Ideological Infection

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Abstract

Many policy problems are inherently dynamic, and outcomes worsen if policy is not adapted to changing circumstances. However, even if everyone agrees on how to address the problem, negotiations do not occur in a vacuum. Thus, ideological conflicts can infect even common-value issues, distorting negotiation dynamics and generating inefficiency. We develop a dynamic bargaining model to study when and how this *ideological infection* emerges. We find that dynamic policy problems are vulnerable to ideological infection precisely because the costs of inaction compound over time. Furthermore, inefficiency is inevitable when players anticipate conflict to rapidly intensify on the ideological dimension. Our findings thus offer a stark warning: even issues with clear common ground may be unable to escape political contagion.

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1 Introduction

Many policy problems are inherently dynamic. A crisis left unaddressed worsens over time, requiring increasingly bold interventions. Infrastructure deteriorates, necessitating greater investment tomorrow. Rapid technological innovations continually reshape society, presenting governments with new and evolving challenges.

Given the growing price of inaction on these issues, one might expect little disagreement among policymakers—at least on the direction policy should move. But policymaking does not occur in a vacuum. Instead, it unfolds in a strategic environment where different issues are often bundled together, linking common-value policies with ideologically divisive ones. Omnibus bills, which package unrelated policies together, are a standard tool in legislative bargaining (Krutz, 2001; Clinton and Lapinski, 2006; Hazama and Iba, 2017; Meßerschmidt, 2021). Similarly, issue linkage is a key tactic in international negotiations (Tollison and Willett, 1979; Keohane, 1984). As a result, policy proposals that would face little opposition in isolation instead often become entangled with more contentious debates.

Bargaining over multiple issues at once can fundamentally alter the ability of negotiating parties to address policy problems. It enables an agenda setter to extract concessions on ideological or contentious policy dimensions by leveraging the urgency of a shared policy problem. In a rapidly evolving world, this creates an incentive to delay striking an efficient agreement today, in order to maximize ideological concessions tomorrow. As a consequence, conflict over an ideological issue may spill over and distort bargaining on an issue where both sides have an interest in cooperation. The risk of this kind of *ideological infection* is not just theoretical. For example, due to the joint interests of the United States and China in addressing climate change, in a 2021 interview China's Foreign Minister Wang Yi described the issue as an "oasis," but went on to state that "surrounding the oasis is a desert, and the oasis could be desertified very soon. China-U.S. climate co-operation cannot be separated from the wider environment of China-U.S. relations." (Stanway, 2021).[1] Likewise, in the context of domestic policymaking in the United States, issue bundling is often blamed for legislative inaction and delays in Congress, with the consequence that 'public policy does not adjust to changing economic and demographic circumstances' (Barber et al. 2015, p. 41).

In this paper, we develop a game-theoretic model to identify the conditions under which an "oasis" turns into a "desert". We find that it is precisely the dynamic nature of the policy problems described above, where the common costs of inaction compound over time, that makes them vulnerable to being infected by other more contentious issues. Furthermore, inefficient outcomes on the common-values policy problem are inevitable when players also anticipate rapidly intensifying disagreements on the ideological dimension. Interestingly, the players delay coming to an efficient agreement today despite knowing their opponent will become more entrenched tomorrow, and thus less willing to yield ideological concessions. In contrast to prior work, we show that inefficiencies can emerge even in policy areas where players know ideological disagreement will never arise, and even if there is no change in proposal power.

In our model, two players repeatedly bargain over both an ideological issue and a common-values issue. The proposer each period offers a policy on each dimension, and the veto player accepts or rejects the entire bundle of policies. On each issue, the policy that is implemented today becomes the status quo tomorrow. To model the dynamic nature of policy issues, we allow the players' preferences on both dimensions to evolve over time. The players' preferences on the common-values dimension are aligned and change in the same way over time. In contrast, on the ideological dimension players disagree on the optimal policy, and this disagreement may grow over time. We further discuss below different interpretations of this preference evolution.

[1] China's vulnerability to climate change has pushed leader Xi Jinping to pledge to peak carbon dioxide emissions before 2030, and achieve carbon neutrality before 2060. For their part, the Biden administration had committed to similarly ambitious goals, pledging to achieve a net-zero emissions economy by 2050.

As highlighted above, the ability to link multiple issues together can alter strategic incentives when bargaining. In a static world, this form of issue linkage generates no detrimental spillovers from the ideological issue to the common-values one. The efficient policy on the dimension of agreement also maximizes the ideological concessions the proposer can obtain. As such, in equilibrium, there is no ideological infection of preferences over the common-values dimension, and hence no impediment to the players adopting the commonly beneficial policy. In a dynamic setting, however, this is not always true. The policy implemented on the common-values dimension today influences the ideological concessions the proposer can extract from the veto player tomorrow. In turn, this generates the potential for players to prefer inefficient policies.

We find that a *necessary* condition for the players' preferences on the common-values dimension to become infected is that the marginal cost of inefficiency on this dimension increases over time. This creates *compounding costs* on the common-values issue, so that any residual inefficiency the players inherit from the past becomes more and more detrimental over time. This condition captures the way in which many policy problems evolve. Underinvestment today becomes even more costly as infrastructure continues deteriorating, or a pandemic that is left unaddressed spreads more and more rapidly.

Increasing marginal costs of inefficiency ensure that, for one or both players, today's price of distorting the common-values policy away from the optimum is smaller than the ideological concessions this residual inefficiency will buy in the future. Absent this compounding, neither player is incentivized to pursue an inefficient policy on the common-values dimension, since any future gain on the ideological dimension is completely offset by the immediate cost of deviating from today's optimal common-values policy. Thus, the equilibrium common-values policy is efficient.

While necessary, increasing marginal costs are not enough to generate inefficiency. The evolution of the ideological dimension are plays a crucial role. To see why, suppose that the conflict on the ideological dimension increases slowly. In this case, the evolution on the common-values dimension (i.e., the worsening of the crisis, the decay in infrastructures) provides the proposer enough leverage to pass his ideal point tomorrow, even if the policy is efficient today. Under certain values of the status quo, this is enough to ensure an efficient policy in equilibrium, even with compounding costs on the common-values dimension.

Conversely, when conflict on the ideological dimension intensifies rapidly, ideological infection of the common-values dimension becomes unavoidable. In this scenario, the proposer prefers to *undershoot* the optimal common-values policy today, e.g., underinvest in infrastructure or only partially address a crisis, to strengthen his bargaining position on the ideological dimension tomorrow. Instead, the veto player prefers to *overshoot* the optimal policy today to constrain the proposer tomorrow, e.g., investing to not only fix but also prevent future infrastructure decay or further crises. As a consequence, the equilibrium common-values policy is always inefficient. Thus, in the presence of compounding costs of inefficiency, rapidly intensifying disagreement on the dimension of conflict is *sufficient* to guarantee that ideological infection emerges.

Finally, we characterize the form the inefficiency takes in equilibrium and the location of the ideological policy. Depending on the environment, inefficiency may manifest as proposer-induced undershooting or veto-induced overshooting. Importantly, we find that, under some conditions, the proposer undershoots the optimal common-values policy *and* chooses an ideological policy more moderate than both its first and second-period ideal points. This last result highlights that the proposer may choose an inefficient policy to preserve leverage for the future even when he still needs leverage today. In this case, a more efficient policy would allow the proposer to reduce costs on the common-values dimension *and* shift today's ideological policy in their preferred direction. However, the compounding costs of inefficiency enable the proposer to better exploit the common-values dimension for ideological concessions in the future.

Our model makes several simplifying assumptions to better illuminate how inefficiency arises in a dynamically changing environment. However, we show that our main results are robust to a number of extensions. First, we find that political turnover can limit the degree of inefficiency by generating uncertainty over who will hold proposal power, but can also create inefficiency when none existed before. Second, our baseline model assumes the players can perfectly predict the evolution of preferences. However, in an extension, we show that just the possibility of rapidly increasing conflict is enough to generate infection with compounding costs on the common-values dimension, even if conflict does not increase in expectation. Our final extension shows that inefficiencies can persist even when bargaining continues over a long time horizon.

1.1 Applications and Implications

We now briefly return to the examples mentioned above where our model may explain how dynamic policy problems become entangled with contentious issues, leading to ideological infection of the common-values issues and policy inefficiencies.

China, the United States, and Climate Change. As mentioned above, US-China climate negotiations during the Biden administration provide an example of ideological infection under evolving policy problems. The warning of China's Foreign Minister underscores that climate cooperation cannot be entirely isolated from other aspects of the bilateral relationship, and suggests that a cooperative approach to climate change might be difficult to sustain if tensions in other areas continue to escalate. Indeed, climate change and the issue of Taiwan are both rapidly evolving. After a period of fragile reconciliation beginning in the late 1980s, tensions between China and Taiwan started intensifying with the election of Tsai Ing-wen, from the traditionally pro-independence Democratic Progressive Party, as Taiwan's first female president in 2016. In this same period, the US has increased its economic relations with Taiwan. These developments have made the issue more salient for both of the bargaining parties. On the other hand, climate-induced disasters have grown increasingly severe and frequent.[2] Thus, even when both countries agree on the urgent need to address an increasingly costly climate crisis, their cooperation on climate policy remains vulnerable because it can be used as leverage to gain concessions on the issue of Taiwan's sovereignty.[3]

Congress, Conflict Expansion, and Ideological Polarization. Two important observations often characterize accounts of American politics. First, political parties are growing increasingly polarized. Looking at the United States Congress, measures of polarization were quite low until the mid 1970s, but have seen a steep increase since that time (Barber et al., 2015). Second, while political conflict between the parties has remained organized along classic dimensions of polarization, other issues 'have been absorbed into it' (Barber et al., 2015, p. 23). Indeed, Lee (2005) describes how partisan divisions now extend to issues such as good government, disaster relief, and transportation programs, areas where we would expect the preferences of 'both parties and all voters [to be] located at a single point' (Stokes 1963, p. 372).[4] As such, political parties appear to be polarized on virtually all policy dimensions, including those with little or no ideological connotation. Consequently, in recent decades we have witnessed a stark decrease in the ability of Congress to legislative efficiently, even on common-values issues (Layman, Carsey and Horowitz, 2006). This case highlights how increasing polarization can generate ideological infection of common-values policy problems, with Congress seemingly unable, or unwilling, to address even the most pressing issues facing the country, instead choosing to 'kick the can down the road (...) and govern by (artificial) crises' (Barber et al. 2015, p. 41).

Broader Implications. These cases highlight the broader implications of our theory for understanding conflict dynamics and political polarization, offering both reason for optimism and cause for concern. On the hopeful side, our findings suggest that polarization does not

[2] https://www.pbs.org/newshour/science/scientists-confirm-global-floods-and-droughts-worsened-by-climate-change.

[3] https://time.com/6295941/us-china-climate-cooperation-challenge/.

[4] The insights of our model would still apply if, for example, there is some disagreement about the optimal degree of disaster relief, but a continuing disaster moves preferences in the same direction.

necessarily stem from deep-seated disagreements but can instead arise from strategic incentives in bundled bargaining. This opens the door for institutional reforms aimed at reducing polarization and improving efficiency. For instance, many countries allow the executive to exercise a line-item veto, rejecting specific provisions of a bill without vetoing it entirely. This mechanism eliminates the proposer's ability to exploit inefficiencies for strategic gain. Notably, the U.S. introduced a line-item veto in 1996 precisely to curb pork-barrel spending, only for the Supreme Court to strike it down as unconstitutional two years later.

Less optimistically, our theory suggests that in an era of rapid change, growing ideological polarization, and intensifying global tensions, the notion that any issue can remain untouched by these forces may be an illusion. Even in the stark case we consider in our model—where no fundamental disagreement exists on the common-values dimension—ideological infection often proves inevitable.

1.2 Contribution to the Literature

Our paper unpacks how the ability to bundle multiple issues together influences bargaining outcomes in a dynamic environment. Other works study the effects of bundling different dimensions, but consider a one-shot interaction or assume bargaining concludes once an agreement is reached (e.g., Fershtman, 1990; Jackson and Moselle, 2002; Chen and Eraslan, 2013; Salam, 2020). Consequently, the inefficiencies we find due to evolving preferences and an endogenous status quo do not arise in these models.

Similar to our paper, Callander and Martin (2017) studies an endogenous status quo bargaining model where policies have an ideological and a (common values) quality component. Quality decays over time but can be restored. However, in Callander and Martin (2017) there is never inefficiency on the equilibrium path of play, i.e., the proposer never underinvests in quality for future leverage. Our analysis highlights two important features of the environment that drive this difference in outcomes. First, we find that the proposer develops a preference for inefficiency only when players anticipate intensifying disagreements on the dimension of conflict in the future. Polarization on the ideological dimension does not change over time in Callander and Martin (2017), and therefore proposer-induced inefficiency cannot arise. Second, in Callander and Martin (2017) parties cannot overinvest in quality today to avoid future decay. Thus, the veto-player induced overshooting inefficiency that emerges in our setting cannot occur in their model.

A small number of other papers have also analyzed how dynamic incentives can lead to inefficient agreements and delay when players bargain over multiple issues. Fox and Polborn (2024) analyze how different veto institutions alter parties' incentives to inefficiently bundle common-values policies with divisive policies. However, in Fox and Polborn (2024) policies do not continue across periods, as such the incentive to maintain leverage for tomorrow, which is crucial in our model, does not exist in their setting.

Incentives to preserve leverage for the future also exist in Acharya and Ortner (2013) and Lee (2020). However, the emergence of inefficiency in these models depends crucially on two assumptions. First, not all issues (or goods) are immediately available for the players to bargain over. Second, the two players place different weights on each issue.[5] Under these assumptions, agreements may be delayed because one player values today's issues less and waits to bundle with his preferred issues when it becomes available in the future. In this sense, in these papers inefficiency emerges when the proposer does not 'need' leverage today. In our model, neither of these features are present, yet parties still agree to inefficient policies. In particular, all issues are available in each period, and equally valued by both players. Rather we assume preferences can evolve over time and inefficiency in multi-issue bargaining arises due to increasing marginal costs on the common-values dimension.[6]

This difference in mechanisms is emphasized by our unique finding that the proposer sometimes pursues an inefficient common-values policy even when doing does not allow him

[5] In Lee (2020), on each issue both players prefer the alternative policy to the status quo, but the payoff they obtain from the alternative is different.

[6] A further difference is that we consider a setting where players can pass policy on the same issue multiple times, while agreement in Acharya and Ortner (2013) and Lee (2020) settles (at least partially) the issue. Additionally, there is no scope for veto-player induced overshooting in either paper. On the other hand, our model abstracts from elections, whereas Lee (2020) explicitly incorporates voters.

to pull the ideological policy at least to his first-period ideal point.[7] Thus, our analysis complements these papers by explaining situations in which issue bundling leads to inefficiency, even when all issues are available to bargain over and doing so is also ideologically costly. From an empirical standpoint, this seems to be the relevant case for the applications we discuss above. For example, the U.S. and China have had the opportunity to bargain over both Taiwan and climate change for many years, and it is unlikely that the U.S. or China is willing to stall an effective environmental agreement because one country has already obtained its current ideal policy on Taiwan (or places very little weight on the issue).

More broadly our research contributes to the extensive political economy literature that explores bargaining with an endogenous status quo (see Eraslan, Evdokimov and Zápal (2022) for a review of this literature). Previous papers in this literature that incorporate multiple policy dimensions have focused on issues of existence (Duggan and Kalandrakis, 2012) and indeterminacy (Anesi and Duggan, 2018) of equilibria. Closer to our work, Penn (2009) allows for multiple dimensions and characterizes how continuing policies can distort preferences. However, proposals are exogenous in her model, which focuses on voting behavior. In contrast, the endogeneity of proposals is a crucial determinant of preferences our model. Chen and Eraslan (2017) also analyzes dynamic bargaining with multiple policy dimensions, but assumes parties can only address one issue at a time. Instead, we specifically focus on the effects of bundling different dimensions.

The mechanism that generates inefficiency in our setting differs from models of endogenous status quo bargaining over one dimension. Previous papers have found inefficiencies that stem from motives such as insurance against turnover (Buisseret and Bernhardt, 2017) or the possibility of developing future conflict on the issue (Riboni and Ruge-Murcia, 2008; Zápal, 2011; Dziuda and Loeper, 2016; Austen-Smith et al., 2019). Instead, in our model, distortions are due to the multidimensionality of the policy space combined with the evolu-

[7]In the Appendix, we formalize the difference between these papers and our mechanism. We shut down compounding costs on the common-values issue but allow the players to weight the dimensions differently. We show that, while the policy outcome can be inefficient, the proposer only ever maintains leverage on the common-values dimension if he also pulls the ideological policy at least as far as his first-period ideal point.

tion of preferences over time. We demonstrate that in a multidimensional world preferences over a common-values issue can become distorted even when the proposer is guaranteed to remain in power indefinitely, and the players are certain they will never disagree on this dimension.[8] As such, the potential for inefficiency to emerge may be even more severe than previously shown.

Finally, our work is also related to studies that analyze multidimensional bargaining where players can make transfers to each other (e.g., Austen-Smith and Banks, 1988; Diermeier and Merlo, 2000). Similar to a transfer, our proposer can use the common-values dimension to obtain favorable policy on the partisan dimension. However, the common-values dimension in our model differs from a transfer because both players benefit from moving policy to the common-values ideal point, therefore inefficiency is costly for the proposer as well.

2 The Model

Players and policies. There are two players, a proposer (P) and veto (V), who interact over two periods, $t \in \{1, 2\}$. The policy space is composed of an ideological dimension $X = \mathbb{R}$ and a common-values dimension $Y = \mathbb{R}$. In every period t, the players bargain to determine a policy outcome $(x_t, y_t) \in X \times Y = \mathbb{R}^2$.

Preferences. The stage utility to player $i \in \{P, V\}$ in period t from a policy outcome (x, y) is $u_{it}(x) + v_t(y)$, where we define $u_{it}(x) = u(x - \hat{x}_t^i)$ and $v_t(y) = v(y - \hat{y}_t)$. We assume u and v are twice differentiable with continuous second derivatives, that u is strictly concave, and that both functions are single peaked at 0. Thus, player i's statically optimal policy in period t is given by its ideal point $(\hat{x}_t^i, \hat{y}_t) \in \mathbb{R}^2$. Additionally, let $u_{it}(\hat{x}_t^i) = v_t(\hat{y}_t) = 0$. The standard quadratic loss function $-(x - \hat{x}_t^i)^2$, for example, satisfies these conditions. In each period,

[8] This distinguishes our work from papers that find strategic polarization on a single dimension that can exhibit conflict (Dziuda and Loeper, 2018), or that study how the potential for future changes in polarization impact the selection of procedural rules (Diermeier, Prato and Vlaicu, 2020).

player P's preferred ideological policy is to the right of player V, $\hat{x}_t^V < \hat{x}_t^P$, and we allow conflict on the ideological dimension to (weakly) increase over time, $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$. To reduce the number of cases, on the common-values issue we assume the shared ideal policy (weakly) increases over time, $\hat{y}_1 \leq \hat{y}_2$.

In our baseline model, the sequence of ideal policies is common knowledge. Thus, the evolution of preferences is deterministic and the parties in our baseline model face no uncertainty. We later discuss the robustness of our results to relaxing this perfect foresight.

Player i's payoff in the dynamic game is given by:

$$\sum_{t \in \{1,2\}} u_{it}(x_t) + v_t(y_t),$$

where for simplicity we assume no discounting.

Political environment. At the start of each period $t \in \{1, 2\}$ player P makes a proposal $(x_t, y_t) \in \mathbb{R}^2$, which consists of a policy on the ideological issue, $x_t \in X$, and a policy on the common-values issue, $y_t \in Y$. Next, player V decides whether to accept or reject the proposal. If the proposal is accepted then the policy outcome in period t is (x_t, y_t). If the proposal is rejected then the policy outcome in period t is $(x_t^q, y_t^q) \in \mathbb{R}^2$, where (x_t^q, y_t^q) is the status quo in period t. Thus, proposals on the two dimensions are bundled together.

The policy outcome in the current period becomes the status quo in the subsequent period. Thus, if (x_1, y_1) is the policy outcome in period 1 then the status quo in period 2 is $(x_2^q, y_2^q) = (x_1, y_1)$. The status quo at the beginning of the game is exogenously set at $(x_1^q, y_1^q) \in \mathbb{R}^2$.

A definition of efficiency. In our analysis below, we will use the following terminology:

Definition 1. *A policy outcome (x_t, y_t) in period t is efficient if it sets the common-values policy at $y_t = \hat{y}_t$. Otherwise, a policy (x_t, y_t) is inefficient.*

According to our definition, an inefficient outcome is always Pareto inefficient as well (both statically and dynamically). Specifically, static Pareto efficiency requires $(x_t, y_t) \in [\hat{x}_t^V, \hat{x}_t^P] \times \{\hat{y}_t\}$, i.e., efficiency on Y *and* policy in the gridlock interval on X. Given our focus on understanding when players cannot agree on the common-values issue, our definition sidesteps that even with efficiency on the common-values dimension the outcome may be Pareto inefficient if the ideological policy x_t is not in the interval $[\hat{x}_t^V, \hat{x}_t^P]$.

Discussion of the model. In our baseline model there is no turnover in proposers, no uncertainty about changes in ideal points, and no asymmetry across players' utility functions (besides ideal points). These assumptions allow us to isolate the mechanism that drives our results, while shutting down features that have previously been shown to cause inefficiency. Additionally, we consider a two-period model in order to obtain sharper results. However, none of these assumptions undermine the core mechanism of our model. We later consider extensions relaxing each of these assumptions.

In order to more clearly illustrate our results, we assume the players share exactly the same ideal policy on the Y dimension. However, our intuitions apply broadly to cases in which players face some disagreement on this dimension, but the status quo is outside of the gridlock interval and the players' ideal points shift in the same direction over time. In other words, the crucial feature of the Y dimension is that players always agree on the optimal direction of policy change. For example, even if the US and China disagree on who should shoulder more of the burden for addressing climate change, they may agree that more needs to be done as the situation worsens.

In contrast, the assumption that $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$ ensures that the X dimension *always* features conflict and thus distinguishes it from a common-values dimension.[9] Absent this assumption, a policy that is in the gridlock interval on X in the first period can become

[9] Likewise, assuming u is concave ensures that the players become more resistant to changes on the X dimension when their ideal points move apart, capturing the idea of increasing conflict. We further discuss this in the Appendix.

unstuck, even without the Y dimension, if the ideal points of both players move closer together or shift in the same direction in the second period. Indeed, our analysis highlights that in a dynamic setting distinguishing policy issues that feature conflict from those with common values depends both on the current location of ideal points and how these ideal points change over time. Figure 1 depicts an example of this evolution.

To clearly illuminate our mechanism we limit the number of degrees of freedom by fixing the shapes of v and u across periods, and only allowing the ideal points to shift. However, under relatively mild assumptions, which ensure that players still become more entrenched on the X dimension between periods, our results are robust even if we allow these shapes to change as well. Thus, the insights of our model can apply to a broad number of ways in which preferences may change. In the Appendix we prove our results while allowing for v and u change over time.

In order to sharpen the intuition behind our results, players in our baseline model can perfectly anticipate how their own ideal points will change in the future. This perfect foresight, however, is not essential to our logic. As we discuss in Section 4, similar incentives exist when the players face uncertainty over the evolution of optimal policies and even if, in expectation, conflict on the ideological dimension remains unchanged.

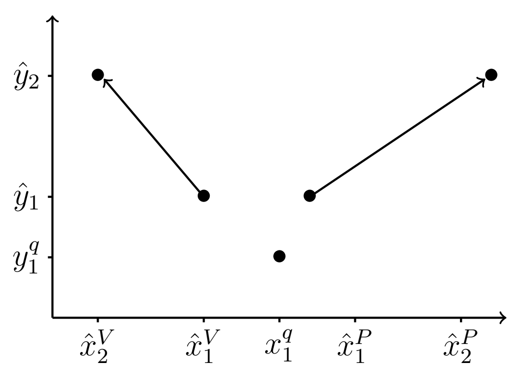

Figure 1: Example of increasing polarization in the evolution of preferences.

Examples of changing preferences. Thus, the two key features of our model are the multidimensionality of the policy space and *the possibility* of changing preferences on these dimensions. We now give some brief examples which microfound how preferences can shift over each policy dimension.

First, consider the evolution of preference over the common-values dimension. One direct interpretation is that ideal points may shift due to changes in the environment, e.g., $v_t(y) = -(y - \theta_t)^2$. For example, the players may anticipate that innovations will make green technologies cheaper, increasing the optimal amount of green energy sources used in the future compared to today.

More generally, changing preferences on Y can capture evolving circumstances that alter the optimal policy intervention, for example due to a worsening crisis or deteriorating infrastructure.[10] In each of these cases, the statically optimal policy changes from one period to the next.[11] To fix ideas, interpret $\hat{y}_t$ as the statically optimal amount of investment needed to fix infrastructure today or address the ongoing crisis today. Even with investments today, infrastructure may continue to deteriorate or the crisis may persist (or a new one emerge). As such, the marginal benefit from investing more resources increases as the problem worsens. In turn, the statically optimal level of investment increases over time.[12]

Next, consider the evolution of the conflict on the X dimension. In general, bargaining players can anticipate changes in their future preferences as a result of the evolution of a policy-relevant state of the world. Suppose that player i's preferences in period t are given by $u_i(x - \beta_i \times \theta_t)$, where $\theta_t \geq 0$, $\beta_i > 0$ and $\beta_{-i} < 0$. Here, the two players agree on the fundamental state of the world θ_t, but disagree on the implications for optimal policies. Then, our assumption of weakly increasing conflict reflects a situation where players anticipate θ_t increasing over time, increasing the distance in their respective induced ideal points. For

[10]Important for our framework is that policymakers can invest to not only correct, but also possibly prevent, this decay.

[11]In the Appendix we also allow the shape of v to change over time, which further captures how the payoffs from not adjusting policy can change over time. In this case, infection can still emerge if $\hat{y}$ is fixed but preferences intensify over time, e.g., $\theta_t v(y)$ with $\theta_2 > \theta_1$.

[12]Depending on the application, it may be more accurate to model a crisis as shifting the status quo, as in Callander and Martin (2017), rather than preferences. However, in either case, the crucial feature for our results is that the marginal cost of inefficiency on Y increases over time. As such, we opt for the more streamlined setting where preferences can change on both dimension.

example, this can capture the observation that globalization has contributed to an increase in polarization, with an intensifying conflict between left- and right-wing parties on issues such as immigration or economic protectionism.[13]

Alternatively, the assumption of increasing conflict can capture that the players foresee the salience of the divisive issue to increase. For example, consider two countries bargaining over a territory of size 1 in each period. Initially, the payoff to controlling a share $x \in [0, 1]$ of the territory is $u(x)$. However, the countries expect the territory to become strategically more important or more valuable in the future, and thus the payoff from controlling share x in the second period increases to $\theta u(x)$, with $\theta > 1$. In this case, countries prefer to control the entire territory in both periods, but the increased value from controlling any share $x \in [0, 1]$ makes each country more sensitive to losing territory. In this example, the players' ideal point remains fixed, but their preferences intensify. Notice that this is a slightly different operationalization of increasing conflict, but it is qualitatively equivalent to the baseline presented above, as we demonstrate in Section 4.

Within the context of legislative bargaining, shifting ideal points can also model (in reduced form) situations in which parties expect increasing polarization among their respective constituents or members, continuing the trend of previous decades. In this view, the party leadership acts as a delegate of the members. As a result, if the party anticipates an evolution in the preferences of its base, it will act *as if* it expects its own preferences to change. As long as the party leadership aims at maximizing the long-run welfare of the party, it will face the dynamic incentives we describe in the model (this is true even if the members themselves do not foresee the preference evolution).

[13] See Rodrik (2021) for a review of the literature on this topic, discussing how globalization and the resulting labor market shocks can increase polarization by driving a 'greater wedge between winners and losers' of this process (p. 164).

3 Analysis

Moving to the analysis, our solution concept is subgame perfect equilibrium and we proceed by backwards induction. In the second period, players only consider their static payoffs. Thus, V accepts any policy (x, y) such that:

$$u_{V2}(x) + v_2(y) \geq u_{V2}(x_2^q) + v_2(y_2^q). \tag{1}$$

V is only willing to accept an ideological policy that moves farther away from its ideal point on X if the proposal improves on the common-values status quo. Consequently, if the inherited status quo is inefficient, $y_2^q \neq \hat{y}_2$, then P can extract concessions on the conflict dimension by proposing a bundle that moves the common-values policy closer to $\hat{y}_2$.

In equilibrium, P chooses its proposal to maximize $u_{P2}(x) + v_2(y)$ subject to (1). Let $\overline{x}(x_2^q, y_2^q)$ be the upper solution to:

$$u_{V2}(x) + v_2(\hat{y}_2) = u_{V2}(x_2^q) + v_2(y_2^q). \tag{2}$$

Lemma 1 characterizes P's optimal second-period proposal.

Lemma 1. *In the second period P proposes $y_2^* = \hat{y}_2$ and $x_2^*(x_2^q, y_2^q) = \min\left\{\hat{x}_2^P, \overline{x}(x_2^q, y_2^q)\right\}$.*

Proposing $y = \hat{y}_2$ maximizes V's utility from the offer on the common-values dimension, and thus maximizes V's willingness to accept a worse payoff on the ideological dimension. As such, the efficient policy $y = \hat{y}_2$ both maximizes P's payoff on the common-values dimension *and* the extent to which P can move the outcome towards its ideal policy $\hat{x}_2^P$. Therefore, the equilibrium policy outcome is always efficient, emphasizing that the ability to bundle dimensions does not lead to inefficiency absent dynamic motives.

Lemma 1 emphasizes that the second-period equilibrium outcome depends on the policy implemented in the previous period. Turning to the first period, then, the players mus balance their static preferences against their dynamic incentives. The players' dynamically optimal policies induced by these strategic considerations are central to our concept of ideological infection. It is therefore useful to introduce the following definition of a player's dynamic ideal point.

Definition 2. *Player i's dynamic ideal point $(\hat{x}_d^i, \hat{y}_d^i)$ solves*

$$\max_{x,y} u_{i1}(x) + v_1(y) + u_{i2}\big(x_2^*(x,y)\big).$$

Thus player i's dynamic ideal point is the policy that i would choose to implement today, anticipating bargaining tomorrow.

Lemma 2 provides an initial characterization of these dynamic ideal points.

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y}_1 \leq \hat{y}_d^V$. On the X dimension $\hat{x}_2^V \leq \hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{x}_1^P \leq \hat{x}_d^P \leq \hat{x}_2^P$.*

Although both players prefer the efficient common-values policy today, their incentives to influence future policy outcomes can lead to divergent dynamic preferences. From equation (1), we see that moving y_2^q further from $\hat{y}_2$ increases $\overline{x}(x_2^q, y_2^q)$, i.e., it increases the proposer's second-period leverage. As such, the proposer's second-period equilibrium payoff increases when the inherited common-values policy is further from $\hat{y}_2$, as shown in Figure 2. Conversely, the veto player's second-period payoff increases when y_1 moves closer to $\hat{y}_2$, as this limits the proposer's leverage in the second period. Consequently, P prefers a policy that weakly *undershoots* the efficient common-values policy, $\hat{y}_d^P \leq \hat{y}_1 < \hat{y}_2$, while V prefers a policy that weakly *overshoots* it, $\hat{y}_1 \leq \hat{y}_d^V < \hat{y}_2$. As we will show, under some conditions these inequalities hold strictly, leading to ideological infection.

Notice that each player's dynamic preferences on the conflict dimension can also be distorted from its static ideal policy. For player i a policy closer to $\hat{x}_2^i$ improves its equilibrium policy payoff tomorrow and worsens the other player's. As such, each player's dynamic ideal point lies in between its first and second-period optimum.

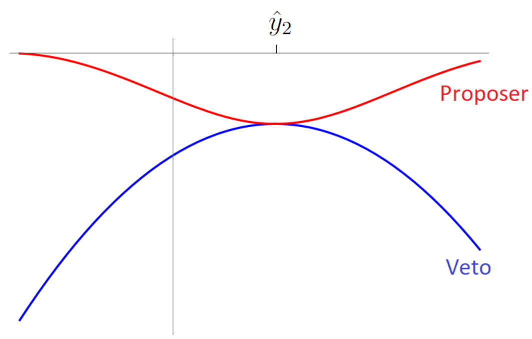

Figure 2: Second-period equilibrium payoffs as a function of y_2^q, if players have quadratic utility on both dimensions.

It is important to note that this distortion of preferences on X would emerge even in a model without the Y dimension. Absent the Y dimension, policy is always stuck (in the gridlock interval) and P's dynamic ideal point is again in $(\hat{x}_1^P, \hat{x}_2^P)$, as this balances its payoff from today versus tomorrow. In contrast, in a model with only the Y dimension there is no difficulty in agreeing to the efficient policy today and tomorrow. Consequently, any preference divergence on the common-values dimension is due solely to the existence of multiple dimensions.

We now formally introduce the concept of ideological infection.

Definition 3. *If $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$ there is no ideological infection. Otherwise, if $\hat{y}_d^P \neq \hat{y}_d^V$ then there is ideological infection. In particular, if $\hat{y}_d^i \neq \hat{y}_1$ then i's preferences are infected.*

Ideological infection emerges when one (or both) players prefer an inefficient common-values policy in the first period. In the subsequent sections, we unpack the conditions under which ideological infection occurs, when infection leads to inefficient policy outcomes, and the form of inefficiency that emerges.

3.1 The Role of Compounding Costs of Inefficiency

We first provide a necessary condition for ideological infection to emerge. Although players have dynamic incentives to distort policy in the first period, any inefficiency exploited to gain an advantage tomorrow imposes a cost on both players today. Thus, the players' dynamic incentives do not necessarily lead to infection of preferences on the common-values dimension. We show that the key feature is whether the gains from moving policy to be more efficient are relatively greater in the second period or the first. Specifically, whether ideological infection can occur in equilibrium depends on the marginal cost of an inefficient policy in the first period versus the second period. We describe this condition with the following definition:

Definition 4. *The costs of inefficiency are compounding over time if the following condition holds:*

$$v_2'(y) > v_1'(y) \text{ for } y \leq \hat{y}_1. \tag{3}$$

Instead, the costs of inefficiency reduce over time if:

$$v_2'(y) \leq v_1'(y) \text{ for } y \leq \hat{y}_1.$$

Notice that if the costs of inefficiency reduce over time then it must be the case that $\hat{y}_1 = \hat{y}_2$, as otherwise $v_2'(\hat{y}_1) > 0 = v_1'(\hat{y}_1)$. Instead, if (3) holds then we must have $\hat{y}_1 < \hat{y}_2$. Thus, in our baseline model, the costs of inefficiency compounding is equivalent to movement in the common-values ideal point.[14]

To see the importance of compounding costs for infection, first consider P's preferences and suppose v is concave. Figure 3 illustrates such a case. Recall that an inefficient policy that undershoots $\hat{y}_1$ imposes costs on both parties in the first period and increases the cost to the veto player for maintaining the status quo in the second period. In turn, this allows P to pull x_2 closer to $\hat{x}_2^P$. Thus, ideological infection of the proposer's preferences requires that the anticipated increase in ideological concessions from the veto player in the second

[14] We adopt the definition of compounding in terms of condition (3), rather than the change in $\hat{y}_t$, because our analysis in the Appendix allows the shape of v to change over time. Under this generalization, the two concepts are no longer equivalent and it becomes clear that increasing marginal costs of inefficiency is the crucial necessary condition for ideological infection.

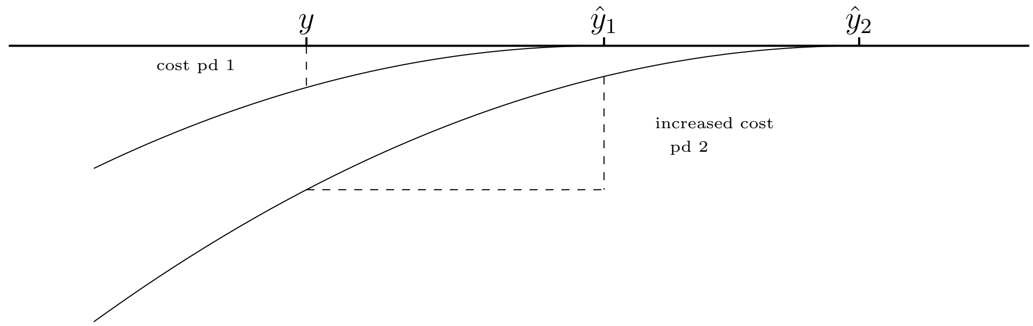

Figure 3: Common-values utility in first and second period.

period is higher than the immediate cost.

Under concavity, compounding costs of inefficiency imply that:

$$|v_2(y) - v_2(\hat{y}_1)| > |v_1(y) - v_1(\hat{y}_1)| \tag{4}$$

for any $y < \hat{y}_1$. In turn, inequality (4) indicates that the increased second-period cost on the veto player is greater than the cost that *both* parties pay for inefficiency in the first period (as shown in Figure 3). This wedge in the cost of residual inefficiency today and tomorrow creates the possibility for P to benefit from a policy that undershoots the static optimum on Y. If instead $v_2'(y) \leq v_1'(y)$ for $y < \hat{y}_1$, then tomorrow's ideological gains are always lower than today's cost of inefficiency, and the proposer's preferences are not infected.

A similar logic explains V's dynamic preferences. If the costs of inefficiency reduce over time, then $\hat{y}_1 = \hat{y}_2$, thus, the efficient first-period policy is also the policy that minimizes the proposer's leverage in the second period. Consequently, the efficient policy is dynamically optimal for the veto player. As such, $\frac{v_2'(y)}{v_1'(y)} \leq 1$ immediately removes any distortion of V's preferences. If instead $\frac{v_2'(y)}{v_1'(y)} > 1$ then the wedge in the cost of residual inefficiency creates the possibility that V benefits from overshooting $\hat{y}_1$, and therefore V's preferences may be infected.

Building on this discussion, Proposition 1 shows that reducing costs of inefficiency over time eliminates ideological infection.

Proposition 1. *If the costs of inefficiency reduce over time then there is no ideological infection,* $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$. *Furthermore, the equilibrium policy outcome is efficient,* $y_1^* = \hat{y}_1$.

Absent ideological infection neither player has a preference for inefficiency, and the players have no difficulty coming to an efficient agreement. Thus, on issues for which the cost of inefficiency reduces over time, efficiency prevails.

3.2 The Role of Increasing Ideological Conflict

Our previous results establish that ideological infection never emerges when the costs of inefficiency reduce over time, and thus some degree of compounding is necessary. We now complete the analysis by showing that whether infection actually emerges in equilibrium also depends on how preferences evolve on the X dimension. Specifically, we find that, with compounding costs of inefficiency, a sufficient condition for infection is rapidly increasing conflict on the ideological dimension.

Throughout this section we maintain the assumption that the costs of inefficiency are compounding over time. Recall that this implies $\hat{y}_1 < \hat{y}_2$.

Assumption 1. *The costs of inefficiency are compounding over time:*

$$v_2'(y) > v_1'(y) \text{ for all } y \leq \hat{y}_1. \tag{5}$$

To characterize the conditions under which each player's preferences are infected we first define two cut-points:

$$\overline{u}_P \equiv u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1), \tag{6}$$

$$\overline{u}_V \equiv u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V), \tag{7}$$

where $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ characterizes V's dynamic ideal point in the case where P does not obtain $\hat{x}_2^P$ in the second period. Proposition 2 now characterizes when each player's preferences are infected.

Proposition 2.

1. *V's preferences are infected if and only if* $u_{V2}(\hat{x}_2^P) < \overline{u}_V$*; and*

2. *P's preferences are infected if and only if* $u_{V2}(\hat{x}_2^P) < \overline{u}_P$.

Furthermore, $\overline{u}_P < \overline{u}_V$.

Whether compounding costs of inefficiency generate infection depends on the anticipated degree of conflict in the future, characterized by $u_{V2}(\hat{x}_2^P)$. To see why, consider the condition for P's preferences to *not* be infected:

$$u_{V2}(\hat{x}_2^P) + v_2(\hat{y}_2) \geq \overline{u}_P = u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) \tag{8}$$

In this case, the increase in conflict on the ideological dimension is low relative to the change on the common-values dimension. Specifically, if condition (8) is satisfied then P has enough leverage in the second period to pass its ideal point, even if the status quo is at the first-period efficient policy $\hat{y}_1$. As such, P has no incentive to undershoot on the common-values dimension, or implement an extreme policy on the ideological dimension. In contrast, if the players anticipate significant conflict in the second period, $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, then P does not have enough leverage to get its preferred policy when the inherited status quo is $(\hat{x}_1^P, \hat{y}_1)$. As a consequence, P's dynamically optimal policy undershoots the efficient $\hat{y}_1$.

Notice that we can rewrite condition (8) as $u_{V2}(\hat{x}_1^P) - u_{V2}(\hat{x}_2^P) \leq v_2(\hat{y}_2) - v_2(\hat{y}_1)$. Thus, under this condition, the conflict on the ideological dimension increases slowly relative to the evolution of the common-values issue. Furthermore, if the players' ideal points on X are the same in period 2 and in period 1 then $u_{V2}(\hat{x}_1^P) = u_{V2}(\hat{x}_2^P)$ and condition (8) always holds. Consequently, infection of the proposer's preferences requires conflict on the ideological dimension to increase over time. Furthermore, this increase needs to be sufficiently rapid relative to the evolution of the common-values dimension.

A similar calculation determines whether the veto's preferences are infected. If there is sufficiently little disagreement in the second period, then the proposer can obtain $\hat{x}_2^P$ even if the first-period policy overshoots $\hat{y}_1$. In turn, inefficiency does not constrain P in the second period, and V's optimal first-period policy is $(\hat{x}_1^V, \hat{y}_1)$. Additionally, because P holds the bargaining power, the anticipated amount of conflict needed to induce infection in V's preferences is lower than the amount needed to infect V, $\overline{u}_V < \overline{u}_P$. Specifically, it is easier for P to move policy to $(\hat{x}_2^P, \hat{y}_1)$ from $(\hat{x}_1^P, \hat{y}_1)$ than from $(\hat{x}_1^V, \hat{y}_1)$.

We note that, if $\hat{y}_2 - \hat{y}_1$ is not too large, then $u_{V2}(\hat{x}_1^P) < \overline{u}_V$. Therefore, infection of V's preferences can emerge even if there is no change on the ideological dimension across periods. However, if $\hat{y}_2 - \hat{y}_1$ is sufficiently large, then infection of the veto player's preferences also requires rapidly increasing disagreement on the dimension of conflict.

The above discussion highlights that a significant increase in the intensity of the ideological conflict from the first to the second period is necessary and sufficient to ensure that *both* parties' preferences are infected (illustrated in Figure 4). As Proposition 3 now establishes, this makes an inefficient equilibrium policy inevitable.

Let $U_q = u_{V1}(x_1^q) + v_1(y_1^q) + u\big(x_2^*(x_1^q, y_1^q)\big)$ denote V's dynamic equilibrium payoff from keeping the status quo. The first-period equilibrium policy then maximizes the proposer's dynamic utility, subject to the constraint that the veto player's dynamic utility is no less than U_q.

Proposition 3.

1. *Assume V's preferences are infected but P's preferences are not. If $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ then the equilibrium policy is efficient. Otherwise, the equilibrium policy is inefficient for almost all $(U_q, \hat{x}_1^V)$.*

2. *If both players' preferences are infected then the equilibrium policy is inefficient for*

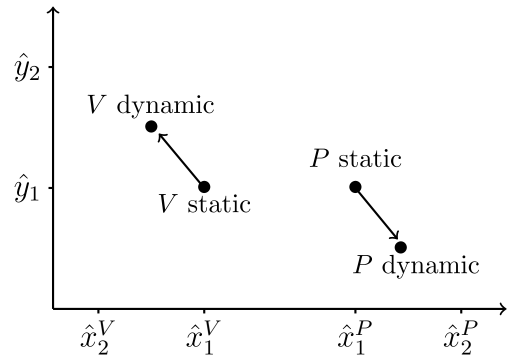

Figure 4: Dynamic ideal points under rapidly increasing polarization.

almost all $(U_q, \hat{x}_1^V)$.

If only the veto player's preferences are infected, i.e., $\overline{u}_P < u_{V2}(\hat{x}_2^P) < \overline{u}_V$, then the equilibrium policy on the common-values dimension may still be efficient. When the initial status quo U_q is bad for the veto player, the proposer can pass its optimal bundle. In particular, following our previous discussion, P can obtain its statically optimal bundle in both periods and the equilibrium is efficient. Otherwise, if U_q is high, then the proposer is constrained in the first period. Thus, to appease the veto player, P proposes an inefficient policy, even though P's own preferences are not infected.

Instead, if the preferences of both players are infected, which occurs when $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, then inefficiency is inevitable. The efficient policy always leaves P with too little or too much leverage in the future. Thus, by Proposition 2, we should expect inefficiency to be most pervasive when players anticipate the conflict to intensify rapidly.

Having established the conditions for the emergence of inefficiency, we conclude this section by analyzing the nature this inefficiency takes in equilibrium and how it influences the policy on the ideological dimension. In particular, the value of U_q is crucial in determining the policy outcome. Suppose that $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, so the policy is (almost) always inefficient.

When U_q is very low, the status quo is highly favorable to the proposer. Consequently, P can pass a policy close to its unconstrained optimum; hence, $y_1^* < \hat{y}_1$ and $x_1^* > \hat{x}_1$, aligning with the proposer's dynamic preferences. A similar symmetric logic holds when U_q is very high. The status quo is very favorable for the veto player, and therefore P has little leverage to pull the conflict-dimension policy close to its first-period ideal. Thus, P needs to propose a policy close to V's dynamic ideal point, binding himself in the future to obtain larger concessions today. The equilibrium in this case is characterized by a veto-player-induced inefficiency. The common-values policy overshoots the first-period ideal, $y_1^* \in (\hat{y}_1, \hat{y}_d^V)$, and the conflict policy remains below P's ideal point, $x_1^* < \hat{x}_1^P$.

More interesting is the case where neither player is initially strongly advantaged by the status quo, i.e., when U_q is intermediate. Intuition may suggest that undershooting on the common-value dimension should only emerge when the proposer does not need leverage in the first period. That is, when P can pass $\hat{x}_1^P$ without moving the Y dimension status quo all the way to the efficient $\hat{y}_1$, and thus chooses to maintain some inefficiency to increase leverage for the future. Our next result shows that this is not always the case in our setting:

Proposition 4. *Assume $u_{V2}(\hat{x}_2^P) < \overline{u}_P$. There exists an open interval $(\underline{U}_q, \overline{U}_q)$, such that, if $U_q \in (\underline{U}_q, \overline{U}_q)$ then $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$.*

When V's payoff from the status quo is intermediate the proposer undershoots on the common-values dimension, $y_1^* < \hat{y}_1$, *and* proposes an ideological policy to the left of both its first- and second-period ideal points, $x_1^* < \hat{x}_1^P$. In this case, the proposer *could* move policy closer to $\hat{x}_1^P$ — potentially even obtaining its first-period ideologically preferred policy. However, doing so requires satisfying the veto player by moving the common-values policy closer to $\hat{y}_2$, reducing P's future leverage. In equilibrium, undershooting occurs despite the proposer needing more leverage today. The reason P is willing to forgo gains today is precisely because there are compounding costs of inefficiency on the Y dimension: the proposer can buy *even more* concessions tomorrow than it can today by implementing the efficient $\hat{y}_1$ and moving x further right. Thus, P chooses to incur immediate costs on the common-values dimension *and* forgoes gains on the ideological dimension.

Figure 5 provides an illustration of the first-period equilibrium under the assumption that the players' have quadratic-loss preferences on each dimension in each period. Under

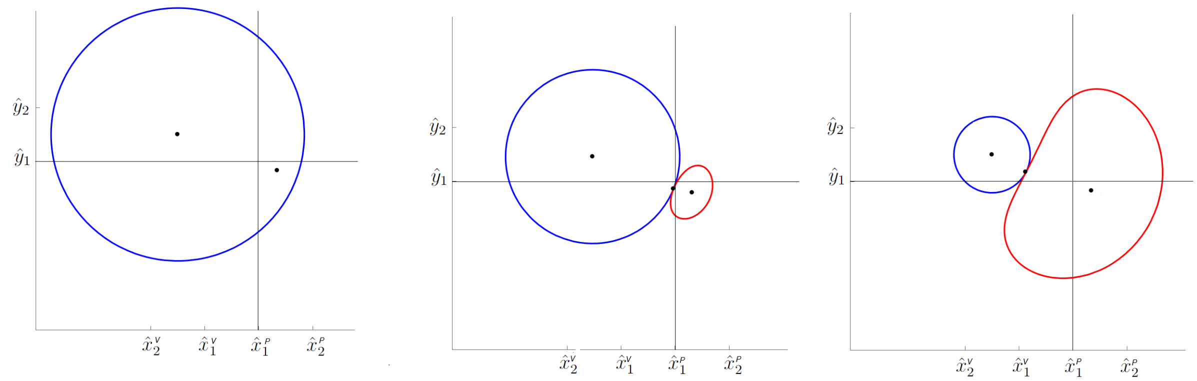

Figure 5: Players' dynamic ideal points and indifference curves with quadratic utility (blue for the veto player, red for the proposer). The left-most panel considers $U_q < \underline{U}$. In the middle panel we have $U_q \in (\underline{U}, \overline{U})$. In the right-most panel we set $U_q > \overline{U}$. Generated from a numerical example where players have quadratic utility over both dimensions.

quadratic utility whether the equilibrium policy undershoots or overshoots $\hat{y}_1$ is fully determined by a unique cutoff in U_q.[15] Specifically, if $U_q \leq \underline{U}$ then $x_1^* > \hat{x}_1^P$ and $y_1^* < \hat{y}_1$, as discussed above, P is able to pull the policy close to its dynamic ideal point when the initial status quo is bad for V. Instead, if $U_q \in (\underline{U}_q, \overline{U}_q)$ then we are in the case of Proposition 4 where P leaves leverage on the table despite needing more today, $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$. Finally, when the status quo is favorable to V, $U_q > \overline{U}_q$, P must tie its hands tomorrow to appease V by proposing $x_1^* < \hat{x}_1^P$ and $y_1^* > \hat{y}_1$.

4 Extensions

In this section we extend the model in several directions. First, we examine the conditions for ideological infection of the players' preferences to emerge if there is turnover in the proposer. Next, we show that infection can additionally still occur if the change in ideal points between periods is stochastic. Finally, we study bargaining over a longer time horizon. This final analysis further emphasizes how the speed at which conflict increases on X relative to the

[15]Providing a full characterization under more general functional forms is challenging, as the equilibrium policy may cross $\hat{y}_1$ multiple times and thus not be monotonic in U_q.

compounding costs of inefficiency on Y matters for the persistence of inefficient policies. As previously mentioned, in the Appendix we also allow for the shape of the players' utility functions to change over time and provide a more general condition that captures the concept of increasing conflict on the ideological dimension.

4.1 Turnover

Up to this point, we have assumed that player P is always the proposer, emphasizing that inefficiency and ideological infection in our setting do not stem from fear of the other player taking power. We now turn our attention to the implications of turnover for our mechanism. Specifically, we assume P is the proposer in period 1 and remains so with probability $\rho \in (0, 1)$ in period 2, while V becomes the proposer with probability $1 - \rho$.

Now, the second-period outcome depends both on the first-period policy *and* the identity of the player selected to be the proposer. If P remains the proposer, the equilibrium outcome is as characterized in the baseline model. Letting $\overline{x}_V(x_2^q, y_2^q) \equiv \overline{x}(x_2^q, y_2^q)$, the second-period outcome is then $x_P^*(x, y) = \min\{\hat{x}_2^P, \overline{x}_V(x, y)\}$. Suppose instead that P becomes the veto player in the second period. Then, the relevant threshold characterizing the set of acceptable policies is $\underline{x}_P(x_2^q, y_2^q)$, which is defined as the lower solution to $u_{P2}(x) = u_{P2}(x_2^q) + v_2(y_2^q)$. Therefore, the second-period outcome if V becomes the proposer is $x_V^*(x, y) = \max\{\hat{x}_2^V, \underline{x}_P(x_2^q, y_2^q)\}$.

We demonstrate that, similar to the baseline model, both players' preferences are always infected when ideological conflict intensifies rapidly.

Proposition 5. *If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, then both players' preferences are infected for almost all values of $\rho \in (0, 1)$.*

To understand these conditions, we focus first on P's preferences. Suppose $\rho = 1$, so that P is certain to remain in power in the second period. The analysis of the baseline model highlights that P's preferences are infected if and only if, by setting $y_1 = \hat{y}_1$ (and $x_1 = \hat{x}_1^P$),

P does not maintain enough leverage to obtain its static optimum in the second period. This condition ensures that any marginal movement away from $\hat{y}_1$ impacts the outcome of the second-period bargaining, generating the distortion. Symmetrically, when $\rho = 0$ and P is certain to lose power in the second period, infection of his preferences emerges whenever implementing $y_1 = \hat{y}_1$ (and $x_1 = \hat{x}_1^P$) in period 1 implies V does not have enough leverage to pass his optimum in the second period. When ρ is between 0 and 1, marginal movements away from $\hat{y}_1$ influence the second-period outcome as long as *at least one* of these conditions is satisfied. As a consequence, either one of these conditions is sufficient for P's preferences to be infected. A similar logic applies to player V. Eliminating the least binding conditions, we obtain that both players' preferences are infected if $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, as stated in Proposition 5.

Notice that the above discussion has an important implication: in our setting, turnover can *generate* ideological infection. When $u_{V2}(\hat{x}_2^P) - u_{V2}(\hat{x}_1^P) > u_{P2}(\hat{x}_2^V) - u_{P2}(\hat{x}_1^V)$, the condition for V to be constrained as a second-period proposer after passing his first-period optimum is less binding than the analogous condition for P. When P is sure to remain in power in the second period, this is irrelevant. However, as described above, when $\rho < 1$ P worries about reducing V's leverage should he be selected as the second-period proposer, and $u_{P2}(\hat{x}_2^V) - u_{P2}(\hat{x}_1^V) < v_2(\hat{y}_1)$ is enough to ensure P's preferences are infected.

However, our final result shows that turnover can mitigate inefficiency on the intensive margin, by reducing the *degree* to which players' preferences are infected.

Proposition 6. *Suppose each player i's dynamic ideal point is such that* $\overline{x}_V(\hat{x}_d^i, \hat{y}_d^i) < \hat{x}_2^P$ *and* $\underline{x}_P(\hat{x}_d^i, \hat{y}_d^i) > \hat{x}_2^V$. *Then,* $\hat{y}_d^P$ *is decreasing in* ρ *and* $\hat{y}_d^V$ *is increasing in* ρ.

Consider the incentives of the proposer. If ρ is high then P is confident of remaining the proposer tomorrow and therefore wants to undershoot in the first period. However, as ρ decreases, P becomes increasing likely to lose power, and moves y towards $\hat{y}_1$ to offset the downside of keeping leverage in case V becomes the proposer. Eventually, the probability of remaining proposer is sufficiently low that P begins to overshoot as insurance against

V becoming the proposer tomorrow. Thus, there is a unique value of ρ for which the proposer's incentives to under and overshoot exactly compensate each other, eliminating infection. For all other values, infection persists. This implies that, for values of ρ below this cutoff, increasing turnover reduces the degree to which P's preferences are distorted. This finding suggests that during times of rapidly intensifying ideological conflict, electoral uncertainty over who will hold power tomorrow can partially mitigate ideological infection and its consequences, but cannot completely eliminate such distortions.

4.2 Stochastic Evolution of Preferences

In our analysis thus far, we have assumed the players can perfectly anticipate how their preferences will evolve over time. This assumption is useful to isolate the mechanism behind our results, but it is an obvious simplification. In this section, we discuss the effects of relaxing this assumption and consider a version of the model where players face uncertainty over their second-period optimal policies. First, we highlight that, with compounding costs of inefficiency, the mere possibility of increased conflict on the X dimension is sufficient to generate ideological infection. Second, analogously to what we established in the baseline model, we show that a necessary condition for ideological infection to emerge is that players *expect* compounding on the Y dimension.

We start by discussing uncertainty over the conflict dimension. Assume that $\hat{x}_{P2} = -\hat{x}_{V2} = \epsilon$, where ϵ is drawn from a continuous distribution G_X with full support on $[0, \infty)$ and density g_X. Thus, polarization may increase or decrease between periods.

For ease of exposition, we focus on infection of P's preferences. Now, in the second period the set of policies V is willing to accept depends on the realization of preferences. Specifically, $\overline{x}(x_2^q, y_2^q; \epsilon)$ solves:

$$u(x + \epsilon) = u(x_2^q + \epsilon) + v(y_2^q).$$

Let $\epsilon^*(x_2^q, y_2^q)$ solve $\overline{x}(x_2^q, y_2^q; \epsilon) = \epsilon$. Realizations for which the players are sufficiently moderate (i.e., $\epsilon < \epsilon^*(x, y)$) allow P to obtain its ideal point in the second period, either because the status quo now falls outside the gridlock interval or because moderation makes V relatively willing to grant concessions on X to obtain better policy on Y. Instead, extreme realizations make V highly reluctant to cede policy on X, making leverage on Y crucial for P. Consequently, P's dynamic preferences are determined by:

$$\max_{x,y} u_{P1}(x) + v_1(y) + \int_{\epsilon^*(x,y)}^{\infty} u_{P2}\big(\overline{x}(x, y; \epsilon)\big) g_X(\epsilon_X) d\epsilon.$$

In turn, $(\hat{x}_d^P, \hat{y}_d^P)$ solves:

$$u'_{P1}(x) + \int_{\epsilon^*(x,y)}^{\infty} \frac{u'_{P2}\big(\overline{x}(x, y; \epsilon)\big)}{u'_{V2}\big(\overline{x}(x, y; \epsilon)\big)} u'_{V1}(x) g(\epsilon) d\epsilon = 0 \tag{9}$$

$$v'_1(y) + \int_{\epsilon^*(x,y)}^{\infty} \frac{u'_{P2}\big(\overline{x}(x, y; \epsilon)\big)}{u'_{V2}\big(\overline{x}(x, y; \epsilon)\big)} v'_2(y) g(\epsilon) d\epsilon = 0. \tag{10}$$

The first-order condition demonstrates that P's preferences are still infected with uncertainty over $\hat{x}_{P2}$ and $\hat{x}_{V2}$ when there is compounding on the common-values dimension. Evaluating (10) at $y = \hat{y}_1$ we have $v'_1(\hat{y}) = 0$. Furthermore, $v'_2(\hat{y}_1) > 0$ and $\frac{u'_{P2}\left(\overline{x}(x,y;\epsilon)\right)}{u'_{V2}\left(\overline{x}(x,y;\epsilon)\right)} < 0$ for all realizations above ϵ^*, as such the LHS of (10) is strictly negative at $y = \hat{y}_1$. Thus, just the possibility of rapid polarization on the X dimension is sufficient to generate infection on the Y dimension. Notice, this is true even if the mean of G_X is $\hat{x}_{P1}$, and so in expectation there is no change in polarization (i.e., the shock to the preferences from one period to the next has mean zero).

We now consider uncertainty over the optimal second-period policy on the common-values dimension, instead than over X. Suppose that $\hat{y}_2 = \hat{y}_1 + \epsilon$, where ϵ is drawn from a continuous distribution G_Y with density g_Y and full support over the real line.

Now the set of policies V accepts in the second period is $\overline{x}(x_2^q, y_2^q; \epsilon)$, which solves:

$$u_{V2}(x) = u_{V2}(x_2^q) + v(y_2^q - \hat{y}_1 - \epsilon).$$

Let $\underline{\epsilon}(x_2^q, y_2^q)$ and $\overline{\epsilon}(x_2^q, y_2^q)$ be the lower and upper solutions, respectively, to $\overline{x}(x_2^q, y_2^q; \epsilon) = \hat{x}_2^P$. Extreme shocks shift $\hat{y}_2$ far from y_2^q, which gives P enough leverage to obtain its ideal point in the second period. Therefore, P's dynamic ideal point $(\hat{x}_d^P, \hat{y}_d^P)$ solves

$$\max_{x,y} u_{P1}(x) + v_1(y) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} u_{P2}\big(\overline{x}(x, y; \epsilon)\big) g_Y(\epsilon) d\epsilon.$$

Notice, if the support of G was such that $\epsilon \in \big(\underline{\epsilon}(\hat{x}_1^P, \hat{y}_1), \overline{\epsilon}(\hat{x}_1^P, \hat{y}_1)\big)$ with probability 0 then P's preferences are not infected. Similar to the case of $u_{V2}(\hat{x}_2^P) + v_2(\hat{y}_2) \geq u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ in Proposition 2, P anticipates having sufficient leverage tomorrow to obtain $\hat{x}_2^P$, even if the efficient policy is implemented today. When instead G_Y has full support on $\mathbb{R}$ then $(\hat{x}_d^P, \hat{y}_d^P)$ must solve:

$$u'_{P1}(x) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} \frac{u'_{P2}\big(\overline{x}(x, y; \epsilon)\big)}{u'_{V2}\big(\overline{x}(x, y; \epsilon)\big)} u'_{V1}(x) g_Y(\epsilon) d\epsilon = 0 \tag{11}$$

$$v'_1(y) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} \frac{u'_{P2}\big(\overline{x}(x, y; \epsilon)\big)}{u'_{V2}\big(\overline{x}(x, y; \epsilon)\big)} v'_2(y; \epsilon) g_Y(\epsilon) d\epsilon = 0. \tag{12}$$

The first order conditions highlight that the results of this enriched model align with our baseline findings that a necessary condition for inefficiency is that players expect changes in the way they evaluate the common-value dimension. In a world with uncertainty, an expectation of no change is equivalent to a case where the shock ϵ has a zero mean, and both the distribution G and the v function are symmetric. Condition (12) shows that, in this case, infection of the proposer's preferences is avoided. Under the assumed symmetry conditions, $\underline{\epsilon}(x, \hat{y}_1)$ and $\overline{\epsilon}(x, \hat{y}_1)$ are centered around 0. In turn, this implies that $\int_{\underline{\epsilon}(x,\hat{y}_1)}^{\overline{\epsilon}(x,\hat{y}_1)} \frac{u'_{P2}\left(\overline{x}(x,\hat{y}_1;\epsilon)\right)}{u'_{V2}\left(\overline{x}(x,\hat{y}_1;\epsilon)\right)} v'_2(\hat{y}_1; \epsilon) g_Y(\epsilon) d\epsilon = 0$ under a symmetric v.

Instead, under a non-zero-mean shock, which captures our cases of substance interest such as a deteriorating crisis or decaying infrastructures, with *some* probability the proposer will dynamically benefit from undershooting on the common-values dimension. As a consequence, the possibility of infection emerges.

4.3 Long-run Outcomes

We now extend our analysis to study when bargaining over a dynamic policy problem and an issue with increasing conflict leads to inefficiency over the long run. Bargaining proceeds as before, but unfolds over a finite number of periods, $t = 1, 2, ..., T$, where throughout we assume T is large.

Let $u(x - \hat{x}_t^i) = -(x - \hat{x}_t^i)^2$ and $v(y - \hat{y}_t) = -(y - \hat{y}_t)^2$. Additionally, we specify the evolution of ideal points as follows: $\hat{y}_t = \gamma t$ and $\hat{x}_t^P = -\hat{x}_t^V = t^\eta$, with $\gamma > 0$ and $\eta > 0$. Thus, the common-values ideal point increases linearly in time, while the evolution of ideal points on the conflict dimension may be concave or convex, and the evolution of these processes are governed by γ and η.

We study subgame perfect equilibria and, given the finite horizon, analyze the model via backwards induction. Notice that at period t any history leading to the same status quo (x_t^q, y_t^q) yields the same continuation game. As such, we focus on strategy profiles where policy proposals only depend on the time period (which also captures the players' ideal points) and the inherited status quo, and acceptance decisions only depend on these factors plus the proposed policy. Of course, in the last period of the game the players always agree on the efficient policy, $y_T^* = \hat{y}_T$, and the ideological policy $x_T^* = \min\left\{\hat{x}_T^P, \overline{x}(x_T^q, y_T^q)\right\}$, where $\overline{x}$ is defined as in the two-period model.

In this setting, the case of $\gamma = 0$ corresponds to no movement in the ideal common-values policy, i.e., Condition (3) fails in each period. For the same logic as in the baseline model, the absence of compounding costs of inefficiency implies there is no ideological infection. The following propositions consider the case where $\gamma > 0$, i.e., the optimal common-values policy changes over time. As we saw in the baseline model, compounding costs of inefficiency are necessary but not sufficient for ideological infection. Specifically, from Proposition 2, P's preferences are not infected when the change in preferences on X is sufficiently slow relative to the change in $\hat{y}$, such that P can obtain its ideal point in both periods. Similarly, the evolution of preferences on the conflict dimension plays a crucial role for whether inefficiency can be sustained in the long-run.

Proposition 7. *If $\eta < \frac{1}{2}$ then there exists $\hat{t} < T$ such that the equilibrium policy outcome is $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large $\hat{t} = 1$.*

When $\eta < \frac{1}{2}$, in which case the evolution of preferences on the X dimension is concave, the ideological conflict is increasing but eventually this increase is very small. Thus, in the long run, the change in conflict on X is slow relative to changes on the common-values dimension, where $\hat{y}_t$ is increasing linearly. Proposition 7 then confirms our insights from the baseline model. The players eventually reach a period where P is able to pass its static ideal policy $(\hat{x}_t^P, \hat{y}_t)$, and from there P has enough leverage to implement his ideal point on the conflict dimension in every subsequent period. Thus, the parties always reach the efficient common-values policy before the end date T. Furthermore, when the evolution of the common-values dimension is sufficiently rapid the parties reach efficiency immediately.

Next, we consider the case of rapidly growing polarization. Similar to the baseline, infection in any non-final period may be inevitable under rapid polarization.

Proposition 8. *If $\eta > \frac{1}{2}$ then $y_t^* \neq \hat{y}_1$ in every period.*

When $\eta > \frac{1}{2}$, so that the evolution of the conflict dimension is convex, the game eventually reaches a state where implementing an efficient policy leaves the proposer with insufficient leverage to obtain its optimal bundle in period $T - 1$. Anticipation of the eventual need for leverage rolls back to the previous periods, and leads to policy outcomes always being inefficient.

5 Conclusion

Addressing policy problems often requires the agreement of multiple parties. However, bargaining parties regularly have trouble reaching an efficient solution, even on issues where they all agree that the situation will grow increasingly worse if there is a lack of action. Our model shows that agreements on these common-values policy problems are vulnerable to being distorted by disagreements on other issues precisely because they worsen over time. Furthermore, if parties anticipate being more entrenched on the conflict dimension in the future, then ideological infection of preferences over the common-values issue is inevitable.

Our analysis provides insight into a number of contexts where parties have failed to adapt policy to deteriorating circumstances. Climate change negotiations between China and the United States have been hampered by disagreements over Taiwan. In the United States, political parties now appear polarized on nearly every issue, including those with little ideological content. Additionally, despite the severe costs, Republicans and Democrats have not always managed to avoid a government shutdown due to strategic incentives to gain an advantage on ideological issues. Our paper uncovers the conditions under which these issues of joint interest become infected by issues of conflict.

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A PROOFS

A.1 Proofs for Baseline Model

As mentioned in the paper, we can generalize the utility functions to allow their shapes to change between periods. In proving the results for the baseline model, we now index u and v by the time period: $u_{it}(x) = u_t(x - \hat{x}_t^i)$ and $v_t(y) = v_t(y - \hat{y}_t)$ and impose the following assumption throughout:

Assumption 2.

$$u'_{V2}(x) \leq u'_{V1}(x) \text{ and } u'_{P1}(x) \leq u'_{P2}(x) \text{ for all } x \in [\hat{x}_2^V, \hat{x}_2^P]. \tag{13}$$

Note that Assumption 2 is automatically satisfied in the baseline model. We discuss further the role of Assumption 2 in Section A.2.

Recall that $U_q = u_{V1}(x_1^q) + v_1(y_1^q) + u_{V2}\big(x_2^*(x_1^q, y_1^q)\big)$ denote V's dynamic equilibrium payoff from keeping the status quo. Thus, in the first period V accepts a proposal (x, y) if:

$$u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x, y)\big) \geq U_q,$$

and rejects otherwise.

Facing this constraint from V, in the first period player P chooses $(x, y) \in \mathbb{R}^2$ to solve the following maximization problem:

$$\max_{x,y} \; u_{P1}(x) + v_1(y) + u_{P2}\big(x_2^*(x, y)\big) \tag{14}$$
$$\text{s.t. } u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x, y)\big) \geq U_q$$

Lemma A.1. *Any (x, y) such that $\overline{x}(x, y) > \hat{x}_2^P$ with $y \neq \hat{y}_1$ or $x > \hat{x}_1^P$ does not solve (14).*

Proof. For a contradiction, assume there exists (x, y) such that $\overline{x}(x, y) > \hat{x}_2^P$ is optimal and $y \neq \hat{y}_1$. Since $\overline{x}(x, y)$ is continuous in y there exists y' closer to $\hat{y}_1$ such that $\overline{x}(x, y') > \hat{x}_2^P$. Furthermore, because $u_{V1}(x) + v_1(y') + u_{V2}(\hat{x}_2^P) > u_{V1}(x) + v_1(y) + u_{V2}(\hat{x}_2^P)$ the policy (x, y') must also satisfy V's acceptance constraint. Evaluating the objective function at (x, y') and (x, y) immediately yields $u_{P1}(x) + v_1(y') + u_{P2}(\hat{x}_2^P) > u_{P1}(x) + v_1(y) + u_{P2}(\hat{x}_2^P)$, contradicting that (x, y) solves problem (14). A similar argument shows that if $\overline{x}(x, y) > \hat{x}_2^P$ and $x > \hat{x}_1^P$ then there exists some profitable deviation $x' \in (\hat{x}_1^P, x)$, which improves the first-period payoffs of P and V without changing second-period payoffs. □

Lemma A.1 establishes an initial characterization of the optimal proposal when $\overline{x}(x, y) > \hat{x}_2^P$. If instead (x, y) is such that $\overline{x}(x, y) \leq \hat{x}_2^P$, then we can write the proposer's problem (14) as:

$$\begin{aligned} \max_{x,y} \;\; & u_{P1}(x) + v_1(y) + u_{P2}\big(\overline{x}(x, y)\big) \\ \text{s.t.} \;\; & u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y) \geq U_q \\ & u_{V2}(x) + v_2(y) \geq u_{V2}(\hat{x}_2^P) \end{aligned}$$

System (15) yields the KKT conditions for this problem:[16]

$$u'_{P1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{P2}\big(\overline{x}(x, y)\big) + \lambda_1 \Big[u'_{V1}(x) + u'_{V2}(x)\Big] + \lambda_2 u'_{V2}(x) = 0 \tag{15a}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{P2}\big(\overline{x}(x, y)\big) + \lambda_1 \Big[v'_1(y) + v'_2(y)\Big] + \lambda_2 v'_2(y) = 0 \tag{15b}$$

$$\lambda_1 \Big[u_{V1}(x) + v_1(y) + u_{V2}\big(\overline{x}(x, y)\big) - U_q\Big] = 0 \tag{15c}$$

$$\lambda_2 \Big[u_{V2}(x) + v_2(y) - u_{V2}(\hat{x}_2^P)\Big] = 0 \tag{15d}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{15e}$$

[16]It is straightforward to show that if the constraint qualification fails at some point (x, y) then it must be that $x \in [\hat{x}_2^V, \hat{x}_\alpha^V)$ and $y \in (\hat{y}_\alpha^V, \hat{y}_2]$. However, clearly this cannot be optimal as a deviation to $x = \hat{x}_d^V$ and $y = \hat{y}_d^V$ is always accepted by V and improves P's dynamic payoff. Thus, the KKT conditions will hold at any solution.

Lemma A.2 establishes that P never proposes an (x, y) that allows it to pass $\hat{x}_2^P$ in the second period. Thus, it simplifies our analysis of System 15 by ruling out corner solutions.

Lemma A.2. *Any (x, y) such that $\overline{x}(x, y) = \hat{x}_2^P$ never solves (14).*

Proof. Suppose there exists (x, y) that solves (14) such that $\overline{x}(x, y) = \hat{x}_2^P$. Thus, (x, y) must solve system (15). In particular, consider condition (15b). Letting $\overline{x}(x, y) = \hat{x}_2^P$, then $u'_{P2}\big(\overline{x}(x, y)\big) = 0$ and this condition becomes:

$$v'_1(y) + \lambda_1\Big[v'_1(y) + v'_2(y)\Big] + \lambda_2 v'_2(y) = 0. \tag{16}$$

First, notice that if such an (x, y) is optimal then we must have $y \leq \hat{y}_1$. If $y \in (\hat{y}_1, \hat{y}_2]$ then P could deviate to $y = \hat{y}_1$, which would maintain $x_2^*(x, y) = \hat{x}_2^P$ and improve the first-period payoff of both players, contradicting that (x, y) solves problem (14).

Second, if $y < \hat{y}_1$ then the LHS of (16) is strictly positive by $\hat{y}_1 \leq \hat{y}_2$, contradicting that (16) holds. Therefore, if (x, y) is such that $\overline{x}(x, y) = \hat{x}_2^P$ and solves problem (14) then we must have $y = \hat{y}_1$.

To finish the proof we now show that $y = \hat{y}_1$ also leads to a contradiction. If $y = \hat{y}_1$ then (16) reduces to:

$$\big(\lambda_1 + \lambda_2\big) v'_2(\hat{y}_1) = 0. \tag{17}$$

We consider two cases depending on $v'_2(\hat{y}_1)$. First, if $v'_2(\hat{y}_1) > 0$ then for (16) to hold requires $\lambda_1 = \lambda_2 = 0$, but $\lambda_2 = 0$ contradicts that $\overline{x}(x, y) = \hat{x}_2^P$. Second, if $v'_2(\hat{y}_1) = 0$ then it must be that $\hat{y}_1 = \hat{y}_2$. Thus, the policy will be stuck at the first-period proposal x in the second period, which implies that $x = \hat{x}_2^P$. In this case, for condition (15a) to hold requires:

$$u'_{P1}(\hat{x}_2^P) + \lambda_1\Big[u'_{V1}(\hat{x}_2^P) + u'_{V2}(\hat{x}_2^P)\Big] + \lambda_2 u'_{V2}(\hat{x}_2^P) = 0. \tag{18}$$

Because $\hat{x}_2^P \geq \hat{x}_1^P$ the LHS of (18) is strictly negative by assumption that P is always further to the right than V, which contradicts that (x, y) solves (14) and completes the argument. $\square$

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y}_1 \leq \hat{y}_d^V$. On the X dimension $\hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{x}_1^P \leq \hat{x}_d^P$.*

Proof. First, we prove the result for the proposer's dynamic ideal point. If P chooses (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$ then the necessary condition $(\hat{x}_d^P, \hat{y}_d^P)$ solves is:

$$u'_{P1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{P2}\big(\overline{x}(x, y)\big) = 0 \tag{19}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{P2}\big(\overline{x}(x, y)\big) = 0. \tag{20}$$

Recall that $\overline{x}(x, y)$ solves (2), the implicit function theorem then yields:

$$\frac{\partial \overline{x}}{\partial x} = \frac{u'_{V2}(x)}{u'_{V2}\big(\overline{x}(x, y)\big)}, \text{ and } \frac{\partial \overline{x}}{\partial y} = \frac{v'_2(y)}{u'_{V2}\big(\overline{x}(x, y)\big)}.$$

As such, the LHS of (19) can be written as: $u'_{P1}(x) + \frac{u'_{V2}(x)}{u'_{V2}(\overline{x})} u'_{P2}\big(\overline{x}(x, y)\big)$, note we will sometimes suppress dependence of $\overline{x}$ on the first-period policy (x, y). Furthermore, in equilibrium we must have $\hat{x}_2^V < x \leq \overline{x}(x, y) \leq \hat{x}_2^P$, which implies $\frac{u'_{V2}(x)}{u'_{V2}(\overline{x})} > 0$ and $u'_{P2}\big(\overline{x}(x, y)\big) \geq 0$. Therefore, $x > \hat{x}_1^P$ is necessary for (19) to hold. Next consider equation (20). After substituting for $\frac{\partial \overline{x}}{\partial y}$, (20) becomes:

$$v'_1(y) + \frac{u'_{P2}\big(\overline{x}(x, y)\big)}{u'_{V2}\big(\overline{x}(x, y)\big)} v'_2(y) = 0.$$

Because $\overline{x}(x, y) \in (\hat{x}_2^V, \hat{x}_2^P]$ we have $\frac{u'_{P2}(\overline{x})}{u'_{V2}(\overline{x})} \leq 0$. Thus, if $y \in (\hat{y}_1, \hat{y}_2]$ then LHS of (20) is strictly negative. Clearly $y > \hat{y}_2$ is never optimal. Hence, a necessary condition for (20) to hold is that $y \leq \hat{y}_1$.

Suppose instead that $(\hat{x}_d^P, \hat{y}_d^P)$ is such that $x_2^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$. In this case, if $x_2^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ is optimal absent the veto player's first-period acceptance constraint, then clearly we must have $(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_1^P, \hat{y}_1)$. Thus, it is always the case that $\hat{x}_d^P \geq \hat{x}_1^P$ and $\hat{y}_d^P \leq \hat{y}_1$.

Now consider the veto player. V's dynamic ideal point $(\hat{x}_d^V, \hat{y}_d^V)$ solves:

$$\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x,y)\big).$$

If $x_2^*(x,y) = \overline{x}(x,y)$ then $(\hat{x}_d^V, \hat{y}_d^V)$ must solve:

$$u'_{V1}(x) + u'_{V2}(x) = 0$$
$$v'_1(y) + v'_2(y) = 0.$$

Thus, $\hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{y}_d^V \geq \hat{y}_1$, as required. If instead it is optimal for V to choose $(\hat{x}_d^V, \hat{y}_d^V)$ such that $x_2^*(x,y) = \hat{x}_2^P$, then clearly it must be that $\hat{x}_d^V = \hat{x}_1^V$ and $\hat{y}_d^V = \hat{y}_1$, completing the argument. $\square$

Proposition 1. *If $v'_1(y) \geq v'_2(y)$ for all $y \leq \hat{y}_1$, then there is no ideological infection, $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$. Furthermore, the policy outcome is efficient, $y_1^* = \hat{y}_1$.*

Proof.

Part 1. To start, we prove part 1, that there is no ideological infection. For a contradiction, assume $v'_1(y) \geq v'_2(y)$ for all $y \leq \hat{y}_1$, but $\hat{y}_d^i \neq \hat{y}_1$ for some $i \in \{V, P\}$.

First, consider player P and suppose $\hat{y}_d^P < \hat{y}_1$. Lemma A.1 implies that if $y_d^P \neq \hat{y}_1$ then $x^*(x,y) = \overline{x}(x,y)$. Thus, from our analysis in Lemma 2, it is necessary that P's dynamic ideal point (x_d^P, y_d^P) solves:

$$u'_{P1}(x) + \frac{u'_{V2}(x)}{u'_{V2}(\overline{x})} u'_{P2}(\overline{x}(x,y)) = 0 \tag{21}$$
$$v'_1(y) + \frac{v'_2(y)}{u'_{V2}(\overline{x})} u'_{P2}(\overline{x}(x,y)) = 0. \tag{22}$$

Using that $y \neq \hat{y}_1$, we can combine conditions (21) and (22) and rearrange to obtain that

$(\hat{x}_d^P, \hat{y}_d^P)$ must satisfy:

$$\frac{v_2'(y)}{v_1'(y)} = \frac{u_{V2}'(x)}{u_{P1}'(x)}. \tag{23}$$

From the proof of Lemma 2, if (x, y) is such that $x_2^*(x, y) = \overline{x}(x, y)$ then $\hat{x}_d^P > \hat{x}_1^P$. Recall that $u_{P1}(x)$ is a translation of $u_{V1}(x)$ and u_{i1} is concave, thus, $u_{V1}'(x) < u_{P1}'(x) < 0$ for $x > \hat{x}_1^P$. Furthermore, by Assumption 2, $u_{V2}'(x) \leq u_{V1}'(x)$. Hence, $u_{V2}'(x) < u_{P1}'(x) < 0$. However, this implies $\frac{u_{V2}'(x)}{u_{P1}'(x)} > 1$ and by assumption $\frac{v_2'(y)}{v_1'(y)} \leq 1$. Therefore, $\frac{u_{V2}'(x)}{u_{P1}'(x)} > \frac{v_2'(y)}{v_1'(y)}$ and (23) cannot hold, contradicting that $\hat{y}_d^P \neq \hat{y}_1$.

Second, we show that L's preferences are also not infected. Suppose not, so $\hat{y}_d^V > \hat{y}_1$. A similar argument as for P in Lemma A.1 yields that if $\hat{y}_d^V \neq \hat{y}_1$ then $\overline{x}(\hat{x}_d^V, \hat{y}_d^V) \leq \hat{x}_2^P$. Thus, consider (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$. In this case, $(\hat{x}_d^V, \hat{y}_d^V)$ solves:

$$u_{V1}'(x) + u_{V2}'(x) = 0 \tag{24}$$

$$v_1'(y) + v_2'(y) = 0. \tag{25}$$

The assumption that $\frac{v_2'(y)}{v_1'(y)} \leq 1$ for all $y \leq \hat{y}_1$ implies that $\hat{y}_1 = \hat{y}_2$, otherwise the assumption would fail at $y = \hat{y}_1 < \hat{y}_2$. Therefore, if $y > \hat{y}_1 = \hat{y}_2$ then $v_1'(y) < 0$ and $v_2'(y) < 0$, which violates (25).

Part 2. Now we show that the policy outcome must also be efficient. To derive a contradiction, suppose that $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$ but $y_1^* \neq \hat{y}_1$. By Lemma A.1 if $y_1^* \neq \hat{y}_1$ then $\overline{x}(x_1^*, y_1^*) \leq \hat{x}_2^P$. Thus, the optimal policy proposal must solve system (15). To prove the result we now consider different cases depending on which constraints are binding.

Case 1: To start, assume $\lambda_2 > 0$, this implies that $\overline{x}(x, y) = \hat{x}_2^P$, which cannot be optimal by Lemma A.2.

Case 2: Second, assume $\lambda_1 = 0$ and $\lambda_2 = 0$. By $\lambda_1 = 0$ the proposer's unconstrained optimal policy is accepted by V. Thus, by Proposition 1 $y_1^* = \hat{y}_d^P = \hat{y}_1$, as required.

Case 3: Finally, consider the case where $\lambda_1 > 0$ and $\lambda_2 = 0$. Solving (15a) and (15b) for λ_1 implies that (x, y) must solve:

$$\frac{v_1'(y) + \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} v_2'(y)}{v_1'(y) + v_2'(y)} = \frac{u_{P1}'(x) + \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} u_{V2}'(x)}{u_{V1}'(x) + u_{V2}'(x)}. \tag{26}$$

Rearranging condition (26), we have that any optimal (x, y) must solve:

$$v_1'(y)u_{P1}'(x) - v_1'(y)u_{V1}'(x) + v_2'(y)u_{P1}'(x) - v_1'(y)u_{V2}'(x) \\ - \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})}\Big(v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x)\Big) = 0. \tag{27}$$

To obtain a contradiction, we show that if $y \neq \hat{y}_1$ then the LHS of (27) is strictly positive. Suppose $y < \hat{y}_1 = \hat{y}_2$, recalling that if $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$ then $\hat{y}_1 = \hat{y}_2$ (an analogous argument proves the case $y > \hat{y}_1 = \hat{y}_2$). Thus, $v_1'(y) > 0$ and $v_2'(y) > 0$.

First, we show that the last term in the LHS of (27) is always positive. To see this, note that $\frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} < 0$, by $\overline{x}(x, y) \in (\hat{x}_2^V, \hat{x}_2^P]$. Thus, a sufficient condition for the last term on the LHS of (27) to be positive is that:

$$v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x) > 0. \tag{28}$$

Clearly, in equilibrium, $x \geq \hat{x}_2^V$. Thus, if $x < \hat{x}_1^V$ then $v_2'(y)u_{V1}'(x) > 0$ and (28) holds. Instead, suppose that $x \geq \hat{x}_1^V$. By Assumption 2 $u_{V2}'(x) < u_{V1}'(x) < 0$. Additionally, by assumption, $v_1'(y) \geq v_2'(y) > 0$. As such, $v_2'(y)u_{V1}'(x) > v_1'(y)u_{V2}'(x)$, and (28) holds. Therefore, $-\frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})}\Big(v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x)\Big) \geq 0$, as claimed.

Second, consider the term: $v_1'(y)u_{P1}'(x) - v_1'(y)u_{V1}'(x)$. By $y < \hat{y}_1$, this term is positive if and only if $u_{P1}'(x) \geq u_{V1}'(x)$, which holds by concavity of $u_1(x - \hat{x}^i)$ and $\hat{x}_1^V < \hat{x}_1^P$.

Finally, to complete the argument that the LHS of (27) is strictly positive, we show that

$v_2'(y)u_{P1}'(x) - v_1'(y)u_{V2}'(x) > 0$. This holds if and only if:

$$v_2'(y)u_{P1}'(x) > v_1'(y)u_{V2}'(x). \tag{29}$$

By our previous argument showing that the LHS of (28) is positive, we have $v_2'(y)u_{V1}'(x) > v_1'(y)u_{V2}'(x)$. Thus, a sufficient condition for (29) to hold is that $v_2'(y)u_{P1}'(x) \geq v_2'(y)u_{V1}'(x)$, which again follows from concavity of u_1 and $\hat{x}_1^V < \hat{x}_1^P$. Therefore, the LHS of (27) is strictly positive, contradicting that $y < \hat{y}_1$ is optimal. □

Proposition 2.

1. *V's preferences are infected if and only if $u_{V2}(\hat{x}_2^P) < \overline{u}_V$; and*

2. *P's preferences are infected if and only if $u_{V2}(\hat{x}_2^P) < \overline{u}_P$.*

Furthermore, $\overline{u}_P < \overline{u}_V$.

Proof. To start, we prove part 1 of the proposition. Define $\overline{u}_V$ as:

$$\overline{u}_V \equiv u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V),$$

where $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ solves:

$$\begin{aligned} u_{V1}'(x) + u_{V2}'(x) &= 0 \\ v_1'(y) + v_2'(y) &= 0. \end{aligned} \tag{30}$$

Part 1. We begin by showing that if $u_{V2}(\hat{x}_2^P) < \overline{u}_V$ then V's preferences are infected. In this case, $u_{V2}(\hat{x}_2^P) < \overline{u}_V < u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V)$ which implies $\overline{x}(\hat{x}_\alpha^V, \hat{y}_\alpha^V) < \hat{x}_2^P$. Therefore, V's dynamic payoff from $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ is $\overline{u}_V$. By construction $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ maximizes $u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ and hence maximizes V's dynamic payoff among all policies (x, y) such $x_2^*(x, y) < \overline{x}(x, y)$. Finally, the best possible dynamic payoff to V from any policy

(x, y) such that $x_2^*(x, y) = \hat{x}_2^P$ is $u_{V2}(\hat{x}_2^P)$ which is strictly less than the dynamic payoff from $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ by assumption that $u_{V2}(\hat{x}_2^P) < \overline{u}_V$. From inspection of (30), $\hat{y}_\alpha^V > \hat{y}_1$, and V's preferences are infected.

Next, we prove that if $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ then V's preferences are not infected. Because $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ solves $\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ we have that $\overline{u}_V \geq u_{V1}(\hat{x}_1^V) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1) = u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. Therefore, the assumption $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ also yields that $u_{V2}(\hat{x}_2^P) \geq u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. Thus, the dynamic payoff to V from $(\hat{x}_1^V, \hat{y}_1)$ is $u_{V2}(\hat{x}_2^P)$, which is the greatest possible payoff from any first-period policy such that $x_2^*(x, y) = \hat{x}_2^P$. Instead the best policy for V such that $\overline{x}(x, y) \leq \hat{x}_2^P$ solves:

$$\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$$
$$\text{s.t. } u_{V2}(x) + v_2(y) \geq u_{V2}(\hat{x}_2^P).$$

Recall that (x_α^V, y_α^V) solves this problem when the constraint does not bind, however, $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ and hence the constraint must be binding at the solution. Therefore the best policy (x, y) for V such that $\overline{x}(x, y) \leq \hat{x}_2^P$ must set $\overline{x}(x, y) = \hat{x}_2^P$, and clearly $u_{V1}(x) + v_1(y) + u_{V2}(\hat{x}_2^P) < u_{V1}(\hat{x}_1^V) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_2^P)$. Consequently, $(\hat{x}_d^V, \hat{y}_d^V) = (\hat{x}_1^V, \hat{y}_1)$ and V's preferences are not infected.

Part 2. Now we prove part 2 of the proposition. Recall that $\overline{u}_P = u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$.

We first show that if $u_{V2}(\hat{x}_2^P) \geq \overline{u}_P$ then P's preferences are not infected. By definition of $\overline{u}_P$, if P chooses $(\hat{x}_1^P, \hat{y}_1)$ in the first period it can implement $(\hat{x}_2^P, \hat{y}_2)$ in the second. As this is the unique sequence of policies that yields P its first-best payoff, P's preferences are not infected.

Next, we show that if $u_{V2}(\hat{x}_2^P) < \overline{u}_P$ then P's preferences are infected. To show a contradiction, assume that $\hat{y}_d^P = \hat{y}_1$. We break the argument into two parts. First, let $\overline{x}(x, \hat{y}_d^P) = \hat{x}_2^P$, then from System (15) for (x, y) to satisfy the KKT conditions it must be that $x < \hat{x}_1^P$. However, this implies $u_{V2}(x) + v_2(\hat{y}_1) \leq u_{V2}(\hat{x}_2^P)$ and $u_{V2}(x) + v_2(\hat{y}_1) > u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) = \overline{u}_P$, which contradicts that $u_{V2}(\hat{x}_2^P) < \overline{u}_P$.

Second, let $\overline{x}(\hat{x}_d^P, \hat{y}_d^P) < \hat{x}_2^P$. By System (15) $\hat{x}_d^P$ must solve:

$$\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))}v'_2(\hat{y}_1) = 0. \tag{31}$$

However, $\overline{x}(x,\hat{y}_1) \in (\hat{x}_2^V, \hat{x}_2^P)$, thus $\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} < 0$. Additionally, by Assumption 1 $v'_2(\hat{y}_1) > 0$. Therefore, the LHS of (31) is strictly less than 0, which contradicts that $\hat{y}_2^P = \hat{y}_1$.

Part 3. To conclude the proof we now demonstrate that $\overline{u}_P < \overline{u}_V$. We again note that $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ is the unique maximizer of $u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ and thus $\overline{u}_P = u_{V1}(\hat{x}_1) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) < u_{V1}(\hat{x}_1) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1) < u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V) = \overline{u}_V$. □

Proposition 3.

1. *Assume V's preferences are infected but P's preferences are not. If $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ then the equilibrium policy is efficient. Otherwise, the equilibrium policy is inefficient for almost all $(U_q, \hat{x}_1^V)$.*

2. *If both players' preferences are infected then the equilibrium policy is inefficient for almost all $(U_q, \hat{x}_1^V)$.*

Proof. We first prove the second part of the result. Assume that the preferences of both players are infected, thus, $\overline{u}_P > u_{V2}(\hat{x}_2^P)$ by Proposition 2.

By Lemmas A.1 and A.2 if the equilibrium policy is such that $x_2^*(x_1^*, y_1^*) = \hat{x}_2^P$ then $x_1^* < \hat{x}_1^P$ and $y_1^* = \hat{y}_1$. However, by $x_1^* < \hat{x}_1^P$ we have $u_{V2}(x_1^*) + v_2(\hat{y}_1) > u_{V2}(\hat{x}_1) + v_2(\hat{y}_1) = \overline{u}_P$. Thus, the assumption $\overline{u}_P > u_{V2}(\hat{x}_2^P)$ implies $u_{V2}(x_1^*) + v_2(\hat{y}_1) > u_{V2}(\hat{x}_2^P)$, which contradicts that $\overline{x}(x_1^*, \hat{y}_1) \geq \hat{x}_2^P$. Therefore, the equilibrium policy (x_1^*, y_1^*) must be such that $\overline{x}(x_1^*, y_1^*) <$

$\hat{x}_2^P$ and thus solve System (15). From (15) if $y = \hat{y}_1$ then x must solve:

$$u_{V1}(x) + u_{V2}\big(\overline{x}(x, \hat{y}_1)\big) = U_q \tag{32}$$

The LHS of (32) is strictly decreasing in x for $x > \hat{x}_1^V$, thus, there must be a unique $x' > \hat{x}_1^V$ that solves this equality.

Solving for λ_1 from (15a) and (15b) and rearranging we have that the equilibrium x must also solve:

$$\frac{u'_{P2}(\overline{x}(x, \hat{y}_1))}{u'_{V2}(\overline{x}(x, \hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)} = 0. \tag{33}$$

Using this condition, define $f : \mathbb{R}^2 \to \mathbb{R}$ as $f(x, \hat{x}_1^V) = \frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)}$. Then

$$Df(x, \hat{x}_1^V) = \left(\frac{\partial \overline{x}}{\partial x} \frac{u''_{P2}(\overline{x})u'_{V2}(\overline{x}) - u'_{P2}(\overline{x})u''_{V2}(\overline{x})}{u'_{V2}(\overline{x})^2} - \frac{u'_{V1}(x)u''_{P1}(x) - u'_{P1}(x)u''_{V1}(x)}{u'_{V1}(x)^2}, -\frac{u'_{P1}(x)u''_{V1}(x)}{[u'_{V1}(x)]^2} \right),$$

Notice that if $x = \hat{x}_1^P$ then (33) cannot hold. Moreover, $-\frac{u'_{P1}(x)u''_{V1}(x)}{[u'_{V1}(x)]^2} = 0$ if and only if $x = \hat{x}_1^P$. Therefore, if $(x, \hat{x}_1^V)$ is such that $f(x, \hat{x}_1^V) = 0$ then $Df(x, \hat{x}_1^V)$ has rank $1 = \min\{1, 2\}$. Thus, 0 is a regular value of f and by the Transversality Theorem (De la Fuente, 2000, Theorem 2.5) the set of x that solve (33) is measure 0 for almost all $\hat{x}_{V1}$. Fix such a $\hat{x}_{V1}$. The unique solution to (32) is strictly increasing in U_q, while solutions to (33) do not change in U_q, since we can change U_q without changing anything in (33) by changing the initial status quo policy. Thus for almost all $(U_q, \hat{x}_{V1})$ the solution to (32) does not coincide with any solutions to (33).

Next, we prove the first part of the proposition. Suppose only V's preferences are infected, thus, $\overline{u}_V > u_{V2}(\hat{x}_2^P) > \overline{u}_P$ by Proposition 2. For $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ clearly $(x, y) = (\hat{x}_1^P, \hat{y}_1)$ is optimal, as P passes its ideal point in each period. Next, assume $U_q > u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$. Thus, V rejects $(\hat{x}_1^P, \hat{y}_1)$, which implies P chooses (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$

and $\lambda_1 > 0$. For a contradiction suppose that $y_1 = \hat{y}_1$. By $U_q > u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ the veto player must also reject any policy $(x_1, \hat{y}_1)$ where $x_1 > \hat{x}_1^P$. Thus, $x_1 < \hat{x}_1^P$. Solving for λ_1 and rearranging x must also solve

$$\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)} = 0, \tag{34}$$

and the same argument as above yields that the equilibrium policy can be efficient for only a measure zero set of parameters $(U_q, \hat{x}_1^V)$. □

Proposition 4. *Assume $u_{V2}(\hat{x}_2^P) < \overline{u}_P$. There exists an open interval $(\underline{U}_q, \overline{U}_q)$, such that, if $U_q \in (\underline{U}_q, \overline{U}_q)$ then $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$.*

Proof. Since $u_{V2}(\hat{x}_2^P) < \overline{u}_P$ the optimal proposal must solve system (15), and the implicit function theorem delivers that we can view solutions (x_1^*, y_1^*) as continuous functions of U_q. For U_q sufficiently large we have $x_1^*(U_q) < \hat{x}_1^P$ and $y_1^*(U_q) > \hat{y}_1$, and for U_q sufficiently small we have $x_1^*(U_q) > \hat{x}_1^P$ and $y_1^*(U_q) < \hat{y}_1$, because the equilibrium policy must be close to V's and P's dynamic ideal points, respectively. Thus, there must exist some U'_q such that $y = \hat{y}_1$. Specifically, let U'_q be the first such value of U_q where $y_1^* = \hat{y}_1$.

First, we show that $U'_q > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. Suppose not, so that $U'_q \leq u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. The veto constraint must bind, otherwise P could choose its unconstrained optimal which sets $y < \hat{y}_1$. Thus, if $y = \hat{y}_1$ then, from our earlier analysis, the equilibrium proposal must satisfy:

$$\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} = \frac{u'_{P1}(x)}{u'_{V1}(x)}. \tag{35}$$

Furthermore, because $U'_q \leq u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$ we must have $x > \hat{x}_1^P$, otherwise if $x < \hat{x}_1^P$ then P could profitably deviate to $x = \hat{x}_1^P$, which the veto player would accept. Thus, if $y = \hat{y}_1$ then $x > \hat{x}_1^P$. If $x > \hat{x}_1^P$ then $\frac{u'_{P1}(x)}{u'_{V1}(x)} > 0$. However, $\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} < 0$, contradicting that (35) holds.

Therefore, there is some $U_q' > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}(x_2^*(\hat{x}_1^P, \hat{y}_1))$ such that $y < \hat{y}_1$ for $U_q < U_q'$. Furthermore, for any $U_q \in \Big(u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big), U_q'\Big)$ if $y < \hat{y}_1$ and $x > \hat{x}_1^P$ then it is profitable for V to reject because $U_q' > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. Thus, $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$ for $\Big(u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big), U_q'\Big)$, as claimed. □

A.2 Changing Shapes of the Utility Functions

We now discuss the role of Assumption 2 for our results. Recall that Assumption 2 requires:

$$u_{V2}'(x) \leq u_{V1}'(x) \text{ and } u_{P1}'(x) \leq u_{P2}'(x) \text{ for all } x \in [\hat{x}_2^V, \hat{x}_2^P]. \tag{36}$$

Assumption 2 states that for each player the marginal cost of moving policy away from its ideal point on X becomes greater over time. This assumption is consistent with our consideration of X as a dimension of disagreement on which conflict is increasing is over time. Thus, under this definition of increasing ideological conflict the central result from our baseline model continues to hold. Even though both players expect their opponent to become more entrenched on the ideological dimension in the future, and thus less willing to compromise, compounding costs on the common-values dimension generate incentives to delay coming to an efficient agreement.

If the above condition fails, then the utility function over the X dimension can become "flatter" over time despite the ideal points pulling further apart. In this case, our model delivers a less surprising result. If players become less sensitive over time to changes on the conflict dimension, and thus more willing to compromise tomorrow, this naturally creates incentives to delay agreeing to an efficient policy today, even absent compounding costs of inefficiency. That is, even if preferences on the Y dimension do not change, the proposer may want to undershoot the efficient policy to preserve leverage because the veto player becomes close to indifferent over policies on the X dimension in the second period. As such, any amount of inefficiency is more valuable and P's preferences are infected. Here, the key condition for P's preferences to not be infected is that, for all $x > \hat{x}_1^P$ and $y \leq \hat{y}_2$:

$$\frac{u'_{V2}(x)}{u'_{P1}(x)} \geq \frac{v'_2(y)}{v'_1(y)}. \tag{37}$$

Under the increasing conflict condition, $\frac{u'_{V2}(x)}{u'_{P1}(x)} > 1$ for all $x > \hat{x}_1^P$, thus, $v'_2(y) \leq v'_1(y)$ is sufficient for (37) to hold. If the increasing conflict condition fails, then it is possible that $\frac{u'_{V2}(x)}{u'_{P1}(x)} < 1$ for $x > \hat{x}_1^P$. Consequently, (37) can fail and infection of the proposer's preferences can occur even if, for example, $v_1(y) = v_2(y)$. However, Condition (37) still maintains a similar flavor as the original result, whereby infection of the proposer's preferences is avoided as long as the marginal cost of inefficiency is relatively smaller tomorrow than today. We also note that the same logic and discussion applies if we relax concavity of u and only require u to be quasi-concave over X: pulling apart the players' ideal points can make them less sensitive to changes over the relevant policy region (even if the shape of u does not change in this case). Overall, given our substantive interest in situations where the actors are becoming more antagonistic and policy problems worsen over time, we have focused our analysis on the case where the u_ts are concave and condition (36) holds.

Finally, now that we allow the shape of the utility functions to change over time we point out that Condition (3) does not consider the case where $\hat{y}_1 = \hat{y}_2$ but $v'_2(y) > v'_1(y)$ for $y < \hat{y}_1$. This streamlines the presentation of our results, but it is not crucial for infection of P's preferences.[17] That is, a similar logic of compounding can lead to infection of P's preferences if the ideal point on Y remains the same but the marginal cost of inaction increases over time (except at $\hat{y}_1 = \hat{y}_2$), e.g., $v_2(y) = \theta v_1(y)$ with $\theta > 1$. In this case, V's preferences are not infected, as it is not possible for y to overshoot $\hat{y}_1$. However, P is still incentivized to undershoot $\hat{y}_1$ because the compounding makes V more willing to yield concessions in period 2.

For example, consider the following numerical specification: $u_t(x - \hat{x}_t^i) = -(x - \hat{x}_t^i)^2$, $\hat{x}_1^P = -\hat{x}_1^V = 1$, $\hat{x}_2^P = -\hat{x}_2^V = 7$, $v_1(y) = -\frac{1}{4}(y-1)^2$, and $v_2(y) = -(y-1)^2$. If the

[17]Furthermore, we note that Condition (3) allows the change in $\hat{y}_1$ to be arbitrarily small.

efficient policy is implemented in the first period, $y = \hat{y} = 1$, then the optimal x for P is the midpoint $\frac{\hat{x}_1^P + \hat{x}_2^P}{2} = 4$. P's dynamic payoff in this case is $-(4-1)^2 - (4-7)^2 = -18$. Instead, consider the policy $x = 3.5$ and $y = -2.4$. In the second period, the veto player is willing to accept $\overline{x}(3.5, -2.4) \approx 4.03$. Then P's dynamic payoff from the inefficient policy $(3.5, -2.4)$ is $-(3.5-1)^2 - \frac{1}{4}(-2.4-1)^2 - (4.03-7)^2 \approx -17.93 > -18$.

A.3 Proofs for Turnover Extension

With turnover, P's problem in the first period can be written as:

$$\max_{x,y} \; u_{P1}(x) + v_1(y) + \rho u_{P2}\big(x_P^*(x,y)\big) + (1-\rho)u_{P2}\big(x_V^*(x,y)\big) \tag{38}$$
$$\text{s.t. } u_{V1}(x) + v_1(y) + \rho u_{V2}\big(x_P^*(x,y)\big) + (1-\rho)u_{V2}\big(x_V^*(x,y)\big) \geq U_q$$

Proposition 5. *If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, then both players' preferences are infected for almost all values of $\rho \in (0,1)$.*

Proof. We consider the case of player P. A similar argument extends the result to player V. The argument proceeds as follows. First, we show that if $(\hat{x}_d^P, \hat{y}_d^P)$ is such that $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \overline{x}_V(\hat{x}_d^P, \hat{y}_d^P)$ or $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \underline{x}(\hat{x}_d^P, \hat{y}_d^P)$ then it must be that $\hat{y}_d^P \neq \hat{y}_1$ for almost all values of ρ. Second, we show that if $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ and $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^V$ then the only possible solution for player P's preferences to not be infected requires $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_1^P, \hat{y}_1)$, however, this solution is not feasible by $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$.

Step 1. We argue that if $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \overline{x}_V(\hat{x}_d^P, \hat{y}_d^P)$ or $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \underline{x}(\hat{x}_d^P, \hat{y}_d^P)$ then $\hat{y}_d^P \neq \hat{y}_1$ for almost all values of ρ. We break the argument into three parts, depending on if (x,y) is such that one of the players can pass $\hat{x}_2^i$ in the second period.

Part 1. Consider (x,y) such that $u_{V2}(\hat{x}_2^P) \leq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}_2^V) \leq u_{P2}(x) + v_2(y)$.

Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\overline{x}_V(x,y)\big) + (1-\rho)\big(u_{P2}(x) + v_2(y)\big).$$

Letting $a = -x$ and $b = -y$ we can rewrite P's problem as:

$$\max_{(a,b)} u_{P1}(-a) + v_1(-b) + \rho u_{P2}\big(\overline{x}_{V2}(a,b)\big) + (1-\rho)\big(u_{P2}(-a) + v_2(-b)\big).$$

Taking cross-partials of the objective function yields:

$$\begin{aligned}
\frac{\partial^2}{\partial a \partial b} &= \rho\Big(\frac{\partial^2 \overline{x}_V}{\partial a \partial b} u'_{P2}\big(\overline{x}_{V2}(-a,-b)\big) + \frac{\partial \overline{x}_V}{\partial a} \cdot \frac{\partial \overline{x}_V}{\partial b} u''_{P2}\big(\overline{x}_V(-a,-b)\big)\Big) \\
\frac{\partial^2}{\partial a \partial \rho} &= \frac{\partial \overline{x}_V}{\partial a} u'_{P2}\big(\overline{x}_P(-a,-b)\big) + u'_{P2}(-a) \\
\frac{\partial^2}{\partial b \partial \rho} &= \frac{\partial \overline{x}_V}{\partial b} u'_{P2}\big(\overline{x}_V(-a,-b)\big) + v'_2(-b)
\end{aligned}$$

We have $\frac{\partial^2 \overline{x}_{V2}}{\partial a \partial b} > 0$, $\frac{\partial \overline{x}_{V2}}{\partial a} < 0$, and $\frac{\partial \overline{x}_{V2}}{\partial b} > 0$, which yields, $\frac{\partial^2}{\partial a \partial b} > 0$ and $\frac{\partial^2}{\partial b \partial \rho} > 0$. Finally, $u'_{P2}(\overline{x}) < u'_{P2}(x) = u'_{P2}(-a)$ and $\frac{\partial \overline{x}_{V2}}{\partial a} \in (-1, 0)$, thus $\frac{\partial^2}{\partial a \partial \rho} > 0$. Then the usual results on monotone comparative statics (Milgrom and Shannon, 1994) deliver that y^*_d is monotonic in ρ, and thus $y^*_d = \hat{y}_1$ for at most one value of ρ.

Part 2. Consider (x,y) such that $u_{V2}(\hat{x}^P_2) \geq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}^V_2) \leq u_{P2}(x) + v_2(y)$. Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\hat{x}^P_2\big) + (1-\rho)\big(u_{P2}(x) + v_2(y)\big).$$

Thus, such a $(\hat{x}^P_d, \hat{y}^P_d)$ must solve:

$$u'_{P1}(x) + (1-\rho)u'_{P2}(x) = 0$$

$$v'_1(y) + (1-\rho)v'_2(y) = 0,$$

which clearly cannot be satisfied if $y = \hat{y}_1$ when $v_2'(\hat{y}_1) > v_1'(\hat{y}_1) = 0$ and $\rho \in (0,1)$.

Part 3. Consider (x, y) such that $u_{V2}(\hat{x}_2^P) \leq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}_2^V) \geq u_{P2}(x) + v_2(y)$. Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\overline{x}_V(x,y)\big) + (1-\rho) u_{P2}(\hat{x}_2^V).$$

Thus, such a $(\hat{x}_d^P, \hat{y}_d^P)$ needs to solve:

$$\begin{aligned} u_{P1}'(x) + \rho \frac{u_{V2}'(x)}{u_{V2}'\big(\overline{x}_V(x,y)\big)} u_{P2}'\big(\overline{x}_V(x,y)\big) &= 0 \\ v_1'(y) + \rho \frac{v_2'(y)}{u_{V2}'\big(\overline{x}_V(x,y)\big)} u_{P2}'\big(\overline{x}_V(x,y)\big) &= 0. \end{aligned}$$

Because $\frac{u_{P2}'\left(\overline{x}_V(x,y)\right)}{u_{V2}'\left(\overline{x}_V(x,y)\right)} < 0$ and $\rho \neq 0$, again it is clear that the second equality cannot be satisfied when $y = \hat{y}_1$ if $v_2'(\hat{y}_1) > v_1'(\hat{y}_1) = 0$.

Step 2. By the proof of step 1, if P's preferences are not infected then $(\hat{x}_d^P, \hat{y}_d^P)$ must be such that $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ and $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^V$. Thus, if P is not infected then $(\hat{x}_d^P, \hat{y}_d^P)$ solves:

$$\max_{x,y} \; u_{P1}(x) + v_1(y) + \rho u_{P2}(\hat{x}_2^P) + (1-\rho) u_{P2}(\hat{x}_2^V) \tag{39}$$

$$\text{s.t. } u_{V2}(\hat{x}_2^P) \geq u_{V2}(x) + v_2(y) \tag{40}$$

$$u_{P2}(\hat{x}_2^V) \geq u_{P2}(x) + v_2(y) \tag{41}$$

P's dynamic ideal point $(\hat{x}_d^P, \hat{x}_2^V)$ needs to solve the KKT conditions of this problem, which are given by:

$$u'_{P1}(x) - \lambda_1 u'_{V2}(x) - \lambda_2 u'_{P2}(x) = 0 \tag{42}$$

$$v'_1(y) - \lambda_1 v'_2(y) - \lambda_2 v'_2(y) = 0 \tag{43}$$

$$\lambda_1 \big[u_{V2}(\hat{x}_2^P) - u_{V2}(x) - v_2(y)\big] = 0 \tag{44}$$

$$\lambda_2 \big[u_{P2}(\hat{x}_2^V) - u_{P2}(x) - v_2(y)\big] = 0. \tag{45}$$

First, suppose λ_1 or $\lambda_2 \neq 0$ and $y = \hat{y}_1$. Then equation (43) reduces to $-(\lambda_1 + \lambda_2)v'_2(\hat{y}_1) < 0$. Thus, if P's preference are not infected it must be that neither constraint is binding.

Second, let $\lambda_1 = \lambda_2 = 0$. Then, $(\hat{x}_d^P, \hat{x}_2^V)$ must solve $u'_{P1}(x) = 0$ and $v'_1(y) = 0$ and thus $(\hat{x}_d^P, \hat{x}_2^V) = (\hat{x}_1^P, \hat{y}_1)$. However, by assumption, either: (i) $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$; or (ii) $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ holds this immediately yields that $(\hat{x}_1^P, \hat{y}_1)$ is not a feasible solution, specifically, if violates (40). If instead $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$ holds then note that $u_{P2}(\hat{x}_1^V) < u_{P2}(\hat{x}_1^P)$, which implies $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1) < u_{P2}(\hat{x}_1^P) + v_2(\hat{y}_1)$. Hence, $(\hat{x}_1, \hat{y}_1)$ is not feasible as it violates constraint (41). Analogous arguments yield that if $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$ then V's preferences must be infected as well. □

Proposition 6. *Suppose each player i's dynamic ideal point is such that $\overline{x}_V(\hat{x}_d^i, \hat{y}_d^i) < \hat{x}_2^P$ and $\underline{x}_P(\hat{x}_d^i, \hat{y}_d^i) > \hat{x}_2^V$. Then, $\hat{y}_d^P$ is decreasing in ρ and $\hat{y}_d^V$ is increasing in ρ.*

Proof. The result follows from the proof of Step 1 Part 1 of Proposition 5. □

A.4 Proofs for Long-run Outcomes Extension

We write player i's continuation payoff from strategy starting at time t as $w_t^i(x_t^q, y_t^q)$, suppressing dependence on the strategy profile. With this notation in hand, a strategy profile constitutes an equilibrium if for all status quo policies (x^q, y^q) and all t, the following conditions hold:

- for any proposal (x, y), V accepts if and only if

$$u_{Vt}(x) + v_t(y) + w_{t+1}^V\big(x, y\big) \geq u_{Vt}(x_t^q) + v_t(y_t^q) + w_{t+1}^V\big(x_t^q, y_t^q\big).$$

- P's proposal, $x_t^*(x^q, y^q)$, solves

$$\max_{(x,y)} \; u_{Pt}(x) + v_t(y) + w_{t+1}^P\big(x, y\big)$$
$$\text{s.t. } u_{Vt}(x) + v_t(y) + w_{t+1}^V\big(x, y\big) \geq u_{Vt}(x_t^q) + v_t(y_t^q) + w_{t+1}^V\big(x_t^q, y_t^q\big).$$

Proposition 7. *If $\eta < \frac{1}{2}$ then there exists $\hat{t} < T$ such that the equilibrium policy outcome is $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large $\hat{t} = 1$.*

Proof. First, we show that if t is sufficiently large then in equilibrium R proposes $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ whenever the status quo is such that $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) \geq u_{V_t}(x_t^q) + v_t(y_t^q)$ and V accepts the proposal.

To start, we establish that V's static payoff in period t from getting P's ideal point $(\hat{x}_t^P, \hat{y}_t)$ is greater than its payoff from getting P's ideal point from the previous period $t-1$ $(\hat{x}_{t-1}^P, \hat{y}_{t-1})$ whenever t is sufficiently large. More precisely, we claim that for t sufficiently large:

$$-\big(t^\eta + t^\eta\big)^2 > -\big((t-1)^\eta + t^\eta\big)^2 - \big(\gamma(t-1) - \gamma t\big)^2. \tag{46}$$

To see that (46) holds for t sufficiently large, note that when $\eta < \frac{1}{2}$:

$$\lim_{t \to \infty} -\big(t^\eta + t^\eta\big)^2 + \big((t-1)^\eta + t^\eta\big)^2 = 0,$$

whereas, $-\big(\gamma(t-1) - \gamma t\big)^2 = -\gamma^2 < 0$ for all t. Thus, under the proposed strategies, for t sufficiently large if V accepts P's static ideal point in period t then it is willing to accept P's static ideal point in period $t+1$.

Clearly, P has no incentive to deviate given the proposed strategies. Next, consider V. V is willing to accept the proposal if rejecting today leads to P making the same form of proposal tomorrow, since given the strategies V accepts which thus yields the same dynamic utility to V and by construction it is statically optimal to accept. Thus, we need to show that if t is sufficiently large and $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) \geq u_{V_t}(x_t^q) + v_t(y_t^q)$ then $u_{V_{t+1}}(\hat{x}_{t+1}^P) + v_{t+1}(\hat{y}_{t+1}) \geq u_{V_{t+1}}(x_t^q) + v_{t+1}(y_t^q)$.

Writing the condition as $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) - u_{V_t}(x_t^q) - v_t(y_t^q) \geq 0$ we have that the same condition will hold in the next period if the LHS of the condition is increasing in t. Differentiating yields

$$2\eta t^{n-1}(x^q + t^n) - 2\gamma(y^q - \gamma t) - 8\eta t^{2n-1}.$$

Rearranging this is positive if and only if

$$y^q < \frac{1}{\gamma}\eta t^{\eta-1}\Big[x^q - 3t^\eta\Big] + \gamma t \tag{47}$$

As $t \to \infty$ the RHS of (47) goes to infinity when $\eta < \frac{1}{2}$ and $\gamma > 0$. Thus, as $y^q \leq \gamma T$ for all t there exists a t' such that for all $t \geq t'$ the statement holds.

This implies that P's unconstrained optimal policy in any period $t \geq t'$ is $(\hat{x}_t^P, \hat{y}^t)$. Thus, if P does not get its unconstrained optimal in a period then P must be choosing (x, y) to make V indifferent between accepting and rejecting. Therefore, if there is an equilibrium such that for all t party P does not propose $(\hat{x}_t^P, \hat{y}_t)$ then V's dynamic payoff is given by:

$$\sum_{t=0}^{\infty} u_{V_t}(x_1^q) + v_t(y_1^q).$$

However, by $\eta < 1/2$ there is some period t'' such that starting in t'' the dynamic payoff to V from getting $(\hat{x}_t^P, \hat{y}_t)$ every period is greater than this dynamic payoff from the initial status quo, which contradicts that this is an equilibrium.

Finally, note that for γ sufficiently large (47) holds for $t = 1$. □

Proposition 8. *Assume $\eta > 1$. There exists $\tilde{t} < T$ such that $y_t^* \neq \hat{y}_t$ in every period $t \in \{\tilde{t}, ..., T-1\}$. If γ is sufficiently small then $\tilde{t} = 1$.*

Proof. We break the argument into several steps.

Step 1. In Step 1 we argue that if T is sufficiently large then at $T-1$ the policy outcome must be inefficient. If $\eta > \frac{1}{2}$ then $\lim_{T\to\infty} -(T^\eta + T^\eta)^2 + \big((T-1)^\eta + T^\eta\big)^2 = -\infty$, while $-\big(\gamma(T-1) - \gamma T\big)^2 = -\gamma^2$ for all T. Therefore, $u_{VT}(\hat{x}_T^P) < u_{VT}(\hat{x}_{T-1}^P) + v_T(\hat{y}_{T-1})$ and $y_T^* \neq \hat{y}_T$ by Propositions 2 and 3.

Step 2. Now, we characterize the continuation payoffs beginning in period $T-1$. Note that P's optimal proposal will be constrained in period T. Thus, if P is constrained in $T-1$, $u_{VT-1}(\hat{x}_d^P) + v_{T-1}(\hat{y}_d^P) + u_{VT}(\hat{x}_d^P) + v_T(\hat{y}_d^P) \leq u_{VT-1}(\hat{x}^q) + v_{T-1}(\hat{y}^q) + u_{VT}(\hat{x}^q) + v_T(\hat{y}^q)$ then $\frac{\partial w_{T-1}^V}{\partial y^q} = v'_{T-1}(y^q) + v'_T(y^q)$ and $\frac{\partial w_{T-1}^V}{\partial x^q} = u'_{VT-1}(x^q) + u'_{VT}(x^q)$. On the other hand, if P is unconstrained then both of these derivatives are 0. Furthermore, the envelope theorem delivers that $w_{T-1}^P(x^q, y^q)$ is differentiable almost everywhere in x^q and y^q, with

$$\frac{\partial w_{T-1}^P}{\partial y^q} = \begin{cases} -\lambda_{T-1}^* \frac{\partial w_{T-1}^V}{\partial y^q} & \text{if P is constrained,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\frac{\partial w_{T-1}^P}{\partial x^q} = \begin{cases} -\lambda_{T-1}^* \frac{\partial w_{T-1}^V}{\partial x^q} & \text{if P is constrained,} \\ 0 & \text{otherwise.} \end{cases}$$

Step 3. We now show that if the period t equilibrium policy is inefficient and continuation payoffs have the analogous properties to those characterized for the $T-1$ case in Step 2, then the equilibrium policy in period $t-1$ is inefficient and continuation payoffs have the same form as in $T-1$. The induction argument together with Steps 1 and 2 as the base case then delivers the proposition.

Suppose $y_t^* \neq \hat{y}_t$ and the derivatives for each player's continuation payoffs $w_t^V(x_t^q, y_t^q)$ and $w_t^P(x_t^q, y_t^q)$ have the same form as at $T-1$. In period $t-1$ P's optimal proposal solves:

$$\max_{(x,y)} \; u_{Pt-1}(x) + v_{t-1}(y) + w_t^P(x,y)$$
$$\text{s.t. } u_{Vt-1}(x) + v_{t-1}(y) + w_t^V(x,y) \geq u_{Vt-1}(x_{t-1}^q) + v_{t-1}(y_{t-1}^q) + w_t^V(x_{t-1}^q, y_{t-1}^q).$$

We break the argument into two parts, depending on whether the veto player's constraint is binding at time $t-1$.

If P chooses a policy where the constraint does not bind then (x_{t-1}^*, y_{t-1}^*) solves:

$$u'_{Pt-1}(x) + \frac{\partial w_t^P}{\partial x} = 0 \tag{48}$$
$$v'_{t-1}(y) + \frac{\partial w_t^P}{\partial y} = 0. \tag{49}$$

Thus, for $\hat{y}_{t-1}$ to be optimal requires that $\frac{\partial w_t^P}{\partial y}|_{y=\hat{y}_{t-1}} = 0$ which implies that P is also unconstrained at time t. If P is unconstrained at time t then $\frac{\partial w_t^P}{\partial x} = 0$, and for (48) to also hold requires $x_{t-1}^* = \hat{x}_{t-1}^P$. However, this contradicts that P is unconstrained at period t, as P's unconstrained optimal policy at time t sets $x_t^* > \hat{x}_t^P > \hat{x}^P$ and $y_t^* > \hat{y}_t > \hat{y}_{t-1}$. Thus, at time t accepting such a (x_t^*, y_t^*) is strictly worse for V than rejecting to keep $(\hat{x}_t^P, \hat{y}_{t-1})$.

Next, suppose that P chooses a policy when the veto constraint is binding. Then $(x_{t-1}^*, y_{t-1}^*, \lambda_{t-1}^*)$ solves:

$$u'_{Pt-1}(x) + \frac{\partial w_t^P}{\partial x} + \lambda\Big[u'_{Vt-1}(x) + \frac{\partial w_t^V}{\partial x}\Big] = 0 \tag{50}$$
$$v'_{t-1}(y) + \frac{\partial w_t^P}{\partial y} + \lambda\Big[v'_{t-1}(y) + \frac{\partial w_t^V}{\partial y}\Big] = 0 \tag{51}$$
$$\lambda\Big[u_{Vt-1}(x) + v_{t-1}(y) + w_t^V(x,y) - u_{Vt-1}(x_{t-1}^q) - v_{t-1}(y_{t-1}^q) - w_t^V(x_{t-1}^q, y_{t-1}^q)\Big] = 0 \tag{52}$$

Since the constraint is binding we have $\lambda > 0$. Solving for λ and rearranging, for (50)-(52) to hold at $y = \hat{y}_{t-1}$ requires that x^*_{t-1} satisfies:

$$\frac{\partial w^V_t}{\partial y}\Big|_{y=\hat{y}_{t-1}} \left(u'_{Pt-1}(x) + \frac{\partial w^P_t}{\partial x}\right) - \frac{\partial w^P_t}{\partial y}\Big|_{y=\hat{y}_{t-1}} \left(u'_{Vt-1}(x) + \frac{\partial w^V_t}{\partial x}\right) = 0 \tag{53}$$

$$u_{Vt-1}(x) + w^V_t(x, \hat{y}_{t-1}) = u_{Vt-1}(x^q_{t-1}) + v_{t-1}(y^q_{t-1}) + w^V_t(x^q_{t-1}, y^q_{t-1}). \tag{54}$$

The LHS of (54) is decreasing in x, thus there is at most one x which solves it. Additionally, because u is quadratic, the LHS of (53) is linear in x. However, the first equality does not depend on the status quo, and thus any perturbation of the initial status quo only changes the solution to (54) and there cannot be an x^*_{t-1} that satisfies (53) and (54) for almost all (x^q_1, y^q_1).

Finally, note that if the constraint is not binding, $u_{Vt-1}(x^*_{t-1})+v_{t-1}(y^*_{t-1})+w^V_t(x^*_{t-1}, y^*_{t-1}) > u_{Vt-1}(x^q_{t-1}) + v_{t-1}(y^q_{t-1}) + w^V_t(x^q_{t-1}, y^q_{t-1})$, then $\frac{\partial w^V_{t-1}}{\partial y^q_{t-1}} = \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}} = 0$ and the envelope theorem delivers that also $\frac{\partial w^P_{t-1}}{\partial y^q_{t-1}} = \frac{\partial w^P_{t-1}}{\partial x^q_{t-1}} = 0$. Instead, if the constraint is binding then $w_{t-1}(x^q_{t-1}, y^q_{t-1}) = u_{Vt-1}(x^q_{t-1}) + v_{t-1}(y^q_{t-1}) + w^V_t(x^q_{t-1}, y^q_{t-1})$. Therefore, $\frac{\partial w^V_{t-1}}{\partial y^q_{t-1}} = v'_{t-1}(y^q_{t-1}) + \frac{\partial w^V_t}{\partial y^q_{t-1}}$ and $\frac{\partial w^V_{t-1}}{\partial x^q_{t-1}} = u'_{Vt-1}(y^q_{t-1}) + \frac{\partial w^V_t}{\partial x^q_{t-1}}$. Again the envelope theorem yields $\frac{\partial w^P_{t-1}}{\partial x^y_{t-1}} = -\lambda^* \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}}$ and $\frac{\partial w^P_{t-1}}{\partial x^q_{t-1}} = -\lambda^* \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}}$. Thus, the derivatives of the continuation payoffs at $t-1$ have the desired form as well.

□

A.5 Different Weights on Dimensions

Here we consider the baseline model where $v(y) \equiv v_1(y) = v_2(y)$, so there is no change in preferences on the Y dimension, but we assume that the two players put different weights on the two dimensions. Specifically, assume that the proposer's stage utility is

$$u_{Pt}(x) + \theta v(y),$$

with $\theta > 0$. As in the baseline, the veto player's stage utility is instead

$$u_{Vt}(x) + v(y).$$

Further, assume that $\hat{y}_1 = \hat{y}_2 = \hat{y}$, so that the optimal policy on the Y dimension remains constant across periods.

Here, we show that a policy pair such that $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}$ can never be sustained in equilibrium. Clearly, if U_q is such that the proposer is unconstrained in the first period, the equilibrium policy must satisfy $x_1^* > \hat{x}_1^P$.

Suppose instead that U_q is sufficiently high that the proposer is constrained in the first period. To establish a contradiction, suppose that $x_1^* < \hat{x}_1^P$, $y_1^* < \hat{y}$ is an equilibrium. Recall that $\bar{x}(x_1^*, y_1^*)$ solves $u_{V2}(x) = u_{V2}(x_1^*) + v_2(y_1^*)$. Further, (x_1^*, y^*) must solve $u_{V1}(x_1^*) + v(y_1^*) + u_{V2}(x_1^*) + v(y_1^*) = U_q$. Let $\tilde{x}$ denote the policy that solves $u_{V1}(x) + u_{V2}(x) = U_q$. Suppose that $u_{V2}(\tilde{x}) \leq u_{V2}(x_1^*) + v(y_1^*)$. Then, it must be the case that a deviation to $(\tilde{x}, \hat{y})$ is profitable for the proposer, as this bundle is passable by definition and improves the proposer's payoff if this inequality is satisfied. Suppose instead $u_{V2}(\tilde{x}) > u_{V2}(x_1^*) + v(y_1^*)$. Then, the above equations imply that $u_{V1}(\tilde{x}) < u_{V1}(x_1^*) + v(y_1^*)$, otherwise $u_{V2}(\tilde{x}) > u_{V2}(x_1^*) + v(y_1^*)$ and $u_{V1}(\tilde{x}) + u_{V2}(\tilde{x}) = U_q$ would imply $u_{V1}(x_1^*) + v(y_1^*) + u_{V2}(x_1^*) + v(y_1^*) > U_q$. Therefore, given concavity and the assumption that $\hat{x}_{V2} \leq \hat{x}_{V1}$, the following holds: $-v(y_1^*) < u_{V1}(x_1^*) - u_{V1}(\tilde{x}) < u_{V2}(x_1^*) - u_{V2}(\tilde{x})$. Thus, $u_{V2}(\tilde{x}) < u_{V2}(x_1^*) + v(y_1^*)$, a contradiction.

Finally, note that, as in Acharya and Ortner (2013) and Lee (2020), the proposer may still want to implement an inefficient policy on the common-values dimension in this setting, $\hat{y}_d^P < \hat{y}_1$. To see this, consider the first-order conditions that $(\hat{x}_d^P, \hat{y}_d^P)$ must solve, assuming $\overline{x}(\hat{x}_1^P, \hat{y}_1) < \hat{x}_2^P$ so that P cannot just achieve its ideal point both periods:

$$u'_{P1}(x) + \frac{u'_{P2}\big(\overline{x}(x,y)\big)}{u'_{V2}\big(\overline{x}(x,y)\big)} u'_{V2}(x) = 0 \tag{55}$$

$$\theta v'(y) + \frac{u'_{P2}\big(\overline{x}(x,y)\big)}{u'_{V2}\big(\overline{x}(x,y)\big)} v'(y) = 0 \tag{56}$$

There is always a solution to these necessary first-order conditions where $y = \hat{y}$ and x solves $u'_{P1}(x) + u'_{P2}(x)$. Indeed, if θ is sufficiently large this solution does maximize P's payoff, as maintaining inefficiency for tomorrow is costly. However, when θ is sufficiently small P weights the costs of inefficiency less than V and $y < \hat{y}$ can instead be better. In this case, from condition (56) we require that $\theta = -\frac{u'_{P2}\big(\overline{x}(x,y)\big)}{u'_{V2}\big(\overline{x}(x,y)\big)}$. When θ is small this implies that x and y are such that $\overline{x}(x,y)$ is close to $\hat{x}_2^P$. Additionally, this condition together with (55) implies that the policy on the X dimension solves $u'_{P1}(x) - \theta u'_{v2}(x) = 0$, and hence x is close to $\hat{x}_1^P$ for θ small. Thus, for θ sufficiently low the inefficient solution yields a better first-period policy on X, a better second-period policy on X, and the cost from the inefficiency is relatively negligible.

For example, suppose $u_{it}(x) = -(x - \hat{x})^2$, with $\hat{x}_1^P = -\hat{x}_1^V = 1$ and $\hat{x}_2^P = -\hat{x}_2^v = 2$. Additionally, let $v(y) = -(y-1)^2$ and $\theta = \frac{1}{8}$. Then $(\hat{x}_d^P, \hat{y}_d^P) \approx (1.43, .058)$ which gives a dynamic payoff of $\approx -.49$, versus the best efficient policy $(x, y) = (1.5, 1)$ which yields $-.5$.