{x}$.

[5]This follows straightforwardly from $\tilde{x} < x_L^{pos}$, and the observation that $|x_L^m| > x_L^{pos}$.

(23) reduces to:

$$-4\overline{\alpha}^2(2\gamma_V-1)^2+4\overline{\alpha}(2\gamma_v-1)(\tilde{x}+x_L)-4\tilde{x}x_L+32\overline{\alpha}^2\psi\gamma_V(\overline{\alpha}(2\gamma_V-1)-\tilde{x})(\overline{\alpha}\gamma_V-\overline{\alpha}-x_L)<0 \quad (24)$$

Recall that $\tilde{x} > \overline{\alpha}(2\gamma_V - 1)$. Further, notice that if the conditions in Proposition 1 are satisfied ($\overline{\alpha}^2 > \frac{1}{8\psi\gamma_V}$), then the LHS is increasing in x_L. Thus, there exists a $\underline{\widetilde{x_L}} \leq 0$ s.t. (26) is satisfied if and only if $x_L < \underline{\widetilde{x_L}}$. Thus, asymmetric gambling equilibria exist if and only if $x_L < x_L^{Asym} = \min\{\underline{\widetilde{x_L}}, \overline{\widetilde{x_L}}\}$.

This concludes the proof of Proposition 2 □

Corollary 1: *Both parties' expected utility in any asymmetric equilibrium is (weakly) higher than in all symmetric equilibria.*

Proof. This follows straightforwardly from two facts: i) no policy to the right of the right-wing party's global optimum (x_R^m) can ever be sustained in equilibrium, and ii) asymmetric equilibria exist if and only if $x_L^{Pos} \geq x_R^m$, where x_L^{Pos} is the left-wing party's local optimum in the positive range. Thus, the left-wing party's utility increases over $[\overline{\alpha}(2\gamma_V - 1), x_R^m]$. This guarantees that (under the parameter values such that asymmetric equilibria exist) the party's utility is (weakly) higher than in every symmetric equilibrium. □

Corollary 2.

- *Suppose $\gamma_V > \frac{1}{2}$ (i.e., the left-wing party is the unpopular one). Then, the left-most platform that can be sustained in a symmetric gambling equilibrium is decreasing in γ_V, and the right-most platform is increasing in γ_V;*

- *Suppose instead that $\gamma_V < \frac{1}{2}$ (i.e., the right-wing party is the unpopular one). Then, the left-most platform that can be sustained in a symmetric gambling equilibrium is increasing in γ_V, and the right-most platform is decreasing in γ_V.*

Proof. Suppose that $\gamma_V > \frac{1}{2}$. From the proof of Proposition 2 and Corollary 1A, we know that $x_L^{Min} = max\{2\overline{\alpha}(2\gamma_V - 1) - x_L^{Pos}, \hat{x}\}$. Suppose that $2\overline{\alpha}(2\gamma_V - 1) - x_L^{Pos} > \hat{x}$. Then, the left-most platform that can be sustained in equilibrium is decreasing in γ_V iff:

$$4\bar{\alpha} + 8\bar{\alpha}^2\psi(x_L + \bar{\alpha}(1-2\gamma_V)) < 0 \tag{25}$$

Which reduces to:

$$x_L < \frac{2\bar{\alpha}^2\psi(2\gamma_V - 1) - 1}{2\bar{\alpha}\psi} \tag{26}$$

From the proof of Proposition 1 we know that gambling equilibria exists if and only if the following condition is satisfied:

$$x_L < \frac{-\bar{\alpha}(2\gamma_V - 1) - 8\bar{\alpha}^3\psi\gamma_V(1-\gamma_V)}{8\bar{\alpha}^2\psi\gamma_V - 1} \tag{27}$$

It is easy to verify that the RHS in condition (27) is strictly smaller than the RHS in (26). As such, (26) is never binding and $x_L > \underline{x_L}$ is sufficient to guarantee that $\frac{\partial x_L^{Min}}{\partial \gamma_V} < 0$.

Suppose instead that $2\overline{\alpha}(2\gamma_V - 1) - x_L^{Pos} < \hat{x}$, where (from Corollary 1A) $\hat{x} \leq 0$ is such that $E[U_L(\hat{x})] = E[U_L(2\overline{\alpha}(2\gamma_V - 1) - \hat{x}]$. Solving for this condition, we get that

$$\hat{x} = \frac{\overline{\alpha}(2\gamma_V - 1)[x_L - \overline{\alpha}(2\gamma_V - 1) - 8\overline{\alpha}^2\psi\gamma_V(x_L + \overline{\alpha}(1-\gamma_V)]}{x_L - \overline{\alpha}(2\gamma_V - 1)}$$

Which can be rewritten as

$$\hat{x} = \frac{\overline{\alpha}(2\gamma_V - 1)[x_L^{Pos} - \overline{\alpha}(2\gamma_V - 1)]}{x_L - \overline{\alpha}(2\gamma_V - 1)}$$

$\hat{x}$ is decreasing in γ_V if and only if

$$2x_L[x_L^{Pos} - \overline{\alpha}(2\gamma_V - 1)] + (2\gamma_V - 1)[\frac{\partial x_L^{Pos}}{\partial \gamma_V} - 2\overline{\alpha}][x_L - \overline{\alpha}(2\gamma_V - 1) < 0$$

Which is always satisfied when (27) holds.

Thus, (under $\gamma_V > \frac{1}{2}$) the *left-most* platform that can be sustained in a symmetric gambling equilibrium is decreasing in γ_V and the right-most is increasing.

The game is fully symmetric, therefore all the results apply in a symmetric way to the case in which $\gamma_V < \frac{1}{2}$. In particular, following the same logic as in Proposition 2, we can verify that the right-most platform that can be sustained in a symmetric gambling equilibrium under $\gamma_V < \frac{1}{2}$ is $x_R^{Max} = min \in \{2\overline{\alpha}(2\gamma_V - 1) - x_R^{Neg}, \widehat{x}^R\}$. Here, $x_R^{Neg} = x_R + 8\overline{\alpha}^2\psi(1-\gamma_V)[\overline{\alpha}\gamma_V - x_R]$ is the right-wing party's local maximum in the negative number, and $\widehat{x}^R = \frac{\overline{\alpha}(2\gamma_V-1)[x_R^{Neg} - \overline{\alpha}(2\gamma_V-1)]}{x_R - \overline{\alpha}(2\gamma_V-1)}$ satisfies $E[U_R(\widehat{x}^R)] = E[U_R(2\overline{\alpha}(2\gamma_V-1) - \widehat{x}^R)]$. Further, following the same steps as in Proposition 1, we obtain that gambling equilibria require $x_R > \frac{8\overline{\alpha}^3\psi\gamma_V(1-\gamma_V)+\overline{\alpha}(1-2\gamma_V)}{8\overline{\alpha}^2\psi(1-\gamma)-1}$. Then, as above, we can show that under this condition we always have $\frac{\partial x_R^{Max}}{\partial \gamma_V} < 0$. Thus, when $\gamma_V < \frac{1}{2}$, the *right-most* platform that can be sustained in a symmetric gambling equilibrium is decreasing in γ_V, and the left-most is increasing.

This concludes the proof.

□

Corollary 2A:

- *Suppose $\gamma_V > \frac{1}{2}$ (i.e., the left-wing party is the unpopular one). Then, the left-most platform that can be sustained in an asymmetric gambling equilibrium is decreasing in γ_V;*
- *Suppose instead that $\gamma_V < \frac{1}{2}$ (i.e., the right-wing party is the unpopular one). Then, the right-most platform that can be sustained in an asymmetric gambling equilibrium is decreasing in γ_V.*

Proof. In an asymmetric gambling equilibrium, the unpopular party can commit to an arbitrarily extreme (and radical) policy. To derive comparative statics, I therefore focus on the focal equilibrium in which both parties commit to their global bliss point. First, suppose $\gamma_V > \frac{1}{2}$. Then, the unpopular left-wing party commits to its global maximum x_L^m, which satisfies:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -(\mathrm{x_1} - \mathrm{x_L})^2 - (1 + 4\overline{\alpha}\psi\mathrm{x_1})(\overline{\alpha}(2\gamma_\mathrm{V} - 1) - \mathrm{x_L})^2 + 4\overline{\alpha}\psi\mathrm{x_1}(\underline{\alpha} - \mathrm{x_L})^2 \\ \text{subject to} \quad & x_1 \in [-\frac{1}{4\overline{\alpha}\psi}, 0] \end{aligned} \tag{28}$$

Notice that when $x_1 < 0$, the probability of an informative outcome is $-4\overline{\alpha}\psi x_1$.

It is easy to verify that $x_L^m = \max \in \{-\frac{1}{4\overline{\alpha}\psi}, \min \in \{0, x_L + 8\overline{\alpha}^2\psi\gamma_V(x_L + \overline{\alpha}(2\gamma_V - 1)\}\}$ is (weakly) decreasing in γ_V. The larger the voter's right-wing bias, the larger L's gain from facilitating voter learning, the more extreme its optimal policy. Similarly, we can verify that under $\gamma_V < \frac{1}{2}$, $x_R^m = \min \in \{\frac{1}{4\overline{\alpha}\psi}, \max \in \{0, x_R - 8\overline{\alpha}^2\psi(1-\gamma_V)(\overline{\alpha}\gamma_V - x_R\}\}$ is (weakly) decreasing in γ_V. Thus, the unpopular party always moves in the opposite direction as the voter.

□

Corollary 3A: *The maximum amount of platform polarization sustainable in a gambling equilibrium is increasing as γ_V moves away from $\frac{1}{2}$.*

Proof. Here, I focus on $\gamma_V > \frac{1}{2}$. The results apply symmetrically to the case in which $\gamma_V < \frac{1}{2}$.

For the case in which $x^L > x_L^{Asym}$, the proof follows straightforwardly from Corollary 2. Suppose instead that $x^L < x_L^{Asym}$, once again focusing on the focal equilibrium in which both parties commit to their global maximum. Here, we must establish that

$$frac\partial x_R^m \partial\gamma_V - \frac{\partial x_L^m}{\partial\gamma_V} > 0 \tag{29}$$

Which reduces to

$$2\overline{\alpha}(2\gamma_V - 1) - x_L - x_R > 0 \tag{30}$$

Given $x^L < x_L^{Asym}$, we know that $x_L^{Pos} > x_R^m$. Further, recall that ideological beliefs imply that $x_R^m \geq x_R$. Therefore, $2\overline{\alpha}(2\gamma_V - 1) - x_L > x_L^{Pos}$ is sufficient to ensure that polarization is increasing in γ_V. It is easy to verify that $2\overline{\alpha}(2\gamma_V - 1) - x_L > x_L^{Pos}$, given the assumption that $-x_L \geq \overline{\alpha}$.

It is worth emphasizing: this Corollary implies that, even though in an asymmetric gambling equilibrium *both* parties may move in the opposite direction as the voter, the unpopular party's platform always shifts more (resulting in an increased polarization). Therefore, Implication 1 in the main body continues to hold (i.e., $\beta_3 < 0$). □

Appendix B: Robustness and Extensions

Parties' Beliefs and Ideology

I have so far assumed that each party assigns probability (arbitrarily close to) 1 to the true state of the world being in line with its own ideology, i.e. each believes information would *always* move the voter's future preferences closer to its own. Here I relax this assumption and show that, while the baseline's results survive under less restrictive conditions, heterogeneous priors are necessary for the existence of gambling equilibria.

Proposition 1 (A). *There exist unique* $\overline{\alpha}^{NMon}$, $x_L^{g^{Bel}}$, $x_R^{g^{Bel}}$ *and* $\tilde{\gamma} < \gamma_V$ *such that gambling equilibria exist if and only if:*

- *Learning the true state has a sufficiently large impact on the voter's preferences:* $\overline{\alpha} > \overline{\alpha}^{NMon}$
- *The parties are sufficiently extreme:* $x_L < x_L^{g^{Bel}}$ *and* $x_R > x_R^{g^{Bel}}$
- *The parties are sufficiently ideological in their beliefs:* $\gamma_L < \tilde{\gamma} < \gamma_R$

Proof. As in Proposition 1, necessary condition for the conjectured equilibria to be sustained is that $x_L^{pos} > \overline{\alpha}(2\gamma_V - 1)$:

$$x_L - 8\overline{\alpha}^2\psi(x^L(\gamma_V - \gamma_L) + \overline{\alpha}\gamma_V(1 - \gamma_V)) > \overline{\alpha}(2\gamma_V - 1) \tag{31}$$

The above can be satisfied only if the LHS is decreasing in x_L. Thus we obtain

$$x^L < \frac{-\overline{\alpha}(2\gamma_V - 1) - -8\overline{\alpha}^3\psi\gamma_V(1 - \gamma_V)}{8\overline{\alpha}^2\psi(\gamma_V - \gamma_L) - 1} \tag{32}$$

$$\gamma_L < \gamma_V - \frac{1}{8\overline{\alpha}^2\psi} \tag{33}$$

and

$$\overline{\alpha}^2 > \frac{1}{8\psi\gamma_V} \tag{34}$$

However this is not sufficient. It is also necessary for the right-wing party's utility to be strictly increasing at $x_1 = \overline{\alpha}(2\gamma_V - 1)$.[6] R's expected utility on $[0, x']$ is:

$$\begin{aligned} E[U_R(x_1)] = &-(x_1 - x_R)^2 - (1 - 4\overline{\alpha}\psi x_1)(\overline{\alpha}(2\gamma_V - 1) - x_R)^2 \\ &-4\overline{\alpha}\psi x_1[\gamma_R(\overline{\alpha} - x_R)^2 + (1 - \gamma_R)(\underline{\alpha} - x_R)^2] \end{aligned}$$

Thus $\frac{\partial E[U_R(x_1)]}{\partial x_1} = -2(x_1 - x_R) + 4\overline{\alpha}\psi(2\overline{\alpha}(2\gamma_V - 1) - x_R)^2 - 4\overline{\alpha}\psi(\gamma_R(\overline{\alpha} - x_R)^2 + (1 - \gamma_R)(\underline{\alpha} - x_R)^2)$. The equilibrium condition is therefore:

$$-\overline{\alpha}(2\gamma_V - 1) + x_R + 8\overline{\alpha}^2\psi(x_R(\gamma_R - \gamma_V) - \overline{\alpha}\gamma_V(1 - \gamma_V)) > 0 \tag{35}$$

Which can be rewritten as:

$$x_R > \frac{\overline{\alpha}(2\gamma_V - 1) + +8\overline{\alpha}^3\psi\gamma_V(1 - \gamma_V)}{8\overline{\alpha}^2\psi(\gamma_R - \gamma_V) + 1} \tag{36}$$

Which requires

$$\gamma_R > \gamma_V - \frac{1}{8\overline{\alpha}^2\psi} \tag{37}$$

□

Notice that $\gamma_V > \tilde{\gamma}$. This implies that gambling equilibria can be sustained when the voter and the right-wing party have exactly the same beliefs ($\gamma_R = \gamma_V$), or when the two parties' priors are arbitrarily close ($\gamma_L = \tilde{\gamma} - \epsilon$ and $\gamma_R = \tilde{\gamma} + \epsilon$, where ϵ takes an arbitrarily small value). However, a disagreement between the voter and the unpopular party is always necessary. In other words, the unpopular party must always hold ideological beliefs. Interestingly, the higher the stakes, the

[6] Notice that, given $x_R > \overline{\alpha}$, this is always true under the assumption that $\gamma_R \approx 1$, which was used to derive Propositions 1 and 2.

smaller the minimum disagreement required to sustain gambling in equilibrium (i.e. $\gamma_V - \gamma$ is decreasing in $\overline{\alpha}$).

These results show that ideological beliefs are a crucial part of the story. Extreme preferences are not enough for an instrumentally rational party to be willing to throw out an election. The party must also be convinced that its ideology is in line with the state of the world. Thus, ideological 'extremism' in both beliefs and policy preferences is necessary for gambling behavior to emerge in equilibrium. However, the analysis also reveals that extreme beliefs may to a certain extent substitute for extreme preferences. Specifically, the following comparative statics hold:

Corollary 4A: *As the parties become more ideological in their beliefs, gambling equilibria can be sustained under more and more moderate policy preferences:* $\frac{\partial x_L^{g^{Bel}}}{\partial \gamma_L} > 0$ *and* $\frac{\partial x_R^{g^{Bel}}}{\partial \gamma_R} < 0$.

The intuition is clear: the more ideological a party is in its beliefs, the more it expects to gain from forcing the voter to experiment. As a consequence, the party is willing to gamble under relatively less extreme policy preferences.

Finally, the following Propositions identifie the range of platforms that can be sustained in a gambling equilibrium

Proposition 2 (**A**). *There exists a unique* $\overline{x_L^{Min}}$ *such that any pair of platforms*

1. *symmetric around the voter* ($x_1^{R^*} - \overline{\alpha}(2\gamma_V - 1) = \overline{\alpha}(2\gamma_V - 1) - x_1^{L^*}$), *and*

2. *such that the left-wing party is (weakly) to the right of* $\overline{x_L^{Min}}$ ($x_1^{L^*} \geq \overline{x_L^{Min}}$)

can be sustained in a gambling equilibrium.

Proof. The proof proceeds exactly as for Proposition 2, and is therefore omitted. □

Proposition 3 (**A**). *Further, there exists a unique* $x_L^{Asym^{Bel}}$ *such that, if* $x_L < x_L^{Asym^{Bel}}$, *then any pair of platforms such that*

1. *the right-wing party commits to its global optimum* ($x_1^{R^*} = x_R^m$), *and*

2. *the left-wing party is strictly farther from the voter* ($x_1^{L^*} < 2\bar{\alpha}(2\gamma_V - 1) - x_R^m$)

can also be sustained in a gambling equilibrium.

Proof. The proof proceeds exactly as for Proposition 3, and is therefore omitted. □

Impatient Parties

In the baseline model I assume that parties are fully patient. In this section I relax this assumption, supposing that parties discount their future payoff by a factor $\delta \in (0, 1]$. I show that the qualitative results from Proposition 1 continue to hold as long as the parties are sufficiently patient.

Proposition 4 (A). *There exist unique* $x_L^{g^{Imp}} \leq x_L^g$ *and* $\overline{\alpha}^{Imp} \geq \overline{\alpha}^{NMon}$ *such that Gambling equilibria exist if and only if:*

- *The unpopular party is sufficiently extreme:* $x_L < x_L^{g^{Imp}}$
- *Learning the true state has a sufficiently large impact on the voter's preferences:* $\overline{\alpha} > \overline{\alpha}^{Imp}$
- *The discount factor is sufficiently high:* $\delta > \frac{1}{2}$

Proof. Recall that (given $\gamma_R = 1 - \epsilon$) the right-wing party's global bliss point is always to the right of the voter. Therefore, as in the baseline model, necessary and sufficient condition for gambling equilibria to exist is that L's expected utility is increasing at $x_1 = \bar{\alpha}(2\gamma_V - 1)$. As in the proof of Proposition 1, we can compute the left-wing party's local maxium in the positive number. Gambling equilibria then require that this point is to the right of the voter's (first period) preferred policy:

$$x_L - \delta 8\bar{\alpha}^2\psi(x_L\gamma_V + \bar{\alpha}\gamma_V(1 - \gamma_V)) > \bar{\alpha}(2\gamma_V - 1) \tag{38}$$

The above can be satisfied if and only if the LHS id decreasing in x_L. Thus, we obtain:

$$x_L < \frac{-\bar{\alpha}(2\gamma_V - 1) - 8\delta\bar{\alpha}^3\psi\gamma_V(1 - \gamma_V)}{8\delta\bar{\alpha}^2\psi\gamma_V - 1} \tag{39}$$

And

$$\bar{\alpha}^2 > \frac{1}{8\delta\psi\gamma_V} \tag{40}$$

Recall that $\overline{\alpha} < \frac{1}{4\overline{\alpha}\psi}$, therefore the above requires $\delta > \frac{1}{2}$.

Finally, notice that $x_L^{g^{Imp}}$ is increasing in δ and $\overline{\alpha}^{nm^{Imp}}$ is decreasing in δ. This implies that $x_L^{g^{Imp}} \leq x_L^g$ and $\overline{\alpha}^{Imp} \geq \overline{\alpha}^{NMon}$, where x_L^g and $\overline{\alpha}^{NMon}$ are the thresholds for the existence of gambling equilibria in the baseline model with fully patient parties. Unsurprisingly, this indicates that gambling equilibria are harder to sustain the less patient the unpopular party is. □

Electoral Volatility

In this section I analyze an extension of the baseline model in which the voter's preferences may be subject to a shock across periods. In particular, the voter's second-period bliss point has two components: a policy one, function of the voter's posterior beliefs about the state of the world, and an ideological one, function of an idiosyncratic shock. Formally, the voter's second-period bliss point is $\overline{\alpha}(2\mu_V - 1) + \nu$,[7] where ν is an ideological shock that realizes and is publicly observed at the beginning of the second period, and is drawn from a continuous distribution with mean ξ and variance σ^2.

The analysis shows that the presence of this shock increases the unpopular left-wing party's incentives to gamble when the expected shock moves the voter's preferences to the right. In contrast, if the shock will (in expectation) move the voter to the left, gambling equilibria become harder to sustain. Formally, the following holds:

Proposition 5 (A). *The likelihood that gambling equilibria exist (in the sense of set inclusion) increases in the magnitude of the expected shock $|\xi|$ when $\xi > 0$, and decreases in $|\xi|$ otherwise.*

Proof. First of all, notice that with $x_R > \overline{\alpha}$ and $\gamma_R = 1 - \epsilon$ the right-wing party's global optimum is always to the right of the voter. Thus, analogously to what established in the baseline model, necessary and sufficient condition for gambling equilibria to exist is that the left-wing party' expected

[7] Recall that μ_V is the voter's posterior that the state of the world takes value $\overline{\alpha}$.

utility is increasing at $x_1 = \overline{\alpha}(2\gamma_V - 1)$. Using the bias-variance decomposition of the quadratic loss function, we can write the left-wing party's expected utility (conditional on $x_1 \geq 0$) as:

$$-(x_1 - x_L)^2 - (1 - 4\overline{\alpha}\psi x_1)(\overline{\alpha}(2\gamma_V - 1) + \xi - x_L)^2 - 4\overline{\alpha}\psi x_1(-\overline{\alpha} + \xi - x_L)^2 - \sigma^2 \tag{41}$$

Thus, necessary and sufficient for gambling equilibria to exist is

$$-2(\overline{\alpha}(2\gamma_V - 1) - x_L) + 4\overline{\alpha}^3\psi(2\gamma_V - 1)^2 - 4\overline{\alpha}^3\psi + 8\overline{\alpha}^3\psi(\xi - x_L)2\gamma_V > 0 \tag{42}$$

This concludes the proof. □

A Longer Time Horizon

In this section I analyze an extended version of the baseline model, whereby the game is repeated for an infinite number of periods and parties discount future payoffs by a factor $\delta \in (0, 1)$. I maintain the assumptions (already adopted in the baseline model) that $x_R > x'$ and $\gamma_L = 1 - \gamma_R = \epsilon$. Furthermore I assume, for purposes of presentation, that $x' < 2\overline{\alpha}(2\gamma_V - 1)$.

Notice that, given the stark learning process in this model, in each period the voter's beliefs are either equal to her prior γ_V (i.e., she observed an uninformative outcome in the previous period), or are degenerate at 0 or 1 (i.e., she observed a fully informative outcome). Denote $\hat{t}$ the first period in which the voter observes an informative outcome. It is easy to see that each period following $\hat{t}$ is equivalent to a one-shot Downsian game. Thus, in equilibrium the parties will always converge on the voter's preferred policy in any period following $\hat{t}$. Restricting attention to equilibria in Markov strategies, any gambling equilibrium would therefore have the following features:

- In each period $t \leq \hat{t}$, the two parties commit to the same pair of policies x_R^* and x_L^* s.t. $x_L^* < \overline{\alpha}(2\gamma_V - 1) < x_R^*$, and the popular party R is elected with probability 1;
- In each period $t > \hat{t}$, the parties converge on the voter's preferred policy.

The following proposition establishes that gambling equilibria exist if and only if the discount factor is sufficiently high, and the unpopular party is sufficiently extreme:

Proposition 6 (**A**). *There exist unique* $\underline{\delta} > 0$ *and* ${x_L}^{g^{Inf}}(\delta) < 0$ *s.t. gambling equilibria exist if and only if* $\delta > \underline{\delta}$ *and* $x_L < {x_L}^{g^{Inf}}(\delta)$.

Proof. First of all, notice that (exactly as in the two-periods baseline) there can be no equilibrium in which $x_R^* > x'$.[8] The left-wing party could in fact propose x' and win, thereby strictly increasing its immediate payoff and leaving the probability of the voter learning the true state (and thus the party's own future expected payoff) unchanged. Additionally, since $\gamma_R = 1 - \epsilon$ and $x_R > x'$, it is easy to see that when $x_R^* \leq \min\{x', \overline{\alpha}\}$ the right-wing party has no profitable deviation from the conjectured equilibrium.[9] Thus, fixing $x_R^* \in (\overline{\alpha}(2\gamma_V - 1), \min\{x', \overline{\alpha}\})$, we only need to verify that the left-wing party has no profitable (single) stage deviation from the conjectured gambling strategy.

In the conjectured equilibrium, the left-wing party's expected discounted utility (as a function of the party's own beliefs over the state of the world) in any subgame starting in period $t \leq \hat{t}$ is

$$E[U_L(x_R^*)] = -(x_R^* - x_L)^2 - \frac{\delta 4\overline{\alpha}\psi x_R^*}{1-\delta}(\underline{\alpha} - x_L)^2 + \delta(1 - 4\overline{\alpha}\psi x_R^*)V_L \tag{43}$$

Where

$$V_L = -(x_R^* - x_L)^2 - \frac{\delta 4\overline{\alpha}\psi x_R^*}{1-\delta}(\underline{\alpha} - x_L)^2 + \delta(1 - 4\overline{\alpha}\psi x_R^*)V_L \tag{44}$$

Thus, the conjectured strategy is robust to single-stage deviations if and only if the left-wing party cannot find a winning policy $x_t < x_R^*$ s.t.

[8] Recall that x' is the smallest positive policy guaranteeing that the resulting outcome is always fully informative.

[9] For simplicity, I restrict attention to equilibria in which the parties adopt symmetric strategies. This is without loss of generality, since the range of (winning) platforms that could be sustained in asymmetric equilibria is a strict subset of those that can be sustained in symmetric ones.

$$-(x_t-x_L)^2-\frac{\delta 4\overline{\alpha}\psi x_t}{1-\delta}(\underline{\alpha}-x_L)^2+\delta(1-4\overline{\alpha}\psi x_t)V_L \geq -(x_R^*-x_L)^2-\frac{\delta 4\overline{\alpha}\psi x_R^*}{1-\delta}(\underline{\alpha}-x_L)^2+\delta(1-4\overline{\alpha}\psi x_R^*)V_L \tag{45}$$

In what follows I will assume, for purposes of presentation, that $x' < 2\overline{\alpha}(2\gamma_v-1)$. This guarantees that the left-wing party can never find a winning platform to the left of zero, therefore we only need to check for possible deviations in the range of positive policies. It is easy to see that the LHS is strictly concave in x_t on $[0, x']$. Therefore, necessary and sufficient condition for for the existence of gambling equilibria is that the we can find a policy $x_R^* \in (\overline{\alpha}(2\gamma_V-1), \min\{x', \overline{\alpha}\}]$ such that the LHS is is increasing in x_t at $x_t = x_R^*$:

$$-2(x_R^*-x_L)-\frac{\delta 4\overline{\alpha}\psi}{1-\delta}(-\overline{\alpha}-x_L)^2-\delta 4\overline{\alpha}\psi V_L \geq 0 \tag{46}$$

Substituting V_L, we obtain

$$-2(x_R^*-x_L)-\frac{\delta 4\overline{\alpha}\psi}{1-\delta}(-\overline{\alpha}-x_L)^2-\delta 4\overline{\alpha}\psi\left[\frac{-(x_R^*-x_L)^2-\frac{\delta}{1-\delta}4\overline{\alpha}\psi x_R^*(-\overline{\alpha}-x_L)^2}{1-\delta(1-4\overline{\alpha}\psi x_R^*)}\right] \geq 0 \tag{47}$$

Which can be rewritten as

$$-2(x_R^*-x_L)+\frac{\delta 4\overline{\alpha}\psi}{1-\delta(1-4\overline{\alpha}\psi x_R^*)}[(x_R^*-x_L)^2-(-\overline{\alpha}-x_L)^2] \geq 0 \tag{48}$$

The condition is never satisfied at $x_L = 0$. Further, it is easy to verify that the LHS is decreasing in x_L if and only if $\delta > \frac{1}{1+4\overline{\alpha}^2\psi}$. Therefore, as long as δ is above this cutoff, there exists a ${x_L}^{g^{Inf}}(\delta) < 0$ s.t. gambling equilibria exist if and only if $x_L < {x_L}^{g^{Inf}}(\delta)$. In particular, ${x_L}^{g^{Inf}}(\delta)$ is such that the condition holds with equality at $x_R^* = \text{argmax}_{x_R^* \in (\overline{\alpha}(2\gamma_V-1), \min\{x', \overline{\alpha}\}]} -2(x_R^*-x_L)+\frac{\delta 4\overline{\alpha}\psi}{1-\delta(1-4\overline{\alpha}\psi x_R^*)}[(x_R^*-x_L)^2-(-\overline{\alpha}-x_L)^2]$. Straightforwardly, the lower δ, the more extreme the party needs to be for gambling equilibria to be sustained. Indeed, as $\delta \to 1$ gambling equilibria exist for all values of

x_L.[10] This stands in contrast with the two-periods baseline, where the condition on x_L is always binding.[11] Finally, notice that, for a sufficiently extreme left-wing party, any pair of symmetric platforms s.t. $x_R^* \in (\overline{\alpha}(2\gamma_V - 1), \min\{x', \overline{\alpha}\}]$ can be sustained in equilibrium.

□

A Look at a Forward Looking Voter

I have so far worked under the assumption that the voter is myopic, and fully discounts the future. While there are substantive reasons to defend such an assumption, it is important to highlight that the results survive with a forward looking, and fully patient, voter. In this section I analyze the model presented above, but allow the voter to have a positive discount factor $\delta_V > 0$.

Proposition 7 (A). *There exist unique $\overline{\alpha}^{NMon}$, $\tilde{\gamma}$, $x_L^{g^\dagger}$ and $x_R^{g^\dagger}$ such that gambling equilibria exist if and only if the following conditions are satisfied:*

- *Learning the true state has a sufficiently large impact on the voter's preferences: $\overline{\alpha} > \widehat{\overline{\alpha}}$*
- *The parties are sufficiently ideological in their beliefs: $\gamma_L < \gamma < \gamma_R$*
- *The parties are sufficiently extreme: $x_L < x_L^{g^\dagger}$ and $x_R > x_R^{g^\dagger}$*

Proof. Analogously to what established in the baseline model, necessary and sufficient condition for gambling equilibria to exist is that the parties' expected utility is increasing at $x_1 = x_V^m$, where x_V^m is the forward looking voter's preferred policy in period one. First of all we must find the voter's optimum x_V^m. This is the policy that solves the following maximization problem:

$$\begin{aligned} \underset{x_1}{\text{maximise}} \quad & -\mathrm{\gamma_V(x_1 - \overline{\alpha})^2 - (1-\gamma_V)(x_1 - \underline{\alpha})^2 - \delta_V(1 - 4\overline{\alpha}\psi x_1)[\gamma_V(\overline{\alpha}(2\gamma_V - 1) - \overline{\alpha})^2 + (1-\gamma_V)(\overline{\alpha}2\gamma_V)^2]} \\ \text{subject to} \quad & x_1 \leq \frac{1}{4\overline{\alpha}\psi} \end{aligned} \tag{49}$$

[10]Recall that throughput the paper I impose $x_L \leq \underline{\alpha} = -\overline{\alpha}$ (this is to ensure that the voter's preferred policy is always between the two parties' static bliss points).

[11]i.e., x_L must be *strictly* lower than $\underline{\alpha}$ for gambling equilibria to exist.

$x_V^m = \min\{\frac{1}{4\alpha\psi}, \overline{\alpha}(2\gamma_V - 1) + 8\delta\overline{\alpha}^3\psi\gamma_V(1-\gamma_V)\}$. Given $\overline{\alpha} < x' = \frac{1}{4\overline{\alpha}\psi}$, $x_V^m = \overline{\alpha}(2\gamma_V - 1) + 8\delta\overline{\alpha}^3\psi\gamma_V(1-\gamma_V)$. Thus, necessary and sufficient conditions for the existence of gambling equilibria are:

$$x_L - 8\overline{\alpha}^2\psi(x^L(\gamma_V - \gamma_L) + \overline{\alpha}\gamma_V(1-\gamma_V)) - \overline{\alpha}(2\gamma_V - 1) - 8\delta\overline{\alpha}^3\psi\gamma_V(1-\gamma_V) > 0 \tag{50}$$

And

$$x_R + 8\overline{\alpha}^2\psi(x_R(\gamma_R - \gamma_V) - \overline{\alpha}\gamma_V(1-\gamma_V)) - \overline{\alpha}(2\gamma_V - 1) - 8\delta_V\overline{\alpha}^3\psi\gamma_V(1-\gamma_V) > 0 \tag{51}$$

These reduce to

$$x^L < \frac{-\overline{\alpha}(2\gamma_V - 1) - (1+\delta_V)8\overline{\alpha}^3\psi\gamma_V(1-\gamma_V)}{8\overline{\alpha}^2\psi(\gamma_V - \gamma_L) - 1} \tag{52}$$

$$\gamma_L < \gamma_V - \frac{1}{8\overline{\alpha}^2\psi} \tag{53}$$

$$\overline{\alpha}^2 > \frac{1}{8\psi\gamma_V} \tag{54}$$

$$x^R > \frac{\overline{\alpha}(2\gamma_V - 1) + (1+\delta_V)8\overline{\alpha}^3\psi\gamma_V(1-\gamma_V)}{8\overline{\alpha}^2\psi(\gamma_R - \gamma_V) + 1} \tag{55}$$

$$\gamma_R > \gamma_V - \frac{1}{8\overline{\alpha}^2\psi} \tag{56}$$

□

Characterizing the full range of platforms that can be sustained in a gambling equilibrium is more challenging than when considering a myopic voter. This is due to the fact that a forward looking voter's expected utility may not be single peaked. Indeed, if the value of information is sufficiently large, the voter's expected utility will have a second (local) maximum in the negative numbers (denoted as x_V^{neg} in Figure 1). Thus, for any platform $x > x_V^m$ there may exist multiple

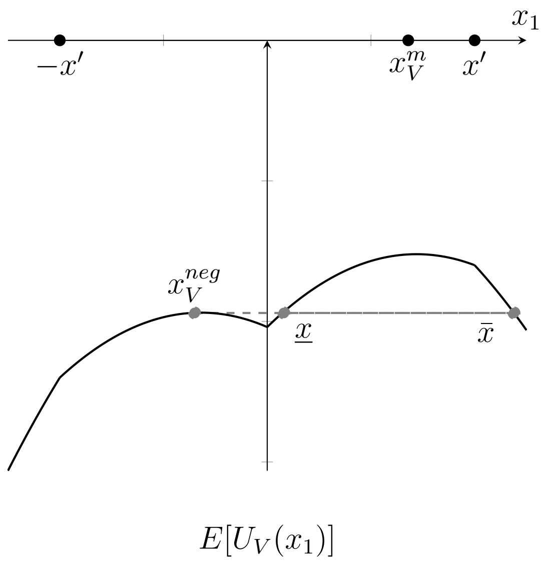

Figure 1: Forward looking voter's expected utility as a function of first-period policy

negative policies that leave the voter weakly better off. This makes it hard to identify pairs of platforms such that the left-wing party has no profitable deviation.

However, there must always exist a range of positive policies that provide the voter with strictly higher utility than x_V^{neg}. In particular, there always exist a pair of policies $\underline{x} \in [0, x_V^m)$ and $\overline{x} > x_V^m$ such that $E[U_V(\underline{x})] = E[U_V(\overline{x})] = E[U_V(x_V^{neg})]$, and $E[U_V(x)] > E[U_V(x_V^{neg})]$ for any $x \in (\underline{x}, \overline{x})$ (see Figure 1). The existence of this range allows us to partially characterize the equilibrium correspondence.

Proposition 8 (A). *Any pair of platforms satisfying:*

1. $E[U_V(x_1^{L^*})] = E[U_V(x_1^{R^*})]$
2. $\underline{x} \leq x_1^{L^*} \leq x_V^m \leq x_1^{R^*} \leq min \in \{\overline{x}, x_L^{pos}, \tilde{x}\}$, *where* $\tilde{x}$ *is the maximum of* R*'s expected utility in the range* $[0, x']$

can be sustained in a a gambling equilibrium.

Proof. The proof proceeds as for Proposition 2 and is therefore omitted. □